

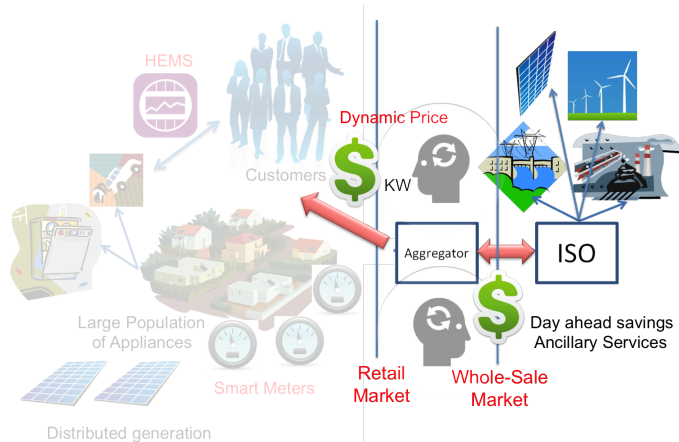
Ex-ante modeling, Load Scheduling and Differentiated Service Models for Populations of Flexible Loads

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GSR: Mahnoosh Alizadeh

CERTS Review Meeting 2014

Main objectives of the project



- **Objectives:** Modeling control mechanisms and economic value for aggregated load flexibility
- **Evaluation:** Theory and validation by simulations

Residential/personal appliances potential

- **Observation:** Costs include that of recruiting customers
- The “Internet of things” will make personal appliances easy to monitor and control (think of app “WhatsOn?”)



- Our tasks for this year model is the information needed to:
 - 1 **Deliverable 1:** **Ex-ante** plan and **Real-time** control (EV + TCL)
 - 2 **Deliverable 2:** **Pricing** a specific flexible use (EVs)

Ex-ante decisions:

- How much power $B(t)$ to purchase and how much ancillary service capacity $M(t)$ to offset costs in the forward market
- Solve for the minimum cost forward:

$$\min_{B(t), M(t)} \sum_{t \in \Omega} \mathbb{E}\{\mathcal{C}^F(L(t), B(t), M(t))\} \quad \text{s.t. } L(t) \in \mathcal{L}^{DR}(t), \quad (1)$$

where $\mathcal{L}^{DR}(t)$ the feasible set of loads ex-ante

Real-time decisions:

- Control $L(t)$ to follow the schedule $(B(t), M(t))$
- Minimize its real-time cost (here myopic):

$$\min_{L(t)} \mathcal{C}^R(L(t), B(t), M(t)) \quad \text{s.t. } L(t) \in \mathcal{L}^{RT-DR}(t) \quad (2)$$

where $\mathcal{L}^{RT-DR}(t)$ is the feasible set of loads in real time

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Part I

Modeling DR flexibility Ex-ante and in Real-time

Existing aggregate ex-ante models

- **Tank model:** Fill the flexible demand tank by the end of the day
[Lambert, Gilman, Lilienthal, '06], [Lamadrid, Mount, Zimmerman, Murillo-Sanchez, '11], [Papavasiliou, Oren '10]
 - Proposed for the system operator planning does not well capture inter-temporal constraints
- **Detailed model:** Model each individual appliance constraints
[Joo, Ilic, '10], [Huang, Walrand, Ramchandran, '11], [Foster, Caramanis, '13]
 - Good for local planning but not scalable for large populations
- **Quantized Population Models:** Cluster appliances and derive an aggregate occupancy model *[Chong85], [Mathieu, Koch, Callaway, '13] and our work...*
 - Good for both!

Smart appliances capabilities

- Deferrable loads with dead-lines ✓
- Interruptible rate constrained EVs with deadlines and V2G ✓
- Thermostatically Controlled Loads ✓

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Unified modeling approach

- 1 **State-space** parametric description of the set $\mathcal{L}_i(t)$ of possible load injections of specific appliance i
- 2 **Event-driven**: Appliances are available for control after t_i with initial state S_i ; (arrival is $a_i(t) = u(t - t_i)$ unit step)
- 3 **Divide and conquer**: Define a representative set $\mathcal{L}_q^v(t)$ for a given appliances category (v), quantizing possible parameters (q) and, if continuous, quantize the state (x)
- 4 **Aggregate and conquer**: Describe total flexibility $\mathcal{L}^v(t)$ using:
Aggregate arrival and state occupancy

$$a_x^q(t) = \sum_{i \in \mathcal{P}^{v,q}} \delta(S_i - x) a_i(t), \quad n_x^q(t) = \sum_{i \in \mathcal{P}_E^v} \delta(x_i(t) - x) a_i(t)$$

Aggregate control knob

$d_{x,x'}^q(t) = \#$ appliance moved from x to x' before time t

$\partial d_{x,x'}^q(t) = d_{x,x'}^q(t+1) - d_{x,x'}^q(t) = \# \dots$ at time t

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Non-interruptible Appliances - Individual Flexibility

- Loads that can be shifted within a time frame but cannot be modified after activation, e.g., washer/dryers
- $x_i(t) \in \{0, 1\}$ = state of appliance i (waiting/activated) \rightarrow initial state always 0
- $\partial x_i(t) = x_i(t+1) - x_i(t)$ = state change
- i appliance load = $g_i(t)$, if activated at time 0
- Laxity (slack time) of χ_i

$$\mathcal{L}_i(t) = \{L_i(t) | L_i(t) = \sum_{\tau} \partial x_i(\tau) g_i(t - \tau), x_i(t) \in \{0, 1\}, \\ x_i(t) \geq a_i(t - \chi_i), x_i(t-1) \leq x_i(t) \leq a_i(t)\}.$$

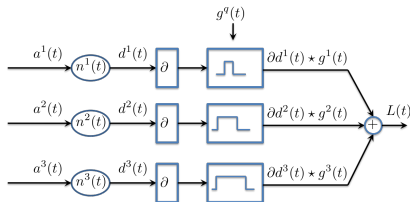
In English:

Load = load shape shifted at time of change of state (off to on)

Non-interruptible Appliances - Aggregate Flexibility

- Appliances clustered: quantized pulses $g^q(t)$ and deadlines χ^q
- $a^q(t)$ = arrivals and $d_{0,1}^q(t) \equiv d^q(t)$ = activations in cluster q

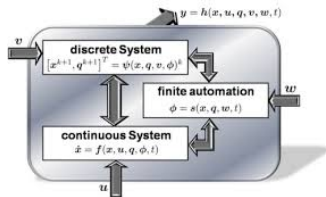
$$a^q(t) = \sum_{i \in \mathcal{P}^{v,q}} a_i(t), \quad d^q(t) = \sum_{i \in \mathcal{P}^{v,q}} x_i(t)$$



$$\mathcal{L}^v(t) = \left\{ L(t) \mid L(t) = \sum_{q=1}^{Q^v} g^q(t) \star \partial d^q(t), d^q(t) \in \mathbb{Z}^+ \right. \\ \left. d^q(t) \geq a^q(t - \chi^q), d^q(t-1) \leq d^q(t) \leq a^q(t) \right\}$$

Generalized to more complex cases....

Hybrid systems: control of the switching events. Discrete switching $x_i(t) \in \{0, 1, \dots, n\}$ + continuous dynamics $\{g_{i,x}(t)\}_{x=1}^n$



- Dimmable Lighting, joint washer/dryer cycle, etc.

Continuous state: EVs- Individual Flexibility

- States are quantized in the set $\mathcal{X}_i = \{0, 1, \dots, E_i\}$
- The possible charging rates in the set $\partial\mathcal{X}_i$ (possibly only one!)
- The deadline for full charge is χ_i

$$\mathcal{L}_i(t) = \{L_i(t) | L_i(t) = \partial x_i(t) a_i(t), x_i(t_i) = S_i, \\ x_i(t) \in \mathcal{X}, \partial x_i(t) \in \partial\mathcal{X}_i, x_i(\chi_i) = E_i\}$$

In English:

Load (power) = rate of change in state of charge $x_i(t)$ (energy)

- The parameters distinguishing $\mathcal{L}_i(t)$ are the battery capacity E_i , the possible charging rates $\partial\mathcal{X}_i$ and the deadline χ_i

Continuous state: EVs- Aggregate Flexibility

$$\mathcal{L}^v(t) = \left\{ L(t) \mid L(t) = \sum_{q=1}^{Q^v} \sum_{\substack{(x, x') \in \mathcal{X}^q, \\ (x' - x) \in \partial \mathcal{X}}} (x' - x) \partial d_{x, x'}^q(t) \right.$$

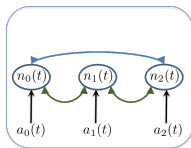
$$\partial d_{x, x'}^q(t) \in \mathbb{Z}^+, \sum_{x \in \mathcal{X}^q} \partial d_{x, x'}^q(t) \leq n_x^q(t)$$

$$\forall t \geq \chi^q \text{ and } x < E^q \rightarrow n_x^q(t) = 0$$

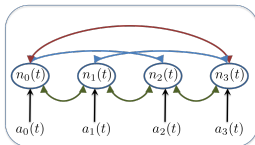
$$\left. n_x^q(t) = a_x^q(t) + \sum_{x' \in \mathcal{X}} [d_{x', x}^q(t-1) - d_{x, x'}^q(t-1)] \right\}$$

- Heterogenous $(\mathcal{X}^q, \partial \mathcal{X}^q, \chi^q) \rightarrow$ different **clusters** $q = 1, \dots, Q$

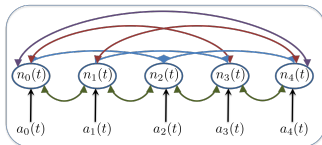
$E = 2$



$E = 3$

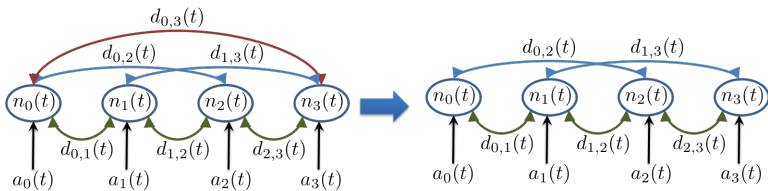


$E = 4$

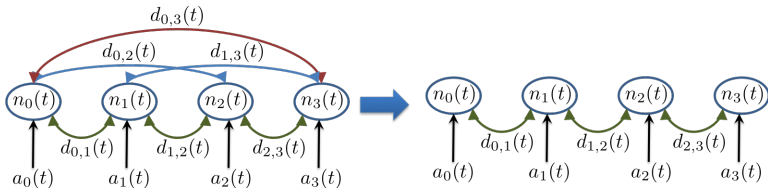


Other classes

- Rate-constrained battery change, e.g., V2G



- Interruptible e.g., pool pump, EV fixed rate charge



TCLs - Individual Flexibility

- $x_i(t)$ temperature in comfort band $[x_i^* - B_i/2, x_i^* + B_i/2]$ in the time window $[\chi_i^s, \chi_i^e]$ of the day.
- TCL cycles on and off $b_i(t) \in \{0, 1\}$ within a time frame $[t_i^s, t_i^e]$ larger or equal than $[\chi_i^s, \chi_i^e]$. TCL i arrival and departure events:

$$a_i(t) = u(t - t_i^s), \quad r_i(t) = u(t - t_i^e).$$

For unit i we have:

$$\mathcal{L}_i(t) = \left\{ \begin{aligned} L_i(t) | \partial x_i(t) &= -k_i x_i(t) + \alpha_i(t) + b_i(t) \xi_i, \\ b_i(t) &\in \{0, 1\}, L_i(t) = b_i(t) \Xi_i, \forall t \in [t_i^s, t_i^e] \\ |x_i(t) - x_i^*| &\leq B_i/2, \forall [t]_{24H} \in [\chi_i^s, \chi_i^e] \end{aligned} \right\}$$

where ξ_i = rate of heat gain Btu/h, Ξ_i is ξ_i in KW/h and the ambient noise $\mathbb{E}[\alpha_i(t)] = x_{amb}(t)k_i$, $x_{amb}(t)$ = ambient temperature

TCLs - Randomized control

- Since $\alpha_i(t)$ is random, switching the control $b_i(t) \in \{0, 1\}$ changes the probability that the appliances move from one state x to x'

$$P_i(x'|x; t; b_i(t)) = \text{Prob}\left(\alpha_i(t) = x' - x(1 - k_i) - b_i(t)\xi_i\right).$$

- We need to cluster based on these probabilities

$$P_i(x'|x; t; b) \mapsto P^q(x'|x; t; b), \quad q = 1, \dots, Q^v$$

- Occupancy of a temperature bin includes those OFF + those ON

$$\begin{aligned}n_x^q(t) &= n_{x,0}^q(t) + n_{x,1}^q(t) \\ &= a_x^q(t) - r_x^q(t) + \sum_{x' \in \mathcal{S}^q} D_{x',x}^q(t-1) - D_{x,x'}^q(t-1)\end{aligned}$$

$D_{x,x'}^q(t) = \#$ appliance moved from x to x' **at time t**

$$\mathbb{E}\{D_{x,x'}^q(t) | n_x^q(t)\} = n_{x,0}^q(t)P^q(x'|x; t; 0) + n_{x,1}^q(t)P^q(x'|x; t; 1)$$

TCLs - Aggregate Flexibility

- The comfort band constraint translates into

$$\forall |x - x^{*q}| > B^q/2 \rightarrow \Pr(n_x^q(t) = 0) \geq \eta,$$

where η is close to one (violations rare)

- Aggregate flexibility of heterogeneous TCLs

$$\mathcal{L}^v(t) = \left\{ L(t) \mid L(t) = \sum_{q=1}^{Q^v} \sum_{x \in \mathcal{S}^q} \Xi^q n_{x,1}^q(t), \quad n_x^q(t) = \sum_{b=0}^1 n_{x,b}^q(t), \right.$$

$$n_x^q(t) = a_x^q(t) - r_x^q(t) + \sum_{x' \in \mathcal{S}^q} D_{x',x}^q(t-1) - D_{x,x'}^q(t-1),$$

$$\mathbb{E}\{D_{x,x'}^q(t) \mid n_x^q(t)\} = \sum_{b=0}^1 n_{x,b}^q(t) P^q(x' \mid x; t; b);$$

$$\forall x : |x - x^{*q}| > B^q/2, \quad \forall [t]_{24H} \in [\chi^{s,q}, \chi^{e,q}) \\ \rightarrow \Pr(n_x^q(t) = 0) \geq \eta \}$$

Real time TCL control: simplified model

- The complexity grows linearly with # of quantization points but exponentially with # of parameters
- Simplified myopic policies based on **EV deadlines**:
Least Laxity First (LLF) and Earliest Deadline First (EDF)
[S. Caron and G. Kesidis, '10], [S. Chen, Y. Ji, and L. Tong, '12], [A. Subramanian, M. Garcia, A. Dominguez-Garcia, D. Callaway, K. Poolla, and P. Varaiya, '12], [G. O' Brien and R. Rajagopal, '13]
- **TLC deadlines based control**: TCL communicates quantized deadline instead of temperature state and switch value
 $(\tau_i(t), b_i(t))$

$$\tau_i(t) = \frac{1}{k_i} \ln \left(\frac{x_i(t) - b_i(t) \frac{\Xi_i}{k_i} - \frac{\alpha_i(t)}{k_i}}{x_i^* - (-1)^{b_i(t)} \frac{B_i}{2} - b_i(t) \frac{\Xi_i}{k_i} - \frac{\alpha_i(t)}{k_i}} \right).$$

- An EDF scheduler maximizes residual future flexibility

Ex-ante:

- To follow the AGC signal the aggregator must be able to
 - ① Increase/decrease demand by a certain step of variable height m from the baseline
 - ② Hold the demand at that value for a certain duration ξ
- We evaluated ξ to be the 97 % quantile of the zero-crossing time from historical AGC signals (19 min. based on PJM signals)
- Capacity estimate for the population 2.05 MWs

$$M' = \sum_{q=1}^Q \min_t M^q(t)$$

where $M^q(t)$ is the maximum deviation m from the baseline that a load in cluster q can tolerate at time t with $0.05m$ error (determined simulating the response of each cluster using $\mathcal{L}^q(t)$)

Regulation through TCL loads

- Real Time the TCLs are controlled for 6 h based on *clustering deadlines* (60 clusters)
- Temperature is Jan 29th 2012 in Davis;
- $\Xi_i = \xi_i \sim U([2000, 4000])$ Btu/h, $k_i \sim U([50, 200])$ W/C, $x_i^* \sim U([69, 75])$, $B_i \sim U([2, 4])$ F

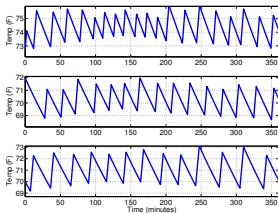
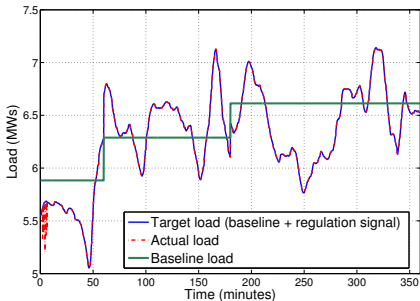


Figure : Simulated response of the TCL population (10000) to regulation signals and three 2 ton A/C units temperatures. The y-axis range is comfort band.

Part II

Pricing specific flexible uses

Price design for specific flexible use

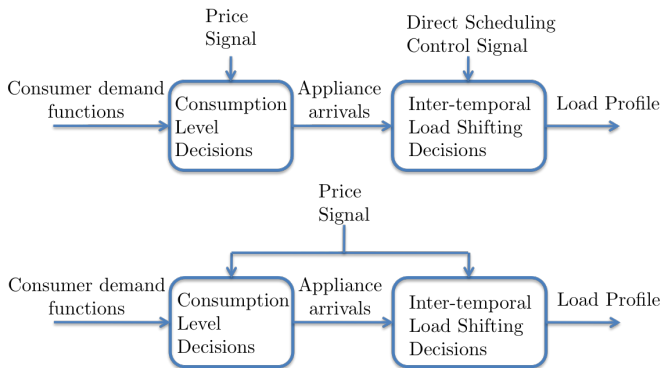


Figure : Differentiated Pricing and Scheduling (top) and Dynamic Retail Pricing (bottom).

Both schemes harness a subset of the *true* flexibility of demand

$$\mathcal{L}^{DR}(t) \subseteq \mathcal{L}(t)$$

DR #1: Dynamic Retail Pricing

- Dynamic retail prices $\mathbf{x}(t) = [\pi^r(t), \dots, \pi^r(t + T)] \in \mathcal{Z}(t)$ (set of regulated prices in $\mathcal{Z}(t)$)
- Possible load shapes:

$$\mathcal{L}^{DR}(t) = \{L(t) | L(t) = f(t; \mathbf{x}(t)), \mathbf{x}(t) \in \mathcal{Z}(t)\} \quad (3)$$

- Here $f(\cdot)$ is the price-response of the population

quantized price response - known

$$f(t; \mathbf{x}(t)) = L^I(t) + \sum_{v=1}^V \sum_{\vartheta \in \mathcal{T}^v} \left[\underbrace{a_{\vartheta}^v(\mathbf{x}(t))}_{\text{unobservable}} \overbrace{\arg \min_{L(t) \in \mathcal{L}_{\vartheta}^v(t)} \sum_{t=1}^T \pi^r(t) L(t)}^{\text{quantized price response - known}} \right]$$

- Price response only observable in aggregate and not for different clusters \rightarrow learning $a_{\vartheta}^v(\mathbf{x}(t))$ from limited observations



DR #2: Pricing for Direct Load Scheduling (DLS)

- An **aggregator** hires appliances and directly schedules their load
- We are one of the first to look at the economic side of DLS
- **Set of differentiated prices** based on plasticity

$$\mathbf{x}^v(t) = \{x_{\vartheta}^v(t), \forall \vartheta \in \mathcal{T}^v\}$$

But how can we have voluntary participation in DLS?

- Differentiated discounts $\mathbf{x}^v(t)$ from a high flat rate \rightarrow **incentives**
- Appliances choose to participate based on incentives $\rightarrow a_{\vartheta}^v(\mathbf{x}^v(t))$

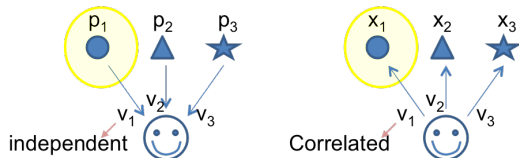
$$\mathcal{L}^{DR}(t) = \mathcal{L}^I(t; \mathbf{x}^v) + \sum_{v=1}^V \sum_{\vartheta \in \mathcal{T}^v} \overbrace{a_{\vartheta}^v(\mathbf{x}^v(t))}^{\text{observable}} \mathcal{L}_{\vartheta}^v(t).$$

- **Reliable**: aggregator observes $a_{\vartheta}^v(\mathbf{x}^v(t))$ after posting incentives and before control - no uncertainty in control unlike retail pricing



Incentive design

- Category v and cluster $q \rightarrow$ intrinsic properties of loads
- Independent incentive design problem for different categories v and clusters $q \rightarrow$ Let's drop q, v for brevity
- Optimal incentives given uncertainty about customer reservations to be recruited?
- The closest problem in literature: “optimal unit demand pricing”



- Customers valuation for different modes correlated (value of EV charge with 1 hr laxity vs. value of EV charge with 2 hrs laxity)

The Incentive Design Problem

- Aggregator designs posted incentives

$$\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_M(t)]^T,$$

- From recruitment of flexible appliances, the aggregator saves money in the wholesale market (utility):

$$\mathbf{u}(t) = [U_1(t), \dots, U_M(t)]^T$$

- Aggregator payoff when interacting with a specific cluster population:

$$Y(\mathbf{x}(t); t) = \sum_{m \in \mathcal{M}} \overbrace{(U_m(t) - x_m(t))}^{\text{Payoff of mode } m} \sum_{i \in \mathcal{P}(t)} \overbrace{a_{i,m}(\mathbf{x}(t); t)}^{\text{indicator of mode } m \text{ selection}} .$$

$a_{i,m}(\mathbf{x}(t); t) = 1$ if load i picks mode m given incentives $\mathbf{x}(t)$

- Goal: maximize payoff $Y(\mathbf{x}(t); t)$
- Problem: we don't know how customers pick modes

Probabilistic Model for Incentive Design Problem

- At best we have statistics \rightarrow Maximize expected payoff
- Probability of load i picking mode m :

$$P_{i,m}(\mathbf{x}(t); t) = \mathbb{E}\{a_{i,m}(\mathbf{x}(t); t)\}.$$

- Incentives posted publically - Individual customers not important
- Define *mode selection average probability* across modes:

$$P_m(\mathbf{x}(t); t) = \frac{\sum_{i \in \mathcal{P}(t)} P_{i,m}(\mathbf{x}(t); t)}{|\mathcal{P}(t)|}$$

$$\mathbf{p}(\mathbf{x}(t); t) = [P_0(\mathbf{x}(t); t), \dots, P_M(\mathbf{x}(t); t)]^T \rightarrow \text{what we need}$$

- Maximize expected payoff across cluster population

$$\max_{\mathbf{x}(t) \succeq \mathbf{0}} \mathbb{E} \left\{ \underbrace{\sum_{m \in \mathcal{M}} (U_m(t) - x_m(t))}_{\text{known}} \underbrace{\sum_{i \in \mathcal{P}(t)} a_{i,m}(\mathbf{x}(t); t)}_{\text{unknown}} \right\} =$$
$$\max_{\mathbf{x}(t) \succeq \mathbf{0}} \underbrace{(\mathbf{u}(t) - \mathbf{x}(t))^T}_{\text{known}} \underbrace{\mathbf{p}(\mathbf{x}(t); t)}_{\text{unknown}}$$

- Mode selection probability $\mathbf{p}(\mathbf{x}(t); t)$, the expected recruitment utility $\mathbf{u}(t)$, and the population size $|\mathcal{P}(t)|$ are daily periodic functions, i.e., $\forall t = iH + h, h = 0, \dots, H - 1, i \in \mathbb{Z}$ that come from a multivariate Gaussian distribution

$$\hat{\mathbf{p}}(\mathbf{x}; h) = \mathbf{B}^h \mathbf{f}(\mathbf{x}) + \mathbf{z}(\mathbf{x}),$$

- Find the point where the probability of improving the payoff beyond its current best value T is highest, i.e.,

$$\max_{\mathbf{x}} Q \left(\frac{T - |\mathcal{P}(h)|(\mathbf{u}(h) - \mathbf{x})^T \mathbf{B}^h \mathbf{f}(\mathbf{x})}{\sigma} \right),$$

where $\sigma^2 = |\mathcal{P}(h)|^2 (\mathbf{u}(h) - \mathbf{x})^T \Sigma(\mathbf{x}) (\mathbf{u}(h) - \mathbf{x})$ and $Q(\cdot)$ denotes the Gaussian Q function.

Numerical results

Stats based on 620 PHEV residential charge events, demand [0, 5 kWhs], 10 clusters, made rate flexible, simple probabilistic model set to match increasing risk as people get close to their travel time

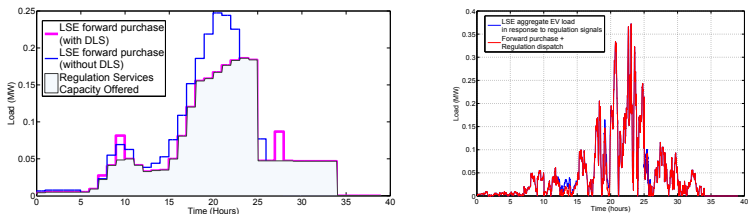


Figure : The performance of 620 PHEVs in following regulation signals.

Regulation service capacity prices are taken to be equal to the ISO New England's day-ahead market clearing prices in the Maine load zone on September 1st

Table : Comparison of the 4 studied incentive design schemes

Method	LSE profit	daily # recruited and payment
Bayesian - Uniform	\$493	707 EVs - 3.2c per EV
Bayesian - Gaussian	\$281	555 EVs - 1c per EV
Black box - kriging	\$653	560 EVs - 2c per EV
Upper bound	\$774	708 EVs - 2.1c per EV

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- M. Alizadeh, A. Scaglione, A. Goldsmith, and G. Kesidis, “Capturing Aggregate Flexibility in Demand Response”, Invited paper, IEEE 53rd Annual Conference on Decision and Control (CDC), Dec. 2014, to appear.

Deliverables and schedule for activities under FY14 funding

- Interface between appliances and aggregator for EVs and TCLs ✓
- Pricing of specific flexibility of EV ✓

Early thoughts on follow-on work for funding in FY15

- Exploring methods to cluster the inter-temporal constraints of solar and wind power to model net-generation flexibility
- Studying congestion in future coupled infrastructures - Traffic and EV charging
- Comparing Dynamic Pricing and Price Differentiated Scheduling via game theoretic analysis