TITLE:

Go Green! How Driving a Hybrid Vehicle Can Help You Cut Down on Pollution and Save Lots of Money – Maybe

SUMMARY:

The purpose of this education module is to introduce students to the advantages and benefits of putting more hybrid electric vehicles on the streets, and also to open their eyes to the economic reality of buying or owning a hybrid electric vehicle. By working through this exercise, students will have practice with ratios and proportions, percents, dimensional analysis, and linear relationships.

AUTHOR:

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COURSE/GRADE LEVEL:

Mathematics, middle school (Grades 7 and 8)

RELEVANT CURRICULUM STANDARDS:

Colorado Model Content Standards for Mathematics, Amended 9-15-05, Colorado Department of Education.

Standard 1

Students develop number sense and use numbers and number relationships in problem-solving situations and communicate the reasoning used in solving these problems.

Benchmarks

4. Use the relationships among fractions, decimals, and percents, include the concepts of ratio and proportion, in problem-solving situations.
6. Use number sense to estimate and justify the reasonableness of solutions to problems involving integers [and] rational numbers.

Standard 2

Students use algebraic methods to explore, model, and describe patterns and functions involving numbers, shapes, data, and graphs in problem-solving situations and communicate the reasoning used in solving these problems.

Benchmarks

1. Represent, describe, and analyze patterns and relationships using tables, graphs, verbal rules, and standard algebraic notation.

2. Describe patterns using variables, expressions, equations, and inequalities in problem-solving situations.
3. Analyze functional relationships to explain how a change in one quantity results in a change in another (for example, how the area of a circle changes as the radius increases, or how a person's height changes over time).
4. Distinguish between linear and nonlinear functions through informal investigations.
5. Solve simple linear equations in problem-solving situations using a variety of methods (informal, formal, graphical) and a variety of tools (physical materials, calculators, computers).

Standard 5

Students use a variety of tools and techniques to measure, apply the results in problem-solving situations, and communicate the reasoning used in solving these problems.

Benchmarks

1. Estimate, use, and describe measures of distance, perimeter, area, volume, capacity, weight, mass, and angle comparison.

Standard 6

Students link concepts and procedures as they develop and use computational techniques, including estimation, mental arithmetic, paper-and-pencil, calculators, and computers, in problem-solving situations and communicate the reasoning used in solving these problems.

Benchmarks

1. Use models to explain how ratios, proportions, and percents can be used to solve real-world problems.
4. Select and use appropriate algorithms for computing with commonly used fractions and decimals, percents, and integers in problem-solving and determine whether the results are reasonable.

FOR THE TEACHER:

Since they were first invented, most automobiles have been powered by engines called *internal combustion engines*. Whether the engines run on gasoline or diesel fuel, they all operate in essentially the same way: a mixture of vaporized fuel and air is introduced into a cylinder, a piston compresses the mixture in the cylinder, and the mixture ignites explosively, pushing the piston down the cylinder and turning the crankshaft. The turning motion of the crankshaft is transferred to the wheels, and the turning motion of the wheels pushes the automobile forward.

Alternatives to internal combustion engines have been tried over the years, but none have outlasted or replaced the gasoline- or diesel-powered internal combustion engine. The Stanley brothers produced steam-powered automobiles between 1902 and 1927, but even their aggressive advertising

campaign could not halt the popularity of the "internal explosion engine," as they called it. Chrysler experimented with turbine-powered vehicles from 1954 to 1979, but abandoned the effort because of difficulties matching the stop-and-go requirements of an automobile with the constant-speed preference of a turbine. Presently, several automotive companies are doing research on fuel cells, which combine hydrogen or methane with oxygen to create electricity without combustion, but the technology is still a few years away from being economically feasible. Electric vehicles have been around for nearly a century, but because of limited energy-storage capacity (batteries) and the resulting limitations on range and power, they have never been popular as replacements for internal combustion engine powered vehicles. In early 2007, an entrepreneur in San Jose, California, announced the introduction of an all-electric sports car.

Rising gasoline prices and the need to cut back on emissions (or pollution) from automobile engines have caused automakers, automotive industry researchers and consumers to take another look at alternatives to the internal combustion engine.

One contemporary, popular alternative is the *hybrid electric vehicle*, which combines an internal combustion engine with an electric motor, a large bank of batteries, and a computer controller. At normal speeds and under normal loads, as determined by the vehicle's computer, the vehicle uses only its electric motor for propulsion. At higher speeds, higher acceleration or heavier loads, the internal combustion engine starts automatically and augments the electric motor's power. The engine turns off automatically when it is no longer needed. When the brake pedal is pressed, the electric motor acts as a generator, converting the kinetic energy from the spinning wheels into electrical energy, which is then stored in the batteries. The internal combustion engine can also be used to recharge the batteries during normal operation if the charge gets too low.

Some current examples of hybrid electric vehicles include the Prius, Camry Hybrid, and Highlander Hybrid from Toyota, Honda Civic Hybrid and Ford Escape Hybrid.

A hybrid electric vehicle combines the power and availability advantages of an internal combustion engine with the zero pollution and zero fuel consumption of an electric vehicle. The computer control allows the vehicle to recover the energy normally wasted through friction during braking, and to use the internal combustion engine only when absolutely necessary.

If the teacher has access to a hybrid electric vehicle, it might be a good idea to take the class outside to look at the hybrid electric vehicle either at the beginning of the unit or during the unit. If it can be done safely, a

demonstration of the vehicle running on electric power alone and on its internal combustion engine might help the students to understand the real-world application of the mathematical concepts being studied.

LEARNING OBJECTIVES:

By the end of this module, students will:

- Know the definitions of internal combustion engine, hybrid electric vehicle, emissions, and pollution.
- Be able to calculate the percentage reduction in emissions achieved by a hybrid electric vehicle.
- Be able to calculate the increase in fuel mileage, or the decrease in fuel consumption, for a hybrid electric vehicle.
- Be able to calculate the payback period for purchase of a hybrid electric vehicle.
- understand the difference between a part-to-part ratio, a part-to-whole ratio, and a rate.
- Be able to set up and solve proportions using rates and ratios.
- Be able to set up and solve percentages as proportions.
- Be able to describe a linear relationship in terms of rate and starting point, using a graph, a table or an equation.
- Be able to find the y-intercept and the x-intercept of a linear relationship, using a graph, a table or an equation.

TIME ALLOTTED:

Students should be able to complete this module in three to five days. The activities in this module can be presented together, or separately during the year following the units in which the mathematical concepts used herein are presented.

RESOURCES / MATERIALS:

The minimum information needed to complete this module can be found in this packet. Additional information, which the students will need to collect, include the following:

- Up-to-date prices of a normal gasoline-powered vehicle and a hybrid electric vehicle.
- Up-to-date prices of gasoline and diesel fuel.
- Average monthly distance traveled by a typical family car, in miles.
- Fuel mileage of a typical family car, in miles per gallon.

Students will also need rulers and graph paper. The graphs and tables provided in this module may be copied and distributed to the students for them to write on.

PREREQUISITE KNOWLEDGE:

Students should already know and be able to use:

- fractions
- ratios, rates, and percentages
- how to perform rate arithmetic, like distance = rate $\times$ time
- proportions
- units, such as miles, hours, and kilograms
- unit rates, such as miles per hour
- how to perform unit arithmetic
- dependent and independent variables
- how to draw and read coordinate graphs

ADDITIONAL KNOWLEDGE:

It will be helpful if students know and are able to use:

- linear functions or linear relationships
- how to set up and solve problems using proportions
- dimensional analysis, or mathematics involving units and unit rates

(If they do not, these activities will introduce the concepts to them.)

LESSON SCOPE AND SEQUENCE:

Activity 1: Reducing Truck Emissions

In this exercise, students will compare the emissions of a normal gasoline-powered vehicle with the emissions of a hybrid vehicle. They will calculate the percent reduction in emissions (or pollution) and the percent of fuel saved by using a hybrid electric vehicle. In the follow-up activity, they will calculate the total amount of fuel saved if a certain percentage of vehicles in Colorado were to be replaced by hybrid electric vehicles.

Activity 2: How Catalytic Converters Work

In this exercise, students will use linear functions to determine the total amount of hydrocarbons emitted by a vehicle depending on its catalyst temperature. For this exercise, students will use a table to create a graph, and then answer questions based on the graph.

Activity 3: How Much Does It Cost?

In this exercise, students will conduct research to find the sale price of a hybrid electric vehicle. They will conduct research to find their family car's fuel mileage and average monthly miles traveled. (As an alternative, the teacher can supply them with a few examples, like the ones provided in this activity.) Students will then calculate the dollar value of fuel savings per month by switching to a hybrid electric vehicle. Finally, using the purchase price of a hybrid electric vehicle and the monthly savings rate, they will create a linear function using an equation a table and a graph, and calculate the payback period for purchasing a hybrid electric vehicle.

ACTIVITY 1: REDUCING TRUCK EMISSIONS

Summary:

In this exercise, students will compare the emissions of a normal gasoline-powered vehicle with the emissions of a hybrid vehicle. They will calculate the percent of emissions (or pollution) eliminated, and the percent of fuel saved by using a hybrid electric vehicle. In the follow-up activity, they will calculate the total amount of fuel saved if a certain percentage of vehicles in Colorado were to be replaced by hybrid electric vehicles.

Activating Prior Knowledge:

Use a hypothetical situation, similar to those found in Section 3.2 of the Connected Mathematics unit *Comparing and Scaling*, to review how to calculate a percent increase or decrease. For example: If a dress normally priced at $50 is on sale for $20, then review with the students how to calculate that the sale price is 40% of the original price, and the discount is a 60% discount. Tell them that in this unit they will utilize this knowledge (how to calculate discounts) in a different setting.

Review the rate equation, $D = RT$, distance = rate $\times$ time, which they learned in the Connected Mathematics units *Variables and Patterns* and *Moving Straight Ahead*. Remind them that although the rate they used with $D = RT$ was miles per hour, rates can have denominators other than time, like hot dogs per student or gallons per mile.

Show the students a picture of a large truck. (A picture is included in the back of this packet if you can't find a better one.) Tell the students that most heavy-duty vehicles run on diesel fuel. Tell them that burning diesel fuel puts a lot of pollutants into the atmosphere, and that scientists have been working on ways to eliminate those pollutants. Ask them if they've ever seen clouds of smoke coming from a truck's exhaust pipe.

New Concepts:

Introduce the students to the word "*emissions*". *Emissions* is the word scientists use for the mixture of gases that comes out of a car's or truck's exhaust pipe.

Ask the students what benefits would come from successfully eliminating or reducing the emissions from a heavy-duty vehicle like a truck.

Introducing the Problem:

Show the students a picture of a hybrid truck. (A picture of a hybrid UPS truck is included in this packet.) Tell the students that, although a hybrid electric vehicle doesn't look any different on the outside, on the inside it includes an electric motor, so that the internal combustion engine doesn't have to do all the work to move the truck forward.

Give each of the students a copy of Tables 1 and 2 (see the handout master at the back of this packet). Table 1 compares the emissions from a normal diesel truck to the emissions from a hybrid electric truck. Table 2 compares the fuel mileage for a normal diesel truck and a hybrid electric truck.

Pollutant	Emissions from a regular truck	Emissions from a hybrid electric truck
Carbon dioxide	1700 grams/mile	1047 grams/mile
Carbon monoxide	8.1 grams/mile	6.25 grams/mile
Hydrocarbons (unburned fuel)	0.20 grams/mile	1.02 grams/mile
Nitrogen oxides	22 grams/mile	10.44 grams/mile

Table 1: Gaseous emissions from heavy-duty diesel and hybrid electric vehicles. (Source: National Renewable Energy Laboratory, Golden, Colorado)

Type of vehicle	Diesel fuel mileage
Internal combustion	3.5 miles/gallon
Hybrid electric	8.8 miles/gallon

Table 2: Fuel consumption from heavy-duty diesel and hybrid electric vehicles. (Source: National Renewable Energy Laboratory, Golden, Colorado)

The Colorado Department of Transportation reports that in 2003, cars and trucks drove more than *26 billion miles* on highways in the state ("Crashes and Rates on State Highways, 2003"). For the purposes of this activity, assume that half of that, or 13 billion miles, are truck miles.

Implementing the Lesson:

Students should use the Tables to answer the following questions.

1. If a single diesel delivery truck is replaced with a hybrid electric truck, what is the percent decrease in carbon dioxide emissions? (This is just like calculating the percent discount on a dress.)
2. Repeat question 1 for carbon monoxide, hydrocarbons, nitrogen oxides and particulates. Put your answers in a table.
3. Trucks drive 13 billion miles a year on Colorado roads. If all the trucks in Colorado were to be replaced with hybrid electric vehicles, how much carbon dioxide pollution would be eliminated?
4. If only 20% of the trucks in Colorado were to be replaced with hybrid electric vehicles, how much carbon dioxide pollution would be eliminated?
5. How much diesel fuel do all the trucks in Colorado consume in one year?
6. If all the trucks in Colorado were to be replaced with hybrid electric vehicles, how much diesel fuel would they consume in a year?
7. Calculate the percent reduction in diesel fuel consumption for question 6.
8. If only 20% of the trucks in Colorado were to be replaced with hybrid electric vehicles, how much diesel fuel would they consume in a year?

ACTIVITY 2: HOW CATALYTIC CONVERTERS WORK

Student handout

Catalyst temperature (°C)	THC (parts per million)
408	650
423	594
480	398
474	391
509	263
525	205

Table 3: Hydrocarbon Emissions vs. Catalyst Temperature
THC = Total hydrocarbons
ppm = parts per million
(Source: National Renewable Energy Laboratory, Golden, Colorado)

Answer the following questions on a separate sheet of paper.

1. In the table, which variable is the *independent variable*?
2. Which variable is the *dependent variable*?
3. How does the concentration of hydrocarbons change if the catalyst temperature goes up by 10 °C? Does the concentration go up or down? By how much?
4. Using your answer to question 3, calculate a rate of change for the hydrocarbon concentration in the exhaust.
5. On a piece of **graph paper**, draw a coordinate graph of these two variables. Make sure the independent and dependent variables go on the proper axis. Plot the points from the table on the coordinate graph.
6. Look at the table and at the graph. Is the relationship between temperature and concentration a linear relationship? How do you know?
7. What would you expect the hydrocarbon emissions to be at 300 °C?
8. What would you expect the hydrocarbon emissions to be when the engine is first started on a crisp autumn day?
9. At what temperature would you expect hydrocarbon emissions to go to zero?

ACTIVITY 3: HOW MUCH DOES IT COST?

Student handout

In this activity, you will do some independent research to calculate the payback period for your family to switch to a hybrid electric vehicle. Write a report answering the following questions. For each question, you must show your work or tell how you got your answer.

1. Describe the vehicle your family uses as "the family car." If you have more than one, choose the one that usually has more than one person in it. For example, you may say: "Our family car is a 2005 Ford Explorer. It has seat belts for seven people."
2. Find out how many miles your family car goes in a month. You can just ask your parents for this number.
3. Find out your family car's fuel mileage, in miles per gallon. You may have to ask your parents for this, or have your parents help you look it up on the Internet. If your family car is a Toyota Camry, Honda Accord or Saturn S400, you will probably have a number between 25 and 32 miles per gallon. If your family car is a van or an SUV, you will probably have a number between 12 and 20 miles per gallon. If your family car is a Hummer or a Ford Expedition, you will probably have a number around 12 miles per gallon.
4. Find out the price per gallon for gasoline or diesel, whichever fuel your family car uses. In the summer of 2007, both gasoline and diesel are hovering around $3.00 per gallon.
5. Using your answers to questions 2, 3, and 4, calculate the cost of fuel for one month.
6. Use newspaper advertisements or contact a car dealership to find the price of a new hybrid electric vehicle to replace your family car. You may need your parents' help for this step. In the summer of 2007, a new 2007 Toyota Highlander Hybrid 4x4 with a third-row seat costs $36,624 from Burt Toyota in Denver. A new 2007 Toyota Prius costs approximately $23,000.
7. Now, suppose that you trade in your family car to buy the hybrid electric vehicle. If you trade in a minivan or SUV, subtract $8000 from the hybrid electric vehicle's price. If you trade in a regular car, subtract $5000 from the hybrid electric vehicle's price. How much did you spend?
8. A Toyota Prius gets approximately 60 miles per gallon. A Highlander Hybrid gets approximately 32 miles per gallon. Use one of these numbers, and your answers to questions 2 and 4, to calculate the cost of fuel for one month for the hybrid electric vehicle.

9. Look at your answers to question 5 and question 8. How much money does the hybrid electric vehicle save you in one month?
10. The answer to question 7 is the amount of money you invested in the hybrid electric vehicle. The answer to question 10 is the rate at which the hybrid electric vehicle saves you money. Make a table with two columns. Label the first column "Months" and the second column "Cost." On the first row of the table, put a zero (0) in the "Months" column and your answer to question 7 in the "Cost" column.
11. In the next row, put a one (1) in the "Months" column. Subtract one month of savings from the cost of the vehicle and write your answer in the "Cost" column. Repeat this step for 2, 3, 4, and on up to 12 months.
12. On a sheet of **graph paper**, plot the data from the table. Make sure you use the correct *independent variable* and *dependent variable*. Make sure your time axis is long enough that a line drawn through your data will eventually cross the axis.
13. Write a rule (or equation) that describes the cost of your hybrid electric vehicle based on how long you have owned it.
14. When the cost reaches zero on the table or in the equation, or when the line crosses the time axis on your graph, you will have found the *payback period* for your purchase: the time it takes for you to get paid back for all the extra money you spent on a hybrid electric vehicle. What is the payback period for your new family car? (Remember to show your work or tell how you found your answer.)
15. Investors usually like a payback period of two years (24 months) or less – they don't like to have their money tied up someplace for longer than that. Based on the payback period in question 14, is a hybrid electric vehicle a good investment for your family?
16. The average new-car buyer keeps a car for four years. This means that if your hybrid electric vehicle has a payback period of less than four years (48 months), it may still be a good investment for your family – it will have paid for itself. So if you use 48 months instead of 24 months as your goal, is a hybrid electric vehicle a good investment for your family?
17. What would the cost have to be, for your vehicle to have a payback period of 48 months?

ACTIVITY 3 – ASSESSMENT RUBRIC

	7 Unsatisfactory or Incomplete	**8 Partially Proficient**	**9 Proficient**	**10 Advanced**
Content	Less than 70% of the questions are answered.	At least 70% of the questions are answered.	At least 90% questions are answered completely.	All questions are answered completely, and answers are plainly marked or circled.
Correctness	The answers do not even come close to the correct answers.	Some of the answers make sense or come close to being correct.	The answers make mathematical sense or are mathematically correct.	The answers are mathematically correct and show evidence of having been verified.
Communication / Clarity	Most answers are given without any explanation.	The answers include brief or cryptic explanations of how they were obtained.	Each answer includes an explanation of how the answer was obtained.	Each answer includes an explanation of the strategies involved in finding the answer.
Comprehension	The answers demonstrate no understanding of the concepts.	The answers demonstrate a partial understanding of the concepts.	The answers demonstrate a nearly complete understanding of the underlying concepts.	The answers demonstrate a mastery of the concepts.

Content: _____

\+ Correctness: _____

\+ Comm/Clarity: _____

\+ Comprehension: _____

= SUBTOTAL: _____ → divide by 4 = TOTAL SCORE: _____