Enabling Cyber Security, Situational Awareness and Resilience in Distribution Grids with High Penetration of Photovoltaics (CARE-PV)

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Technical Objectives

Situational Awareness at the grid edge with high PV penetration

Cyber Intrusions

Assuring energy services at critical locations

Address fundamental challenges in integrating cyber-security mechanisms with state estimation strategies and leveraging this situational awareness to assure energy services at strategic locations
Project Objectives

- Design self-learning grid-interactive inverter technology along with a moving target defense framework
- Develop centralized and decentralized state estimation strategies
- Preemptive voltage monitoring and proactive control to assure service at strategic locations
Technology Innovations (TI)

• **TI 1: Cyber Defense Architecture / Multilevel Security**

• **TI 2: Situational Awareness / Novel State Estimation Strategies**

• **TI 3: Assuring Voltage Services / Probabilistic Voltage Sensitivity Analysis**
Technology Innovations (TI)

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Two-level Cyber-Intrusion Detection Architecture

• Design principles
  • Device level: Mechanisms to identify safe regions of operations and malicious set points
  • System level: Unique moving target detection (MTD) framework that incorporates inverter limits and system losses.
• Learning and performance assessment via series of simulations and experiments
Device level Security

- Development of self-learning inverter that adaptively learns the reduced order model parameters and identifies malicious control signals (based on local data, external weather related data, historical trends and neighborhood inverter data)

The learning algorithm will apply past and current data to form the aggregate model with adjustable parameters, and to identify the normal region.

In response to discrepancies between expected (estimated) and local & communicated data, the smart inverter blocks incoming commands until the source of the mismatch is identified.
Device level Security

- Identification of stable/safe operation regions of individual and cluster of PV inverters
- Quantify PQ set points effects on local point of common coupling voltage
- Set margins for system level MTD

PQ set-points effect on the voltage of a network of inverters
System Level MTD

• Detect FDI attacks at any level on the grid*
• Proactively change setpoints of PV inverters and possibly line impedances
• Develop MTD-based full AC OPF model incorporating inverter constraints
• Develop and test various MTD strategies under different attack models
• Multiple criteria for MTD implementation
  • Stealthiness to strong adversary
  • Economic and reliability considerations

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Observability of Distribution Grid

Currently few measurements are available

The system becomes unobservable

Increasing sensors and smart meters

Imposing stress on communication network/data processing

**Theorem 1:** If there exists a vertex-disjoint set of paths connecting every node in $\mathcal{O}$ with a node in $\mathcal{M}$, the Jacobian matrix associated with the coupled power flow equations is generically invertible.

$\mathcal{M}$: set of metered buses (real power $P$, reactive power $Q$ and voltage magnitude $|V|^2$)

$\mathcal{O}$: set of non-metered buses

**Question:** How to estimate the states using a small number of measurements?
Correlation and Sparse-aware Approaches

Approaches exploit underlying sparsity in data/signals

- Geographical proximity of nodes
- Slow load and PV variations

Spatial Correlation in PV generation

- Static
  - 1D Compressive Sensing
  - Matrix Completion

- Dynamic
  - 2D Compressive Sensing
  - Tensor Completion

Temporal Correlation

- Sparsity

Spatial:
- 1D Compressive Sensing
- Matrix Completion

Spatiotemporal:
- 2D Compressive Sensing
- Tensor Completion

Centralized

Decentralized
**Compressive Sensing**

IEEE 37-node system (unbalanced three-phase network) with 30% PV penetration

- Robustness to bad data, system uncertainty and topology errors

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Compressive Sensing

**Compression Measurement Ratio (CMR)** = \( \frac{\text{Available measurements}}{\text{Maximum number of measurements}} \)

**Integrated Normalized Absolute Error (INAE)** = \( \frac{\sum_{j=1}^{N} |\text{True value}_j - \text{Estimated value}_j|}{\sum_{j=1}^{N} |\text{True value}_j|} \)


Technology Innovations (TI)

- **TI 1: Cyber Defense Architecture / Multilevel Security**

- **TI 2: Situational Awareness / Novel State Estimation Strategies**

- **TI 3: Assuring Voltage Services / Probabilistic Voltage Sensitivity Analysis**
Assuring Voltage Services

How does the stochastic variability of load and generation, cyber attacks or natural outages impact energy services at strategic locations that represent critical infrastructure?

How can we use PV systems to react quickly to assure service at these critical locations?

**Understand the impact of fluctuations and failures occurring in one grid location on another location**

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PVSA*

Dominant Influencer of Voltage Fluctuation (DIVF) *

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- PVSA method predicts that a voltage violation is imminent
- Knowledge of the DIVF will provide a quick way to return the system back into an acceptable state of operation

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Approach

**WHY**

1. PV integration/cyber attacks/natural disasters increase power/voltage variability which affects performance

2. Existing methods of impact assessment are scenario-based

3. Challenges
   1. Uncertainty
   2. Complexity

**HOW**

1. Compute analytical approximations for voltage sensitivity assuming no uncertainty

2. Introduce spatiotemporal uncertainty to quantify voltage sensitivity in a probabilistic framework

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Stochastic Modeling of Distribution Grid

Nodes with PV panels: Actor node (varying power injection)

O: Observation node (monitoring voltage change)
Probabilistic Voltage Sensitivity

**Theorem 1**: For three phase radial distribution system, change in voltage at observation node (\( \Delta V_o \)) due to change in power consumption of an actor node (\( \Delta S_{\theta} \)) is approximated by

\[
\Delta V_o \approx \left[ \frac{\Delta S_{\alpha a}}{V_{\alpha a}} + \frac{\Delta S_{\alpha b}}{V_{\alpha b}} + \frac{\Delta S_{\alpha c}}{V_{\alpha c}} \right] + \left[ \frac{\Delta S_{\beta a}}{V_{\beta a}} + \frac{\Delta S_{\beta b}}{V_{\beta b}} + \frac{\Delta S_{\beta c}}{V_{\beta c}} \right] + \left[ \frac{\Delta S_{\gamma a}}{V_{\gamma a}} + \frac{\Delta S_{\gamma b}}{V_{\gamma b}} + \frac{\Delta S_{\gamma c}}{V_{\gamma c}} \right]
\]

Where \( V_\alpha^* \) is complex conjugate of voltage at actor node;

Subscript: a, b, c represents the three phases;

\( Z \) denotes the self and mutual impedance of shared line between observation node and actor node from source node

**Corollary**: The error in approximating voltage change as stated in Theorem 1, is upper bounded by the following expression:

\[
|\Delta V_o|_{\text{error}} = |\Delta V_o|_{\text{actual}} - |\Delta V_o|_{\text{theor}}
\]

\[
|\Delta V_o|_{\text{error}} \leq \left| \frac{K_1}{V_r} \left( \frac{1}{T_r} - \frac{1}{1 + \Theta_1} \right) \right| + \left| \frac{K_2}{V_i} \left( \frac{1}{T_i} - \frac{1}{1 + \Theta_2} \right) \right|
\]

where \( K_1 = \Delta P R - \Delta Q X ; K_2 = \Delta P X + \Delta Q R ; \Theta_1 = \left( \frac{V}{T} \right)^2 ; \Theta_2 = \frac{1}{C_1} \Delta P \) and \( \Delta Q \) are the change in active and reactive power of actor node; \( R \) and \( X \) are the resistance and reactance of the shared path; \( V^r \) and \( V^i \) are the real and imaginary components of the voltage at actor node.

**Theorem 2**: For three phase radial distribution system, the probability distribution of voltage change at observation node (|\( \Delta V_o \)|) due to random change in power consumption of actor nodes is given by:

\[
|\Delta V_o| = \text{Nakagami}(m, \omega)
\]

\[
\text{shape parameter } m = \frac{\Theta_1 + \Theta_2}{\Theta}, \quad \text{scale parameter } \omega = \sqrt{\Theta_1} + \sqrt{\Theta_2}
\]

\[
\Theta = \frac{2(\Theta_1 + \Theta_2 + 2\Theta_2 \Theta_1 \Theta_2)}{\Theta_1 \Theta_2}, \quad \Theta_1 = C_{\alpha}^2 \Sigma_{\alpha}, \quad \Theta_2 = C_{\gamma}^2 \Sigma_{\gamma}; \quad c = \cos(\Delta V, \Delta V')
\]

Results

Voltage change at node 9, with multiple actor nodes (22, 17, 14, 8, 7) from different phases

- Deterministic case
- Stochastic case

**Deterministic case**

- Phase - a
- Phase - c

**Stochastic case**

- Voltage change distribution

- Complexity of the proposed analytical method is $O(1)$, compared to $O(n^3)$ in classical method.

- Extension to spatio temporal uncertainty case
- DIVF and control strategies to restore services
Testing and Validation – Small Scale Testbed

Primary System → Lateral Feeder → Secondary System

559 nodes in total
Define communication and control infrastructure for automation sensors, smart inverters, smart meters and other sensors.
Testing and Validation – Large Scale Testbed

- Adopt the model to run on NREL’s co-simulation platform
  - Data from Midwest energy to guide distribution system simulation
  - Altering communication behavior to simulate cyber disruptions
  - Craft a series of data attacks (spoofed, corrupted, or missing data)
<table>
<thead>
<tr>
<th>Organization</th>
<th>Expert</th>
<th>Expertise</th>
</tr>
</thead>
<tbody>
<tr>
<td>KSU</td>
<td>Bala Natarajan</td>
<td>State estimation, stochastic modeling, CPS resilience and security</td>
</tr>
<tr>
<td></td>
<td>Mohammad Shadmand</td>
<td>Autonomous PV systems</td>
</tr>
<tr>
<td></td>
<td>Behrooz Mirafzal</td>
<td>Power inverters</td>
</tr>
<tr>
<td></td>
<td>Hongyu Wu</td>
<td>Smart grid cyber security and optimization</td>
</tr>
<tr>
<td></td>
<td>Anil Pahwa</td>
<td>Power Distribution Systems</td>
</tr>
<tr>
<td>Typhoon HIL</td>
<td>Ivan Cevanovic</td>
<td>HIL Testbed and Tech Transfer</td>
</tr>
<tr>
<td>NREL</td>
<td>Yingchen Zhang</td>
<td>Cyber security, state estimation, DMS</td>
</tr>
<tr>
<td></td>
<td>Rui Yang</td>
<td>State estimation, data analytics, optimization</td>
</tr>
<tr>
<td></td>
<td>Govind Saraswat</td>
<td>Hardware-in-the-loop demonstration</td>
</tr>
<tr>
<td>Oracle Utilities</td>
<td>Bradley Williams</td>
<td>DMS implementation and Tech Transfer</td>
</tr>
<tr>
<td>Midwest Energy</td>
<td>Bill Dowling</td>
<td>Commercialization and Tech Transfer</td>
</tr>
<tr>
<td>Enphase Inc</td>
<td>Raghu Belur</td>
<td>Commercialization of smart Inverter</td>
</tr>
</tbody>
</table>
Summary

Challenges
- Cyber intrusions
- Lack of observability and situational awareness
- Assuring energy services for critical infrastructure at strategic locations

Cyber Physical System
- Cyber infrastructure sensors, AMI and communication network

Novel Solutions
- Cyber defense architecture
  - Device-level security: smart self-learning inverter
  - System-level security: MTD
- State estimation integrating cyber effects
- Preemptive voltage monitoring via PVSA
- Proactive control using dominant influence set

Distribution grid
Questions?
## Milestones for BP1

<table>
<thead>
<tr>
<th>SOPO/TWP Milestone #</th>
<th>Milestone Description</th>
<th>Performer</th>
<th>Original Planned</th>
<th>Revised Planned</th>
<th>Actual</th>
<th>Percent Complete</th>
<th>Progress Notes</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Form the IAB and expand the industrial partnership to include 10 utility and vendor companies, along with Sandia National Laboratory. Successful completion of IAB kick-off meeting.</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>Forming IAB and adding at least 3 additional members</td>
<td>Dr. Bala Natarajan, Dr. Anil Pahwa, Dr. Kexing Lai</td>
<td>11/8/19</td>
<td>10/21/19</td>
<td>10/21/19</td>
<td>100%</td>
<td></td>
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</tr>
<tr>
<td>3.1</td>
<td>Defense stealthiness probability (DSP) &gt; 85%</td>
<td>Dr. Hongyu Wu</td>
<td>04/30/20</td>
<td>04/30/20</td>
<td>04/30/20</td>
<td>40%</td>
<td>Finish the preliminary tests related to MTD-based OPF for the single-phase system.</td>
<td>3</td>
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<tr>
<td>3.2</td>
<td>Attack detection probability (ADP) &gt; 70%</td>
<td>Dr. Hongyu Wu</td>
<td>10/31/20</td>
<td>10/31/20</td>
<td>10/31/20</td>
<td>15%</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>3.3</td>
<td>Dynamic model of three-phase feeder &gt; 85% accuracy of dynamic models (less than 15% model mismatch)</td>
<td>Dr. Behrooz Mirazfzal, Dr. Mohammad Shadmand</td>
<td>4/30/20</td>
<td>4/30/20</td>
<td>4/30/20</td>
<td>30%</td>
<td>Towards developing dynamic model of smart PV inverter and feeder, and towards enabling the identification of stable/safe operation regions.</td>
<td>3</td>
</tr>
<tr>
<td>3.4</td>
<td>Stable/reliable operation region &gt; 95% accurate safe operation set-points</td>
<td>Dr. Behrooz Mirazfzal, Dr. Mohammad Shadmand</td>
<td>07/31/20</td>
<td>07/31/20</td>
<td>07/31/20</td>
<td>25%</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>3.5</td>
<td>Full capability to be programmed based on external active and reactive power set-points for fully functional for the rated P and Q set-points</td>
<td>Dr. Behrooz Mirazfzal, Dr. Mohammad Shadmand</td>
<td>10/31/20</td>
<td>10/31/20</td>
<td>10/31/20</td>
<td>25%</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>3.6</td>
<td>Full capability to be programmed for operation within stable/safe operation region (e.g. ANSI c84.1; IEEE 1547-2018) and abnormalities detection for fully functional real-time simulation testbed of an IEEE 33 bus system constrained by the grid-codes</td>
<td>Dr. Behrooz Mirazfzal, Dr. Mohammad Shadmand</td>
<td>10/31/20</td>
<td>10/31/20</td>
<td>10/31/20</td>
<td>45%</td>
<td></td>
<td>3</td>
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## Milestones for BP1

<table>
<thead>
<tr>
<th>Milestone Schedule</th>
<th>Milestone Completion Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SOPO/TWP Milestone #</strong></td>
<td><strong>Milestone Description</strong></td>
</tr>
<tr>
<td>4.1</td>
<td>Average Integrated Normalized Absolute Error (INAE) &lt;20% with 75% of the data.</td>
</tr>
<tr>
<td>4.2</td>
<td>Average INAE of robust estimator at least 10% better than classic weighted least squares (WLS) estimator with same number of measurements.</td>
</tr>
<tr>
<td>5</td>
<td>Voltage violation prediction error &lt;10%.</td>
</tr>
<tr>
<td>6</td>
<td>DMS procured and installed for operational system</td>
</tr>
<tr>
<td>GNG-1A</td>
<td>Successful completion of the BP1 milestones</td>
</tr>
</tbody>
</table>
## Technical Achievements (Task 3.2)

### Enabling the Identification of Stable/Safe Operation Regions

<table>
<thead>
<tr>
<th>#</th>
<th>Month of completion</th>
<th>Performance Metric</th>
<th>Success Value</th>
<th>Assessment Tool / Method of Measuring Success Value</th>
<th>Verification Process</th>
<th>Metric Justification, Additional Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4</td>
<td>12</td>
<td>Stable/reliable operation region</td>
<td>&gt; 95% accurate safe operation set-points</td>
<td>Real-time simulation and stability analysis of the three-phase feeder by adjusting the set points of the smart inverters and comparing the results with the developed dynamic</td>
<td>Real-time simulations via HIL tests</td>
<td>The success value was chosen based on the allowable margin between boundary of stable/reliable operation region</td>
</tr>
<tr>
<td>3.6</td>
<td>15</td>
<td>Full capability to be programmed for operation within stable/safe operation region (e.g. ANSI c84.1; IEEE 1547-2018) and abnormalities detection</td>
<td>Fully functional real-time simulation testbed of an IEEE 33 bus system constrained by the grid-codes</td>
<td>Real-time control hardware in the loop (CHIL) simulator will be used. Several case studies will be simulated to verify the operation of the IEEE 33 bus system based on the grid codes.</td>
<td>Real-time simulations</td>
<td>The success value was chosen to have a realistic distributed high PV penetrated grid.</td>
</tr>
</tbody>
</table>
IEEE 33-node Network

Development of real-time simulation

- IEEE 33-bus radial network has been implemented in Typhoon-HIL (see Fig. 1).

- The developed SCADA system is shown in Fig. 2 and Fig. 3 shows an example of each node measurements.

Fig. 1: IEEE 33-Bus network

Fig. 2: Developed Typhoon HIL SCADA system for IEEE 33-Bus network

Fig. 3: Example of each node measurements
IEEE 33-node Network

Real-time simulation validation

- Typhoon HIL results are compared with results of 33-bus system implemented in EPRI’s Open DSS solver
- The maximum voltage and phase errors found are 1.67% and 0.33° as shown in Fig. 4 and Fig. 5 respectively

Fig. 4: Voltage magnitude data obtained from simulation of IEEE 33-bus system in Typhoon and Open DSS environments

Fig. 5: Voltage phase data obtained from simulation of IEEE 33-bus system in Typhoon and Open DSS environments
Future Work

• The IEEE 33-node real-time simulation testbed developed in Typhoon HIL will be extended to IEEE 37-node system.

• The PV inverters will be added to the 37-node system in Typhoon HIL.

• The realized stable/safe operation region of single inverter will be extended to a cluster of inverters in the distribution network.

• Extend the theory developed regarding the collapse of the point of common coupling voltage for the ith distributed generator with inclusion of the network topology.

• The safe operation and unstable operation of inverters will be distinguished when PQ set-points are manipulated according to the IEEE 1547-2018 standard.
## Technical Achievements (Task 3.2)

### Developing Dynamic Model of Smart PV Inverter and Feeder

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</tr>
</thead>
<tbody>
<tr>
<td>3.3</td>
<td>9</td>
<td>Dynamic model of three-phase feeder</td>
<td>&gt; 85% accuracy of dynamic models (less than 15% model mismatch)</td>
<td>Comparison of dynamic model with simulated models. Analyzing the collected data from the developed dynamic models and simulated models.</td>
<td>Real-time simulations via HIL tests</td>
<td>The success value was chosen based on expected allowable uncertainty in the dynamic models that will not affect significantly the abnormalities detection process considering IEEE 1547 standard.</td>
</tr>
<tr>
<td>3.5</td>
<td>15</td>
<td>Full capability to be programmed based on external active and reactive power set-points</td>
<td>Fully functional for the rated P and Q set-points</td>
<td>Programmable load, grid and PV emulators will be used to verify the functionality of the inverter under grid-tied and standalone modes of operation as well as low voltage ride through test scenarios.</td>
<td>Hardware tests</td>
<td>The success value was chosen based on the flexibility to be used for testing the technology innovations in the project.</td>
</tr>
</tbody>
</table>
• **Added Features to the Self-Learning Inverter:**
  
  • Increased stability margin under weak grid condition using the virtual inductance strategy *(publish in IEEE Transactions on Industrial Electronics)*
  
  • The virtual inductance strategy has been made *adaptive* (work currently *review* for a IEEE Transaction paper)
  
  • A fourth-order state-space model for the self-learning PV inverter has been developed:
    • The developed model is intended to serve as a reference model to evaluate the veracity of the incoming external data in comparison with the local measurements
Direct Phase-Angle Detection

Towards the development of Self-Learning Inverters

- **Added Features to the Self-Learning Inverter:**
  - *Accurate* phase-angle detection is required for grid synchronization and control of grid-tied *self-learning inverters*
  - Classic PLLs require parameter tuning of their internal PI loops and low-pass filters
  - The proposed phase-angle detection *does not require* any parameter tuning
  - A signal reformation algorithm has been developed to accurately detect the phase-angle under *asymmetrical power grids*
Small Scale Test Bed Setup

Hardware Design and Testing of Smart Inverters

• First prototype tested and used to:
  • Tune gate driver circuit components
  • Tune snubber circuit components
  • 37% reduction in signal overshoot

• Based on the tests performed with the first-generation board, a second-generation board has been developed, manufactured, and populated
## Technical Achievements (Task 3.1)

### Developing Moving Target Defense (MTD)-based Optimal Power Flow (OPF)

<table>
<thead>
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<th>Verification Process</th>
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</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>9</td>
<td>Defense stealthiness probability (DSP)</td>
<td>&gt; 85%</td>
<td>At least two cases under each adversary model will be created based on different measurement types and locations in an IEEE 33-bus distribution feeder. In each case, 1000 scenarios will be created with a standard deviation of 1%-3% in Gaussian distributed measurement noise and uncertainty.</td>
<td>Raw test data and detailed report sent to DOE for verification</td>
<td>The success value was chosen based on preliminary results: DSP lower than 85% makes HMTD strategies not superior to the random MTD</td>
</tr>
<tr>
<td>3.2</td>
<td>15</td>
<td>Attack detection probability (ADP)</td>
<td>&gt; 70%</td>
<td>At least two cases under each adversary model will be created based on different locations and boundary conditions of smart PV inverters in an IEEE 33-bus distribution feeder. In each case, 1000 scenarios will be created with a standard deviation of 1%-3% in Gaussian distributed measurement noise.</td>
<td>Report sent to DOE with description of system setup, boundary condition, and attack models</td>
<td>The success value was chosen based on ideal boundary condition of smart PV inverters that can provide desired MTD magnitude</td>
</tr>
</tbody>
</table>
MTD-ACOPF

MTD-ACOPF optimally determines system voltage \( (V, \theta) \), nodal real (P) and reactive power (Q), and the reactance of branches \( x \) that are equipped with D-FACTS devices to minimize the system operating cost and losses

\[
\min_X \quad \lambda_1 L^v(X) + \lambda_2 \sum_{i=1}^{n_g} f^i(p^i_g)
\]

\[
s.t. \quad L^v(X) = \sum_{i=1}^{n_g} S_i^v + S_i^f
\]

\[
g_P(\theta, V, P_g, x) = 0
\]

\[
g_Q(\theta, V, Q_g, x) = 0
\]

\[
h_j(\theta, V, x) \leq 0
\]

\[
h_i(\theta, V, x) \leq 0
\]

\[
\theta_{\text{ref}} \leq \theta \leq \theta_{\text{ref}}
\]

\[
v^\text{min}_i \leq v_i \leq v^\text{max}_i, \quad i = 1, \ldots, n_b
\]

\[
p^\text{min}_i \leq p_i \leq p^\text{max}_i, \quad i = 1, \ldots, n_g
\]

\[
q^\text{min}_i \leq q_i \leq q^\text{max}_i, \quad i = 1, \ldots, n_g
\]

\[
|x_i - x^0_i| \leq \eta x^0_i, \quad i = 1, \ldots, n_{DF}
\]

where \( X = [ \theta \ V \ P_g \ Q_g \ x ] \) are decision variables corresponding to voltage angle, voltage magnitude, generator active generation, generator reactive generation, and the reactance of D-FACTS lines, respectively.
Case studies in the 559-node distribution system

- The reactance of the 36 main feeders and remaining lateral feeders are dispatched to minimize the system losses.

![Fig.1 Dispatched reactance of each branches](image)
## Technical Achievements (Task 4)

### Developing 1-D and 2-D Compressive Sensing Based State Estimation

<table>
<thead>
<tr>
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<th>Month of completion</th>
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</thead>
<tbody>
<tr>
<td>4.1</td>
<td>12</td>
<td>Average Integrated Normalized Absolute Error (INAE)</td>
<td>&lt;20% with 75% of the data</td>
<td>We will assess our centralized static state estimation strategy by running at least 50 distinct test system realizations and average the INAE by comparing estimated and true state values.</td>
<td>Simulation data and Report sent to DOE for verification.</td>
<td>The success value was chosen based on expected loss in measurements in practical systems and the acceptable levels of uncertainty shown in prior literature.</td>
</tr>
<tr>
<td>4.2</td>
<td>15</td>
<td>Average Integrated Normalized Absolute Error (INAE) comparison in the presence of 10% bad data</td>
<td>Average INAE of robust estimator at least 10% better than classic weighted least squares (WLS) estimator with same number of measurements</td>
<td>We will assess our robust static state estimation strategy by running at least 50 distinct test system realizations each with bad data ranging from 1%-10% and compare average INAE between classic WLS and proposed estimator.</td>
<td>Simulation data and Report sent to DOE for verification.</td>
<td>The success value was chosen based on minimum levels of robustness that is suggested in classic estimation literature.</td>
</tr>
</tbody>
</table>
Static 1D-Compressive Sensing

Estimate the signal of interest $\mathbf{x} \in \mathbb{R}^N$ from compressed measurements $\mathbf{y} \in \mathbb{R}^M$, where $M < N$.[1]

$s$ : Sparse signal
$\mathbf{y}$ : Compressed measurement
$\varphi$ : measurement matrix (e.g., Bernoulli entries)
$\psi$ : sparsifying matrix (e.g., wavelet)

$\min ||s||_1 \text{ subject to } \mathbf{y} = \varphi \psi s$
$\hat{x} = \psi s$

The recovered signal

Note: Matrix Completion is also based on sparsity (low rank matrix) with consideration power flow equations that directly estimate the voltages.

Simulation Results

**Indirect method:**
- Reconstruct power data using compressed measurements
- Estimate the voltage states

**Direct method:**
- Reconstruct power and voltage states simultaneously

**Performance measure:** Integrated Normalized Absolute Error (INAE):

\[
\text{INAE} = \frac{\Sigma_{j=1}^{N}|V_j - \hat{V}_j|}{\Sigma_{j=1}^{N}|V_j|} \times 100
\]

- $V_j$: actual voltage at $j^{th}$
- $\hat{V}_j$: estimate of $V_j$
Tasks associated with Static State Estimation in 559-node Test System

<table>
<thead>
<tr>
<th>Estimation Methodology</th>
<th>Indirect Method</th>
<th>Direct Method</th>
<th>Bad data</th>
<th>Multi-timescale data</th>
<th>System Changes</th>
<th>Topology Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D Compressive sensing</td>
<td>Green</td>
<td>Green</td>
<td>Pale</td>
<td>Yellow</td>
<td>Yellow</td>
<td>Yellow</td>
</tr>
<tr>
<td>Matrix Completion</td>
<td>Pale</td>
<td>Pale</td>
<td>Pale</td>
<td>Yellow</td>
<td>Yellow</td>
<td>Yellow</td>
</tr>
<tr>
<td>2D Compressive sensing</td>
<td>Green</td>
<td>Pale</td>
<td>Pale</td>
<td>Yellow</td>
<td>Yellow</td>
<td>Yellow</td>
</tr>
<tr>
<td>Tensor Completion</td>
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<td>Pale</td>
<td>Pale</td>
<td>Yellow</td>
<td>Yellow</td>
<td>Yellow</td>
</tr>
</tbody>
</table>

- **Completed**: Green
- **In progress**: Pale
- **Future work**: Yellow

**Estimation Methodology**
- [Indirect Method](#)
- [Direct Method](#)
- [Bad data](#)
- [Multi-timescale data](#)
- [System Changes](#)
- [Topology Errors](#)
## Technical Achievements (Task 5)

### Deriving and Validating Voltage Sensitivity Analysis Method

<table>
<thead>
<tr>
<th>#</th>
<th>Month of completion</th>
<th>Performance Metric</th>
<th>Success Value</th>
<th>Assessment Tool / Method of Measuring Success Value</th>
<th>Verification Process</th>
<th>Metric Justification, Additional Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>12</td>
<td>Voltage violation prediction error</td>
<td>&lt;10%</td>
<td>We will assess the predictive capability of the extended PVSA using at least 50 different simulation scenarios and evaluate the error in prediction of voltage violations at strategic predetermined locations (indicating critical infrastructure).</td>
<td>Simulation data and Report sent to DOE for verification.</td>
<td>The success value was chosen based on anticipated impact of erroneous predictions.</td>
</tr>
</tbody>
</table>
Project Flow

- Analytical approximation of voltage sensitivity for 3-phase distribution system assuming no uncertainty.

- Error bound for approximation.

- Approximation for Multiple actor node case.

  - Probabilistic Voltage sensitivity for temporal uncertainty in power consumption/injection.

  - Probabilistic Voltage sensitivity for spatial and temporal uncertainty.
Sketch of the proof

Random factor for power change

Constant factor for fixed actor nodes

\[
\Delta S^a = \begin{bmatrix}
\Delta P_1^a \\
\vdots \\
\Delta P_n^a \\
\Delta Q_1^a \\
\vdots \\
\Delta Q_n^a
\end{bmatrix}
\]

\[
c_r(aa) = \begin{bmatrix}
- R_{o1aa} \cos \alpha_1 - X_{o1aa} \sin \alpha_1 \\
|V_1^a| \\
\vdots \\
- R_{onaa} \cos \alpha_n - X_{onaa} \sin \alpha_n \\
|V_n^a|
\end{bmatrix}
\]

\[
\frac{2n+1 \times 1}
\]

\[
\begin{bmatrix}
\Delta V_o^a \\
\Delta V_o^b \\
\Delta V_o^c
\end{bmatrix} = \begin{bmatrix}
c_r(aa) & c_r(ab) & c_r(ac) \\
c_r(ba) & c_r(bb) & c_r(bc) \\
c_r(ca) & c_r(cb) & c_r(cc)
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta S^a \\
\Delta S^b \\
\Delta S^c
\end{bmatrix}
\]
Algorithm Flow

\[ \Sigma \text{ Fixed covariance} \]

\[ \Delta S \text{ Power vector} \]

**Simulation**

Load flow \( V^t \)

\[ \Delta V = V^t - V^0 \]

**Theoretical**

\[ \sigma_1^2 = C_R^T \Sigma C_R, \sigma_2^2 = C_I^T \Sigma C_I \]

\[ \Delta V^r, \Delta V^i \]

\[ \theta = \frac{2(\sigma_1^4 + \sigma_2^4 + 2c^2)}{\sigma_1^2 + \sigma_2^2} \]

\[ |\Delta V|^2 \]

\[ k = \frac{\sigma_1^2 + \sigma_2^2}{\theta} \]

\[ m = k, \omega = \sqrt{k\theta} \]

\[ |\Delta V| \]

\[ |\Delta V| \sim \text{Nakagami}(m, \omega) \]