Essence of Structure Preserving (ESP) Network Reductions for Engineering and Economic Analysis

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Overview

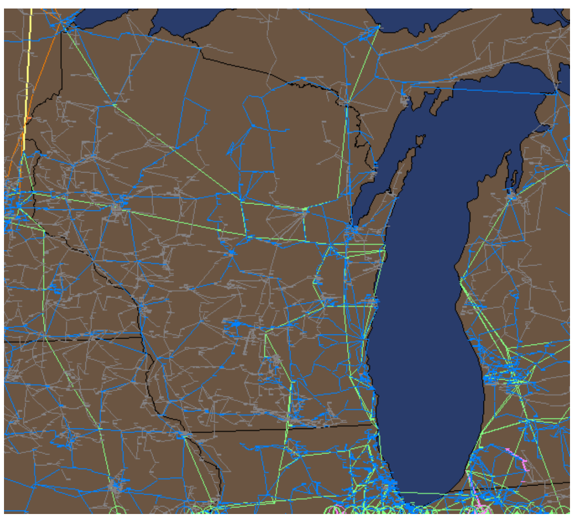
- 1: Supporting Cornell SuperOPF studies of the US power system.
- 2: Porting Modified-Ward reduction method to the public domain.
- 3: Overarching Motivation-develop network equivalents that preserved structure for different applications.
- Inter-zonal-flow preserving network equivalents.
  - Bus Aggregation methods for accurate inter-area power flow calculations.
  - (Review) Bus-Aggregation using PTDF’s
    - Clustering
    - Equivalent branch reactances
    - Accuracy
  - Computation time minimization—effect on accuracy
  - Comparison of PTDF calculation assumptions
  - Results of Reduction (Accuracy)
- Conclusions

Supporting SuperOPF Studies

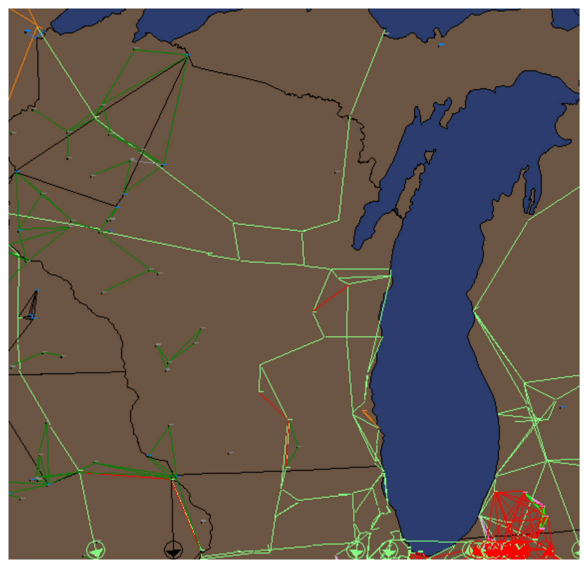
- Supporting Cornell SuperOPF studies.
  - Developing network reductions using Modified Ward method.
  - Using WECC, EI, ERCOT.
  - Different size reduced equivalents.
  - Different base-case reduced equivalents.
  - Validations for accuracy.
    - Branch flows—base case, change-cases.
    - OPF—metrics on LMP, objective function.

Modified Ward Reduction

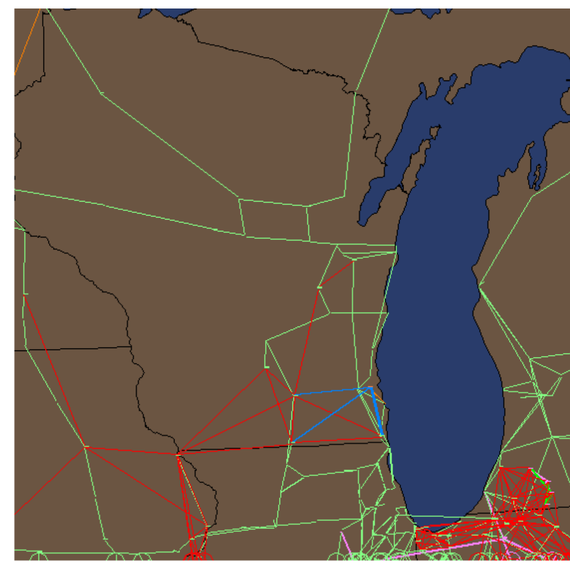
- 1. **Apply the classical Ward equivalencing technique but retain both the selected buses and external generator buses**. This model is used to determine the location of the generators in the final equivalent.
- 2. **Apply the classical Ward equivalencing technique to remove all but the retained buses.**
- 3. **Move generators to “appropriate” retained (boundary) buses.**



(b) Original WI system



(b) after step 1



(c) after step 2

Power system of Wisconsin

The Shortest Path Problem

- 3. (cont'd) **Move generators to the boundary buses which are closest to the generators.**
  - Based on graph theory, the generator moving problem can be formulated as the shortest weighted path problem.
  - Find a path between each generator bus and an internal bus such that the sum of the weights of constituent branches is minimized.
  - This shortest weighted path problem was solved using Dijkstra's algorithm [3, 4].
  - Other ways to measure electrical distance.
- 4. **Move the load to match base-case branch power flows** using inverse power flow algorithm.

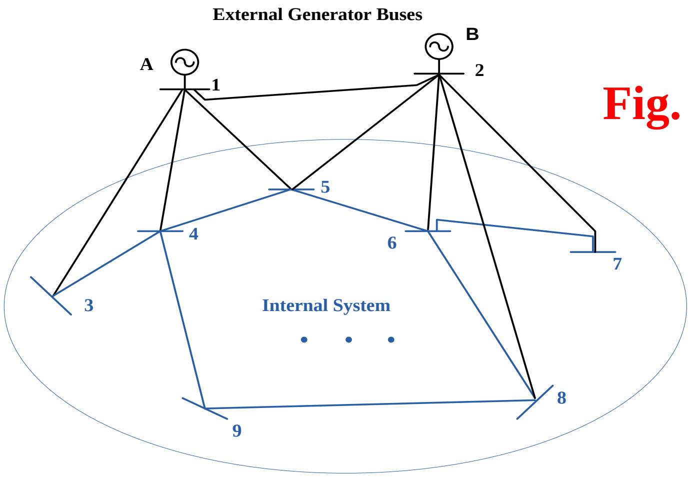


Fig. 1

generalization

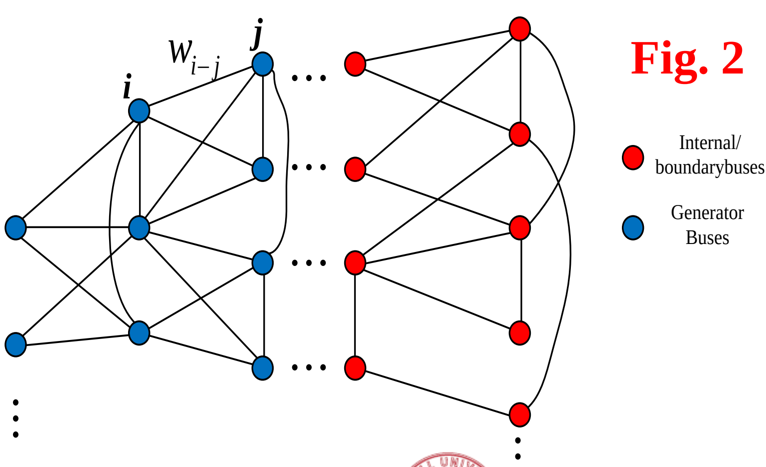
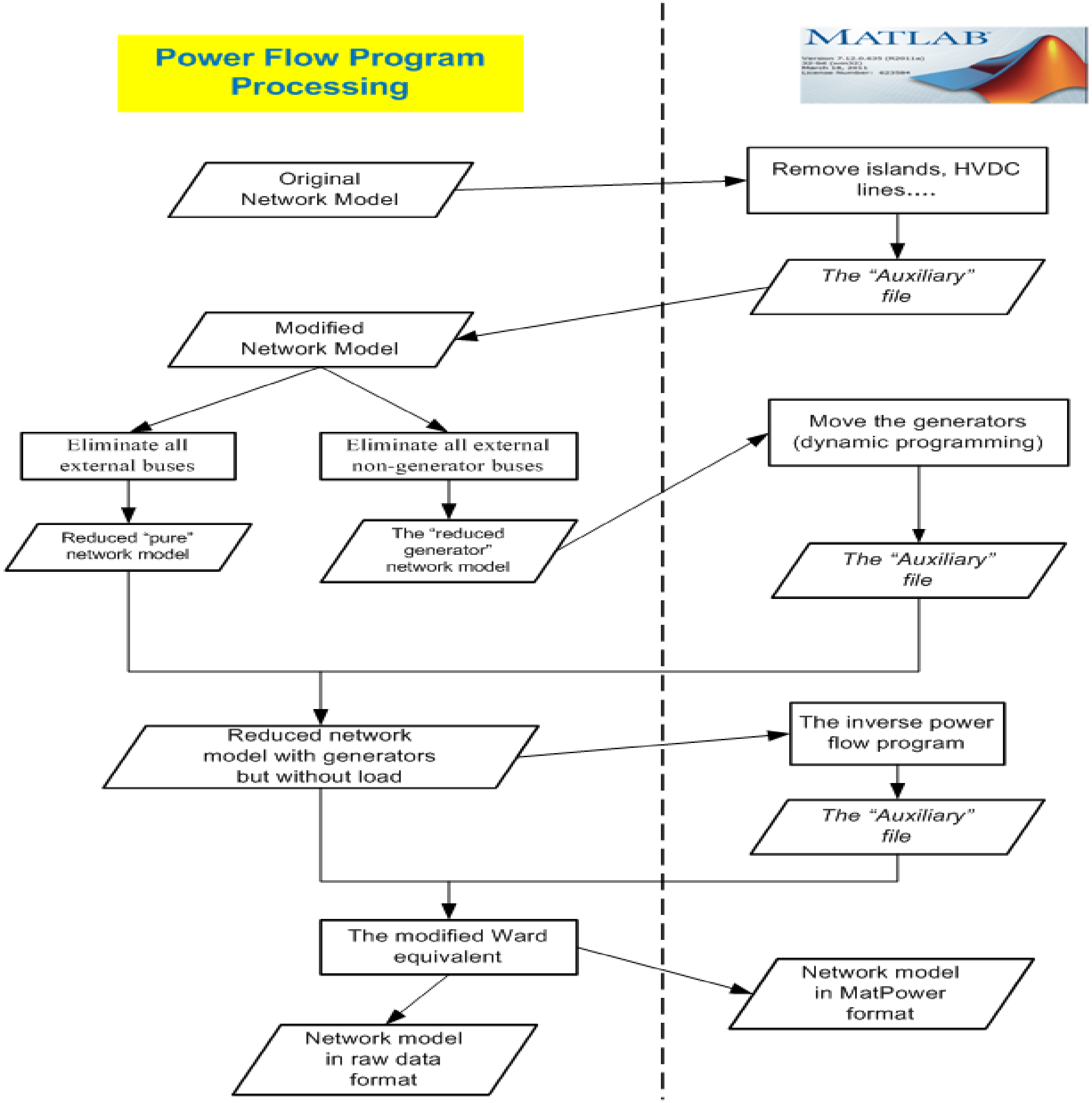


Fig. 2

3. E. W. Dijkstra, "A Note on Two Problems in Connection with Graphs," *Numerische Math*, vol. 1, no. 1, pp. 269-271, 1959.
4. B. V. Cherkassky, A. V. Andrew and T. Radzik, "Shortest Paths Algorithms: Theory and Experimental Evaluation", Mathematical Programming, Ser. A 73(2), pp. 129-174, 1996.

The Whole Process



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The Modified Ward Equivalent

- Thrust #1—Bring Modified-Ward equivalencing procedure to public domain—Bob Thomas. (Continuation of FY12)
- Objectives:
  - User intervention in the process should be minimized.
  - All completed using one power-flow application—MatPower.
  - Testing to prevent crashes.

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Moving Modified Ward Equivalent to Public Domain

- Major tasks for removing the seams between the following discrete steps
  - Data Conversion (Completed)
  - Island Identification & HVDC treatment (In progress)
  - Data Quality Control (In progress)
  - Network Reduction
  - Generator Assignment
  - Load Assignment
  - Output—PSSE? PSSE and PowerWorld??
  - Testing
  - User's Manual

Porting Modified Ward Equivalent to Public Domain

- Data conversion challenges
  - Design application to work with PSSE and PowerWorld input format.
  - Reading in the PSS/E .raw format data
    - Different data formats, e.g., two winding v. three-winding xfmrs.
    - Adapting model differences, e.g., control of switched shunts.
    - Adding fictitious buses, e.g., star point of three-winding transformers.
    - Validation with large datasets is time consuming.

Bus Aggregation
A Different Approach

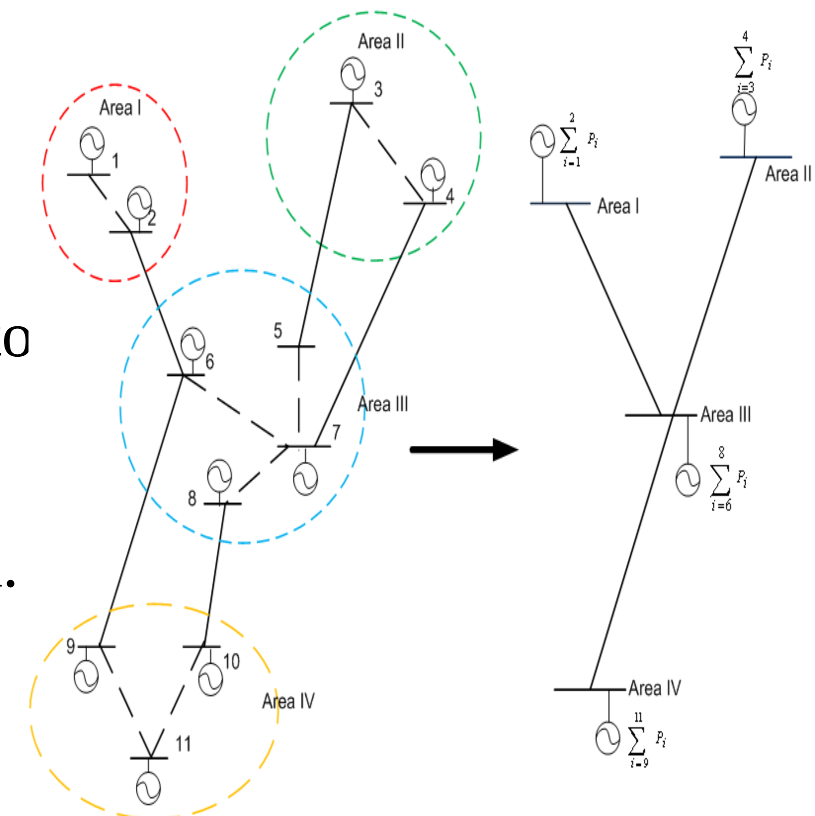
- FY 2012—Bus aggregation techniques: Goal is to preserve inter-zonal power flows to accurately model system behavior under bilateral transactions.
- Difference from modified Ward:
  - dc rather than ac model reduction.
  - Aggregate buses rather than “eliminate” buses.
  - Match base-case *inter-zonal* flows accurately, (rather than retained transmission line flows as in modified Ward.)

Bus Aggregation
A Different Approach

- FY 2013—Continue bus aggregation to its logical conclusion.
- Thrust #2: Improving computational efficiency
- Thrust #3: Improve accuracy

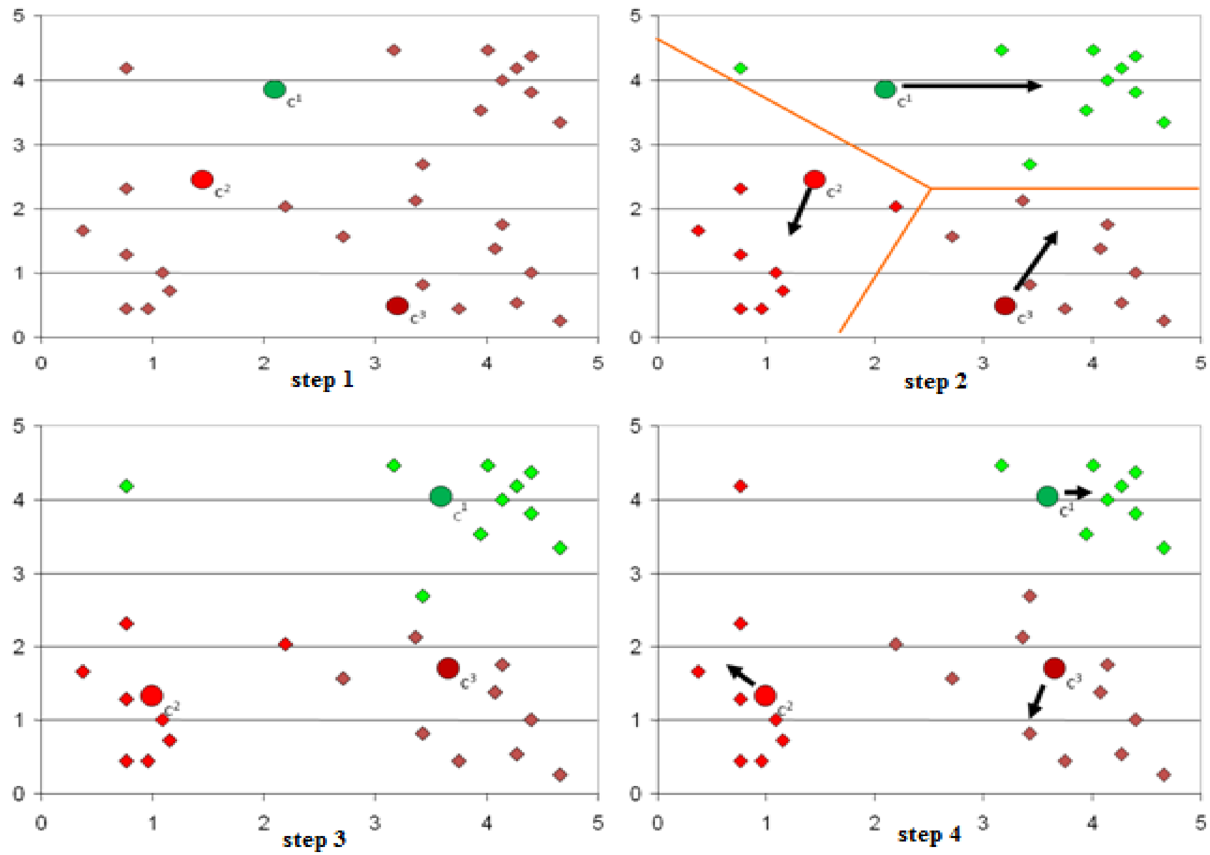
Bus Aggregation

- Basic idea:
  - Divide system into zones (aggregations.)
  - Each zone represented by a single bus
  - Generation & load aggregated to single aggregated bus
  - Intra-zonal lines are neglected
  - Inter-zonal lines are aggregated.



Clustering via k-means++ Algorithm

- The objective of the k-means algorithm works in such a way that within one cluster (t), the Euclidean distance between any bus and this cluster's center is smaller than the Euclidean distance between this bus and any of the other cluster's centers.
- This algorithm includes the following four steps:



Power Transfer Distribution Factor (PTDF)

- Once bus cluster are defined, finding equivalent inter-zonal branch reactances is the linear overdetermined (state estimation) problem.

$$1/x_R = \left[(\Lambda^*)^T \Lambda^*\right]^{-1} \left(\Lambda^*\right)^T \begin{bmatrix} P_{i->j} / \theta^*_{ij} \\ 0 \end{bmatrix}$$

- Number of rows in Λ^* is large $N_{branch} \bullet N_{bus}$: For EI—80,000 × 60,000≈5 billion rows.
- Large computational burden.

The Over-Determined Problem

- To reduce the computational burden, the following features can be used to advantage:
  - Λ is a sparse matrix – very sparse
  - No. of non-zeros in each row of subblock of Λ $[(\Phi_R\, C_R^T - I)diag(c_i)]$ equals the no. of branches connected to bus i.
  - In the equivalent, a bus is connected by 3~4 branches on average.
  - Structural property of the Λ matrix.
  - Ex: A subblock of Λ which contains 6000 equations, typically has only four that are linearly independent.
  - Most equations have minute coefficients.

Equation Selection

- To reduce dimension of Λ, eliminate equations whose coefficients are below a threshold.
- Compare branch reactance and flow errors as a function of threshold values for reduced dc model.

Accuracy v. Threshold

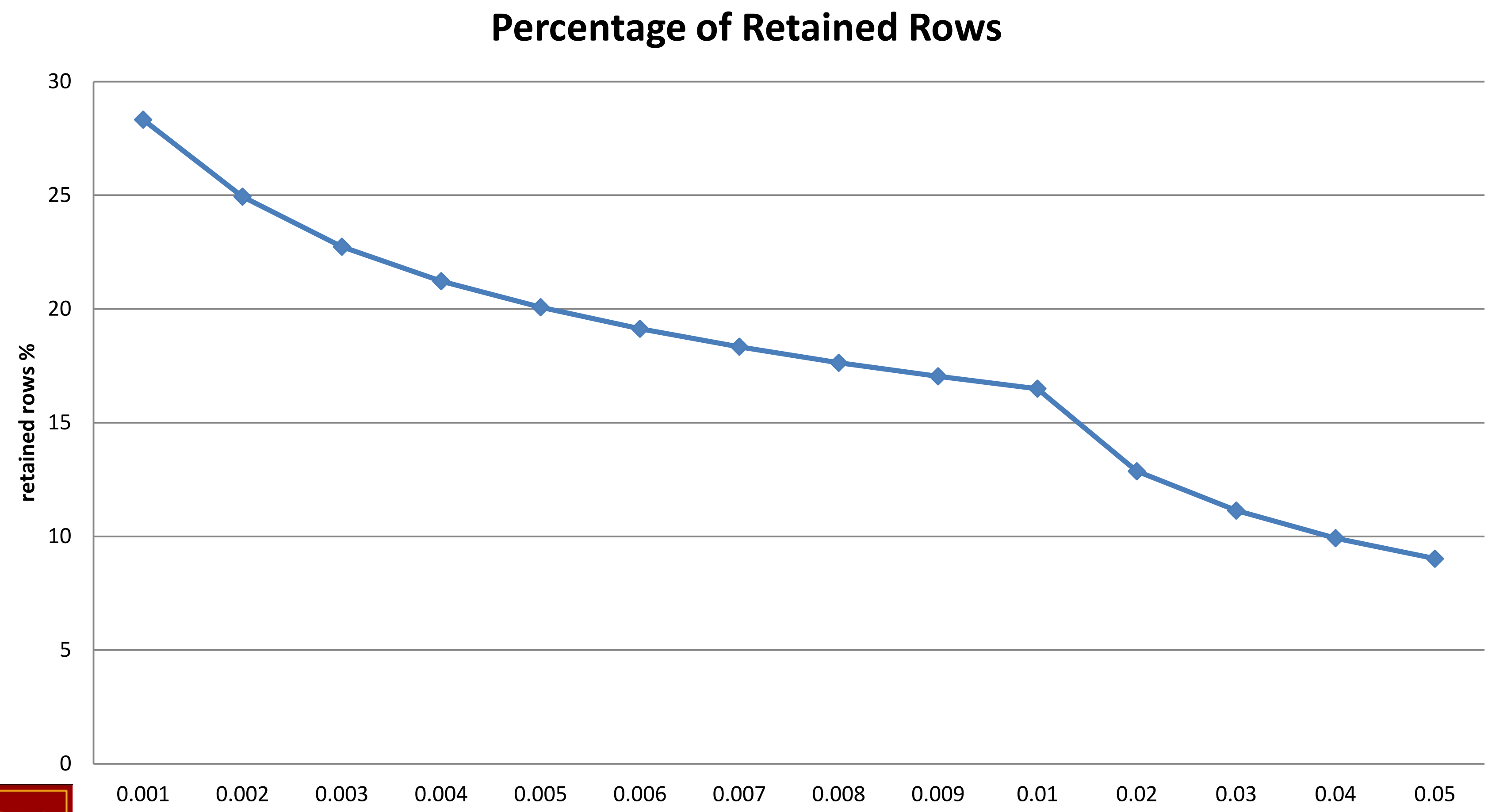
6073-Bus ERCOT System—Bus Aggregation=Shift Factor Similarity

100 Bus Equivalent (48,000 Row Λ Matrix)

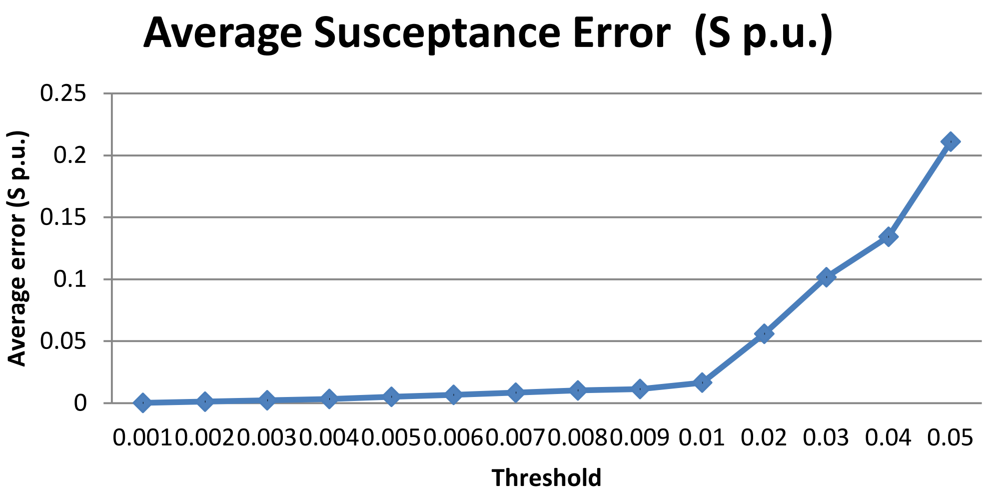
Case No.		1	2	3	4	5	6	7	8	9	10	11	12	13	14
Thresholds		0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.01	0.02	0.03	0.04	0.05
No. of retained rows		13660	12026	10965	10235	9683	9226	8844	8503	8217	7954	6204	5370	4786	4349
No. of retained rows (%)		28.33%	24.94%	22.74%	21.22%	20.08%	19.13%	18.34%	17.63%	17.04%	16.49%	12.86%	11.14%	9.92%	9.02%
Reactance comparison	Max_error (s.p.u.)	0.01	0.08	0.15	0.19	0.32	0.39	0.46	0.50	0.56	0.83	2.01	3.72	4.38	5.89
	Max_error (%)	0.13	0.50	0.57	0.92	1.41	1.79	2.17	2.54	3.11	6.51	34.96	62.98	74.33	78.13
	Avg_error (s.p.u.)	<1e-3	<1e-2	<1e-2	<1e-2	0.01	0.01	0.01	0.01	0.01	0.02	0.06	0.10	0.13	0.21
	Avg_error (%)	<1e-2	0.02	0.03	0.04	0.07	0.10	0.13	0.15	0.17	0.27	1.37	2.27	3.12	4.10
Branch power flow comparison	Max_error (MW)	0.41 (0.01%)	6.18 (0.04%)	8.70 (0.06%)	5.54 (0.17%)	8.81 (0.27%)	15.63 (0.48)	23.21 (0.71%)	26.91 (0.83%)	38.73 (0.29%)	56.98 (0.56%)	685.44 (5.17%)	980.84 (7.4%)	1232.62 (9.3%)	1419.72 (10.72%)
	Max_error (%)	0.15	0.61	1.15	2.01	3.20	5.58	8.24	9.52	11.80	20.26	104.12	165.09	198.76	212.50
	Avg_error (MW)	0.02	0.18	0.29	0.30	0.52	0.65	1.00	1.53	2.06	2.20	24.88	37.46	47.19	52.75
	Avg_error (%)	0.0073	0.03	0.06	0.09	0.15	0.24	0.36	0.45	0.57	0.81	5.96	9.11	11.52	12.80

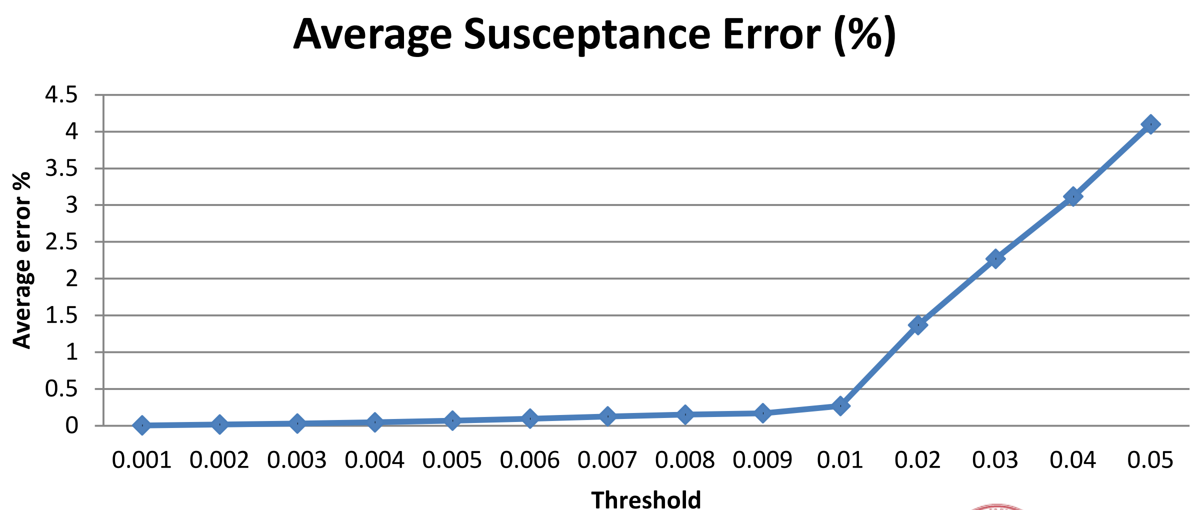
Dimension Reduction of Λ

6073-Bus ERCOT System—Bus Aggregation=Shift Factor Similarity—100 Bus Equivalent



Error in Reduced dc Network Reactance





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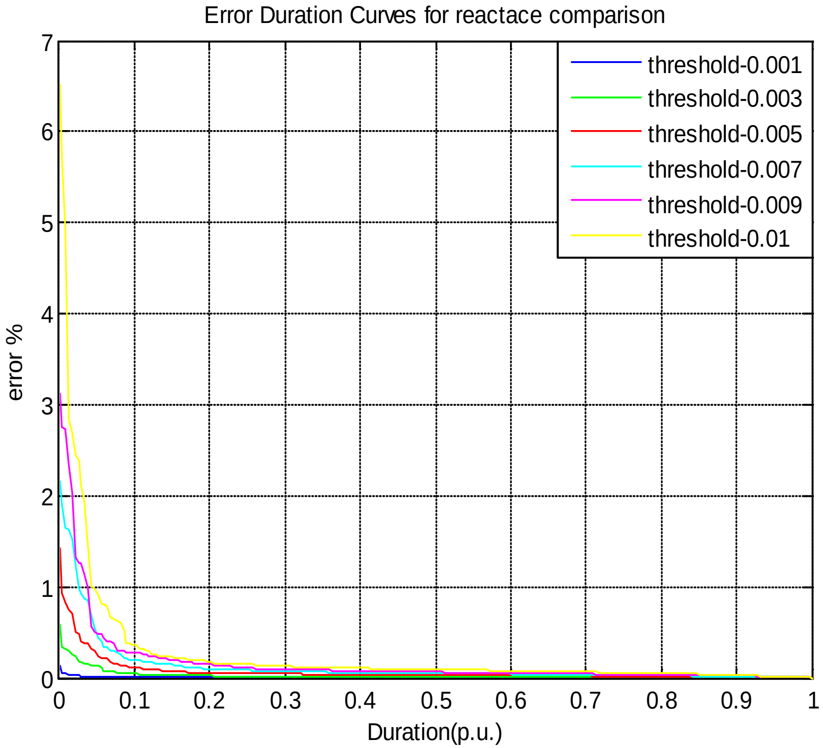
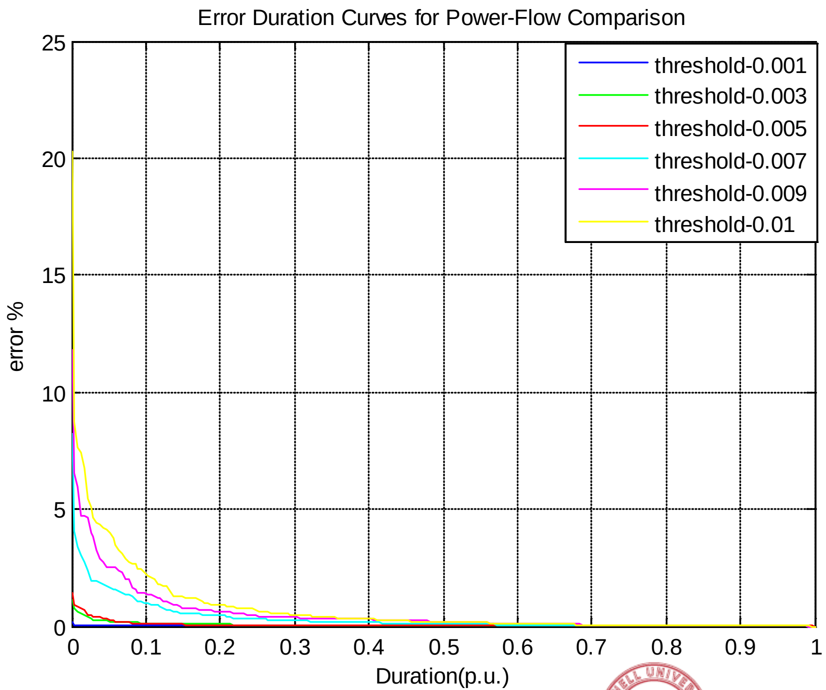
Reactance & Power-Flow Error Duration Curve

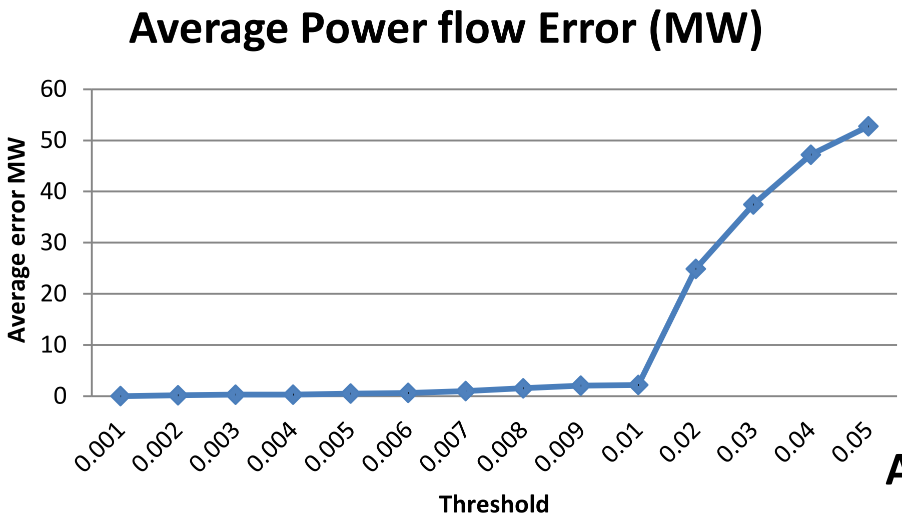
6073-Bus ERCOT System—Bus Aggregation=Shift Factor Similarity—100 Bus Equivalent

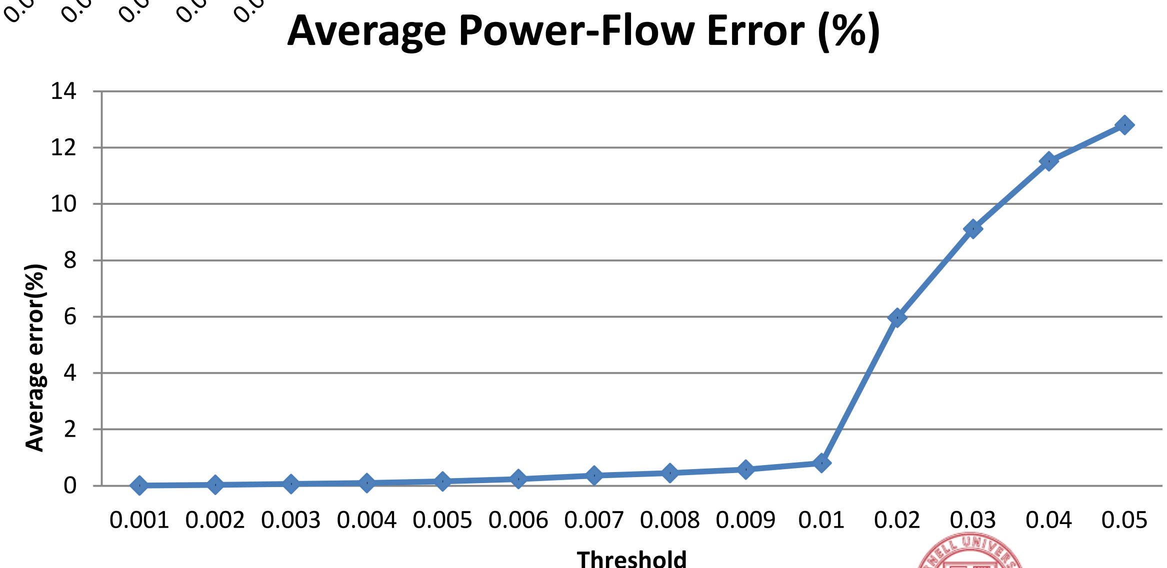




Average Branch Flow Error v. Threshold

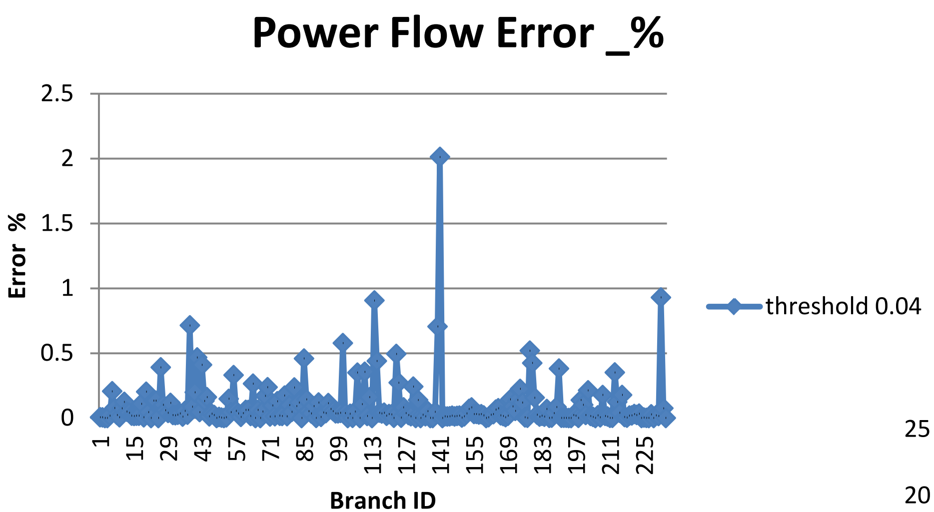
6073-Bus ERCOT System—Bus Aggregation=Shift Factor Similarity—100 Bus Equivalent



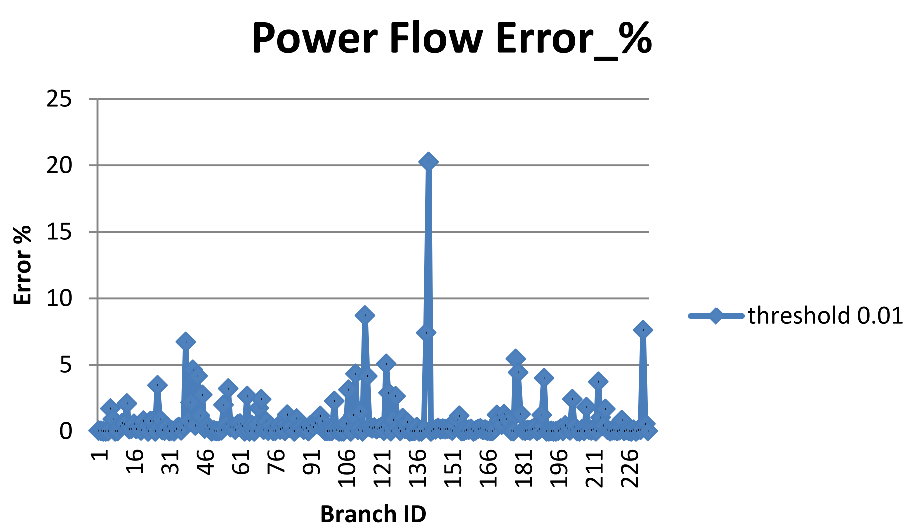


Branch Power Flow Errors

Threshold =0.004



Threshold =0.01



Computation Time v. Threshold

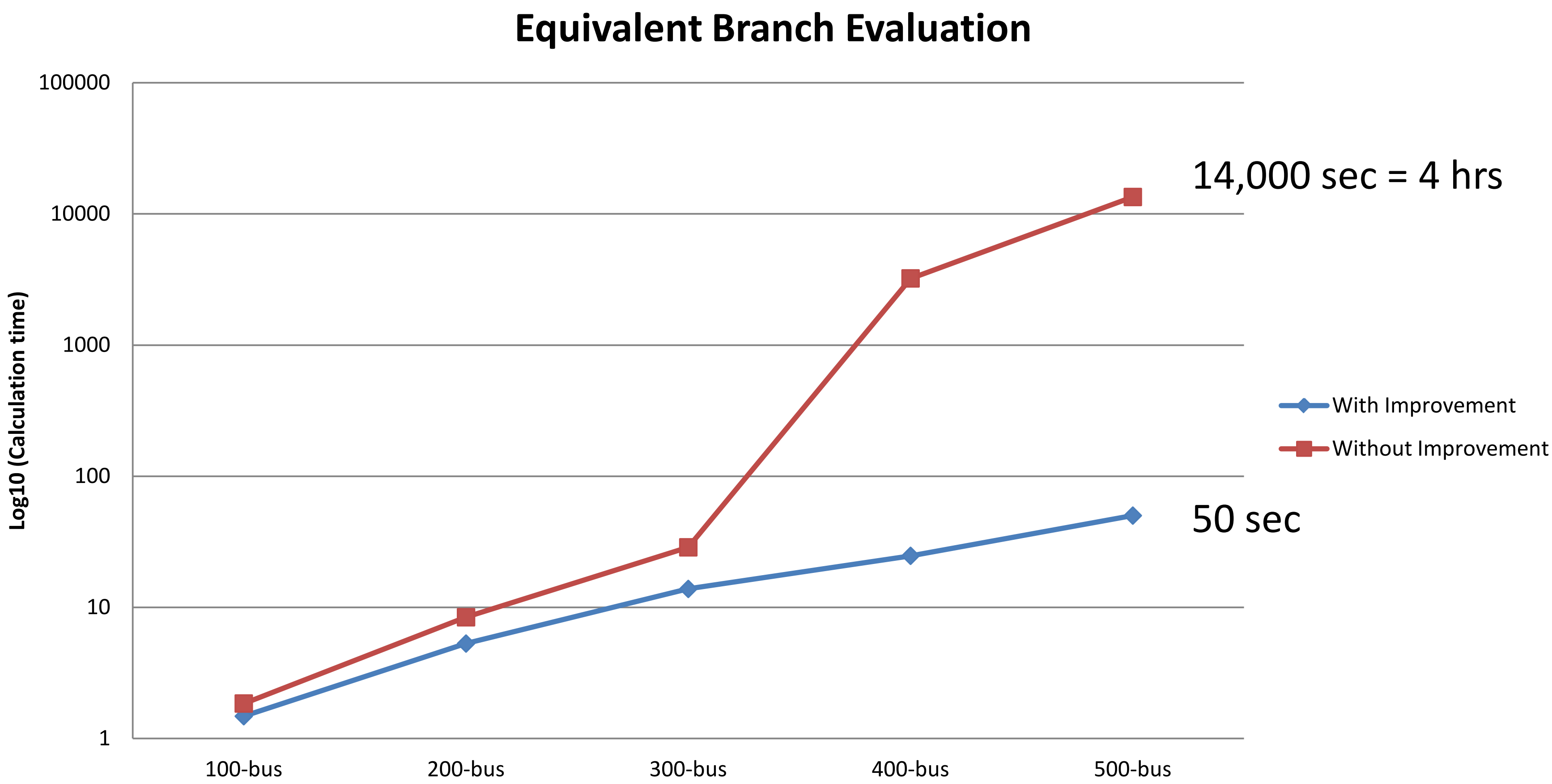
6073-Bus ERCOT System—Bus Aggregation=Shift Factor Similarity

Equivalent Line Evaluation Comp. / Size	Execution Time				Gain	
	Without Threshold (Sec)		With Threshold (Threshold=0.004) (Sec)		Speeding-up Factor	
	(sec*)	Log_{10}	(sec)	Log_{10}	/	Log_{10}
100-bus	1.85	0.267	1.48	0.170	1.25	0.097
200-bus	8.41	0.925	5.3	0.724	1.586	0.201
300-bus	28.71	1.458	13.83	1.141	2.0759	0.317
400-bus	3224.49	3.508	24.73	1.393	130.38	2.115
500-bus	13466.21	4.129	50.11	1.700	268.73	2.429

*These execution times recorded on Intel 3.16 GHz, Core 2 Duo, 4 GB DDR RAM, L2 Cache: 6MB
+Time not recorded. Size of problem required running on parallel processor.

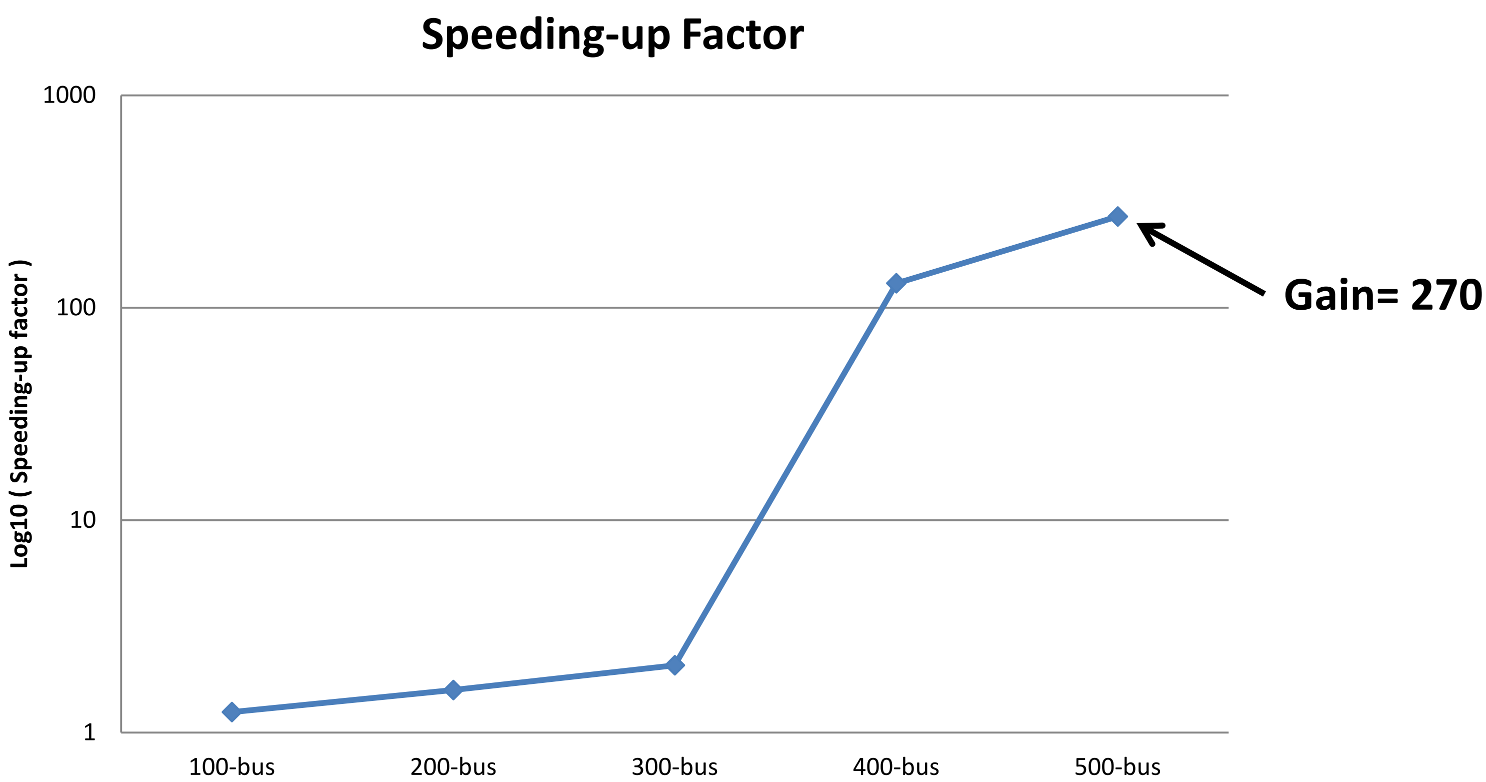
Equivalent Branch Evaluation Gain

6073-Bus ERCOT System



Equivalent Branch Reactance Calculation Gain

6073-Bus ERCOT System



Conclusions

- Using threshold values to eliminate a fraction of the redundant equations in the over-determined dc-network parameter estimation problem:
  - Has negligible effect on line flow accuracy.
  - Yields significant speed advantages.
  - Gives gains that grow exponentially with size of reduced equivalent.

Thrust 3:Improving Bus-Aggregation Based Network Reduction Accuracy

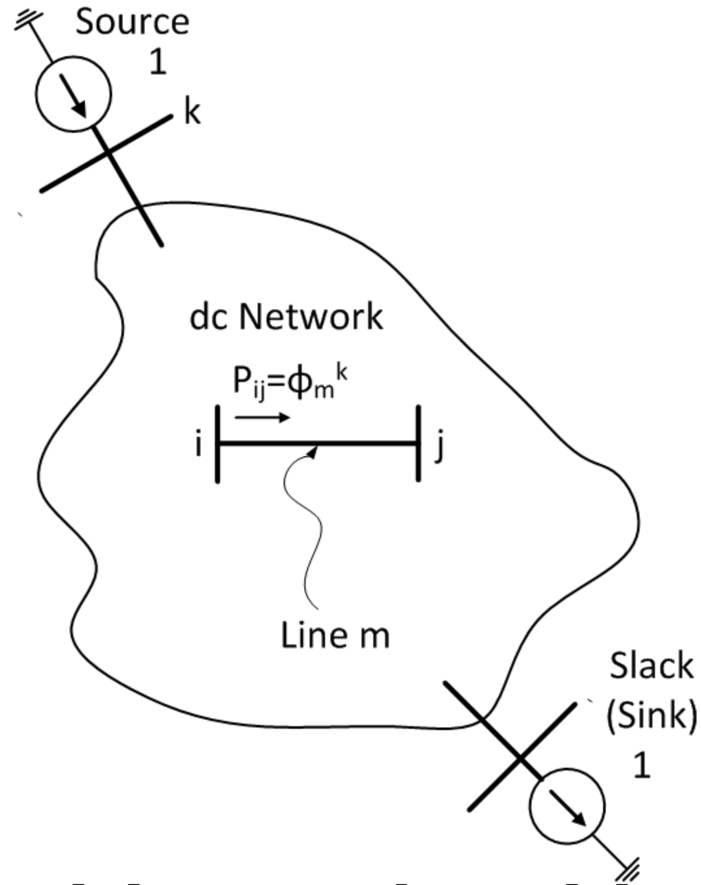
- Thrust 3: Inter-zonal-flow preserving network equivalents.
- Central Concept for Improving Bus Aggregation Techniques for Network Reduction:
  - Don’t use classical dc-model PTDF’s.
  - Calculate accurate ac PTDF’s for unreduced model
  - Calculate “equivalent” dc PTDF’s for unreduced model.
  - Use equivalent dc PTDF’s in the reduction.

Improving Bus Aggregation PTDF's

- PTDF's Nomenclature
  - Classical dc-model PTDF's: $\Phi = B_{branch} B_{bus}^{-1}$
  - ac PTDF's: Account for V, R, sin(θ).
    - Are inconsistent.
  - dc PTDF's: PTDF's created from ac PTDF's by accounting for effect of power loss due to resistance. ⇒ Made consistent (approximately.)

dc PTDF's

- PTDF is an L×N matrix whose coefficients represent the sensitivity of MW flows on every branch due to a unit MW injection at each bus.
- Each column is called a shift factor.



$$
\phi_{dc} = \begin{bmatrix}
\phi_1^1 & \phi_1^2 & \cdots & \phi_1^{N-1} & \phi_1^N \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\phi_i^1 & \phi_i^2 & \cdots & \phi_i^{N-1} & \phi_i^N \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\phi_j^1 & \phi_j^2 & \cdots & \phi_j^{N-1} & \phi_j^N \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\phi_k^1 & \phi_k^2 & \cdots & \phi_k^{N-1} & \phi_k^N \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\phi_L^1 & \phi_L^2 & \cdots & \phi_L^{N-1} & \phi_L^N
\end{bmatrix}_{L\times N}
$$

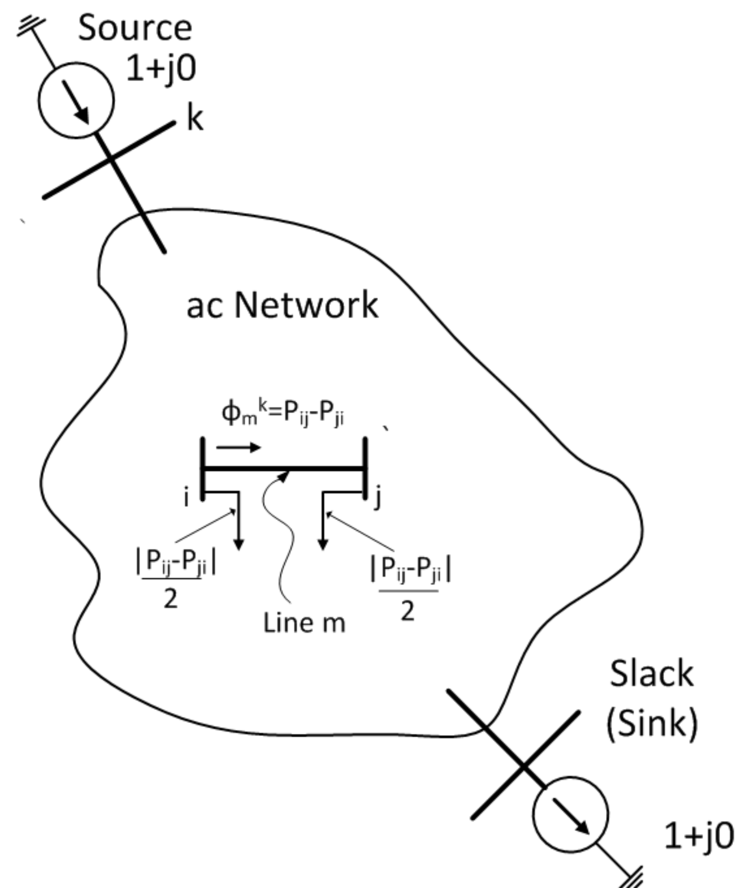
L=# branches
N=# buses

- Classical dc network-model PTDF's easy to calculate.

$$\Phi = B_{branch} B_{bus}^{-1}$$

ac PTDF's

- ac PTDF are a function of V and θ.



- For an incremental injection at a bus, i, the change in voltage (V and θ) is given by the traditional power flow formulation.

$$\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}_{((2N-r)\times 1)} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{bmatrix}_i^{-1} \begin{bmatrix} \Delta P^i \\ \Delta Q \end{bmatrix} = J^{-1}_{((2N-r)\times(2N-r))} \begin{bmatrix} \Delta P^i \\ \Delta Q \end{bmatrix}_{((2N-L)\times 1)}$$

r=# PV buses

$$\begin{array}{ccccc} & 1\ldots i-1, & i, & i+1\ldots N & \\ \Delta P^i = [0 & \ldots 0, & 1, & 0\ldots & 0] \end{array}$$

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ac PTDF's

- For an incremental injection at each bus, one at a time, the change in voltage (V and δ) is given by the $2N$-r × $2N$-r matrix.

$$\begin{bmatrix} \Delta\theta_1 & \Delta\theta_2 \ldots \Delta\theta_N \\ \Delta V_1 & \Delta V_2 \ldots \Delta V_N \end{bmatrix}_{(2N-r)\times N} = J^{-1} \begin{bmatrix} I_{N\times N} \\ 0_{(N-r)\times N} \end{bmatrix}$$

- And then the ac PTDF's are calculated using the differential.

$$\Delta P_{ij} = \frac{\partial P_{ij}}{\partial \delta_i}\Delta\delta_i + \frac{\partial P_{ij}}{\partial \delta_j}\Delta\delta_j + \frac{\partial P_{ij}}{\partial V_i}\Delta V_i + \frac{\partial P_{ij}}{\partial V_j}\Delta V_j$$

$$\phi_{ac} = \begin{bmatrix} \phi_1^{1'} & \phi_1^{2'} & \cdots & \phi_1^{N-1'} & \phi_1^{N'} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \phi_i^{1'} & \phi_i^{2'} & \cdots & \phi_i^{N-1'} & \phi_i^{N'} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \phi_j^{1'} & \phi_j^{2'} & \cdots & \phi_j^{N-1'} & \phi_j^{N'} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \phi_k^{1'} & \phi_k^{2'} & \cdots & \phi_k^{N-1'} & \phi_k^{N'} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \phi_L^{1'} & \phi_L^{2'} & \cdots & \phi_L^{N-1'} & \phi_L^{N'} \end{bmatrix}_{L\times}$$

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dc PTDF's

- **ac PTDF's are not the best for dc network reduction because they are not consistent because of losses.**
  - Consistency: Sum of PTDF's into each non-slack/non-source bus must be 0.
  - PTDF on one end of a branch should equal that on the other end.
- **Get dc PTDF's from ac PTDF's.**

$$\begin{bmatrix} \phi_1^{i'} \\ \vdots \\ \phi_i^{i'} \\ \vdots \\ \phi_j^{i'} \\ \vdots \\ \phi_k^{i'} \\ \vdots \\ \phi_L^{i'} \end{bmatrix}_{(L\times 1)} = \overbrace{\begin{bmatrix} \Delta P_1^i & 0 & \cdots & 0 & \Delta P_2^i & 0 & \cdots & 0 & \cdots & \Delta P_N^i & 0 & \cdots & 0 \\ 0 & \Delta P_1^i & \vdots & 0 & \Delta P_2^i & 0 & \vdots & \cdots & 0 & \Delta P_N^i & 0 & \vdots & \\ & \ddots & 0 & \vdots & 0 & \ddots & 0 & \cdots & \vdots & 0 & \ddots & 0 & \\ 0 & 0 & \Delta P_1^i & 0 & \cdots & \Delta P_2^i & \cdots & 0 & \cdots & 0 & \Delta P_N^i & & \end{bmatrix}}^{L \text{ by } N \bullet L} \left.\begin{bmatrix} \phi_1^1 \\ \vdots \\ \phi_k^1 \\ \vdots \\ \phi_1^2 \\ \vdots \\ \phi_k^2 \\ \vdots \\ \vdots \\ \phi_l^{N-1} \\ \vdots \\ \phi_k^{N-1} \\ \vdots \end{bmatrix}_{(L\bullet N\times 1)}\right\} N \times L$$

$\phi_{dc} = \psi^{-1}\phi_{ac}$

dc PTDF's

$$
\begin{bmatrix} \phi_1^{i'} \\ \vdots \\ \phi_i^{i'} \\ \vdots \\ \phi_j^{i'} \\ \vdots \\ \phi_k^{i'} \\ \vdots \\ \phi_L^{i'} \end{bmatrix}_{(L\times 1)}
=
\overbrace{\begin{bmatrix}
\Delta P_1^i & 0 & \cdots & 0 & \Delta P_2^i & 0 & \cdots & 0 & \cdots & \Delta P_N^i & 0 & \cdots & 0 \\
0 & \Delta P_1^i & \vdots & 0 & \Delta P_2^i & 0 & \vdots & \cdots & 0 & \Delta P_N^i & 0 & \vdots & \\
 & \ddots & 0 & \vdots & 0 & \ddots & 0 & \cdots & \vdots & 0 & \ddots & 0 & \\
0 & 0 & \Delta P_1^i & 0 & \cdots & \Delta P_2^i & \cdots & 0 & \cdots & 0 & \Delta P_N^i & &
\end{bmatrix}}^{L \ \ by\ N\bullet L}
\left.\begin{bmatrix} \phi_1^1 \\ \vdots \\ \phi_k^1 \\ \vdots \\ \phi_1^2 \\ \vdots \\ \phi_k^2 \\ \vdots \\ \vdots \\ \phi_l^{N-1} \\ \vdots \\ \phi_k^{N-1} \\ \vdots \end{bmatrix}_{(L\bullet N\times 1)}\right\} N \times L
$$

- The solution to this equation gives a more *consistent* shift factor for bus *i*.

$$\phi^i{}_{ac(L)} = \psi^i{}_{(L\times N\bullet L)}\ \phi_{dc(N\bullet L)\times 1)}$$

- Add simultaneous equations for all shift factors.

$$\phi_{dc} = \psi^{-1}\, \phi_{ac}$$

- Large sparse matrix equation solution.

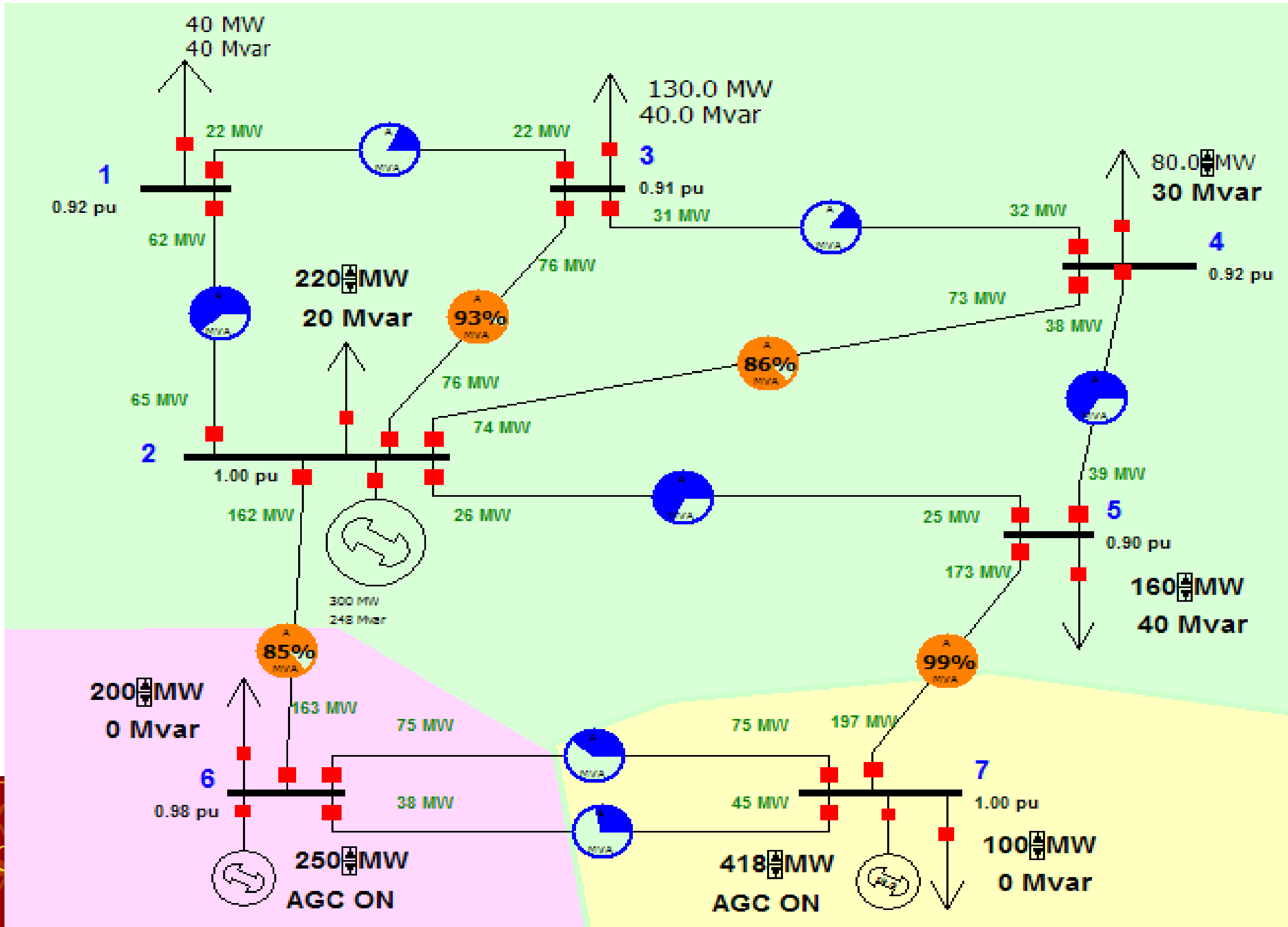
$$
\begin{bmatrix}
\Phi_1'^{1} \\ \Phi_2'^{1} \\ \vdots \\ \Phi_L'^{1} \\ \Phi_1'^{2} \\ \Phi_2'^{2} \\ \vdots \\ \Phi_L'^{2} \\ \vdots \\ \vdots \\ \Phi_1'^{N} \\ \Phi_2'^{N} \\ \vdots \\ \Phi_L'^{N}
\end{bmatrix}
=
\begin{bmatrix}
(1-\Delta P_1^1) \;\; -\Delta P_2^1 \;\; -\Delta P_3^1 \;\; \cdots \;\; -\Delta P_N^1 & 0 & \cdots & 0 \\
0 & (1-\Delta P_1^1) \;\; -\Delta P_2^1 \;\; -\Delta P_3^1 \;\; \cdots \;\; -\Delta P_N^1 & 0 \;\; \cdots & 0 \\
\vdots & \ddots & & \vdots \\
0 & \cdots \;\; 0 & & (1-\Delta P_1^1) \;\; -\Delta P_2^1 \;\; -\Delta P_3^1 \;\; \cdots \;\; -\Delta P_N^1 \\
-\Delta P_1^2 \;\; (1-\Delta P_2^2) \;\; -\Delta P_3^2 \;\; \cdots \;\; -\Delta P_N^2 & 0 & \cdots & 0 \\
0 & -\Delta P_1^2 \;\; (1-\Delta P_2^2) \;\; -\Delta P_3^2 \;\; \cdots \;\; -\Delta P_N^2 & 0 \;\; \cdots & 0 \\
\vdots & \ddots & & \vdots \\
0 & \cdots \;\; 0 & & -\Delta P_1^2 \;\; (1-\Delta P_2^2) \;\; -\Delta P_3^2 \;\; \cdots \;\; -\Delta P_N^2 \\
\vdots & \vdots & \cdots & \vdots \\
\vdots & \vdots & \cdots & \vdots \\
-\Delta P_1^N \;\; -\Delta P_2^N \;\; -\Delta P_3^N \;\; \cdots \;\; (1-\Delta P_N^N) & 0 & \cdots & 0 \\
0 & -\Delta P_1^N \;\; -\Delta P_2^N \;\; -\Delta P_3^N \;\; \cdots \;\; (1-\Delta P_N^N) & 0 \;\; \cdots & 0 \\
\vdots & \ddots & & \vdots \\
0 & \cdots \;\; 0 & & -\Delta P_1^N \;\; -\Delta P_2^N \;\; -\Delta P_3^N \;\; \cdots \;\; (1-\Delta P_N^N)
\end{bmatrix}
\begin{bmatrix}
\Phi_1^{1} \\ \Phi_1^{2} \\ \vdots \\ \Phi_1^{N} \\ \Phi_2^{1} \\ \Phi_2^{2} \\ \vdots \\ \Phi_2^{N} \\ \vdots \\ \vdots \\ \Phi_L^{1} \\ \Phi_L^{2} \\ \vdots \\ \Phi_L^{N}
\end{bmatrix}
$$

Numerical Experiments

- Steps in the reduction
  - Start with unreduced ac network
  - Calculate ac PTDF’s accounting for V and θ
  - Calculate equivalent dc PTDF’s (approximate)
  - Calculate optimum dc unreduced equivalent (aggregate buses individually)
  - Evaluate performance.
  - Calculate dc network PTDF’s from the optimized unreduced network.
  - Calculate bus cluster
  - Calculate reduced equivalent network using the updated dc PTDF’s.
  - Evaluate performance.

7-Bus System—Base-Case Load

Unreduced



7 Bus Unreduced Equivalent
Optimal dc Reactance Nominal Load Conditions

Optimal dc Reactance under Nominal Load Conditions

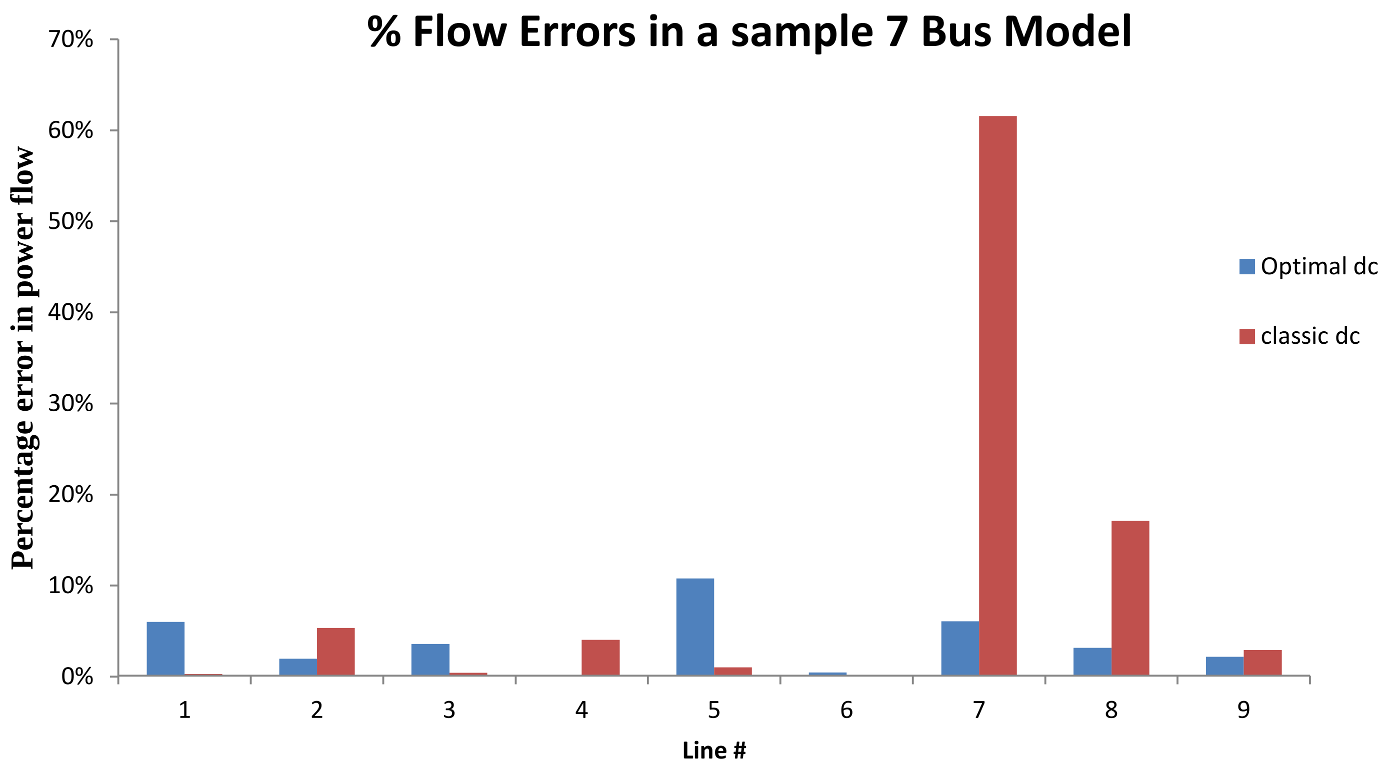
Line from	Line to	R (p.u.)	X (p.u.)	Optimal dc reactance scaled (p.u.)	Difference in reactance (p.u.)	Difference in reactance (%)
1	3	0.02	0.240	0.221	-0.019	-7.76%
1	2	0.06	0.150	0.173	0.023	15.38%
2	6	0.01	0.060	0.054	-0.006	-10.45%
2	5	0.01	0.120	0.114	-0.006	-5.30%
2	4	0.02	0.180	0.169	-0.011	-6.32%
2	3	0	0.180	0.185	0.005	2.52%
3	4	0.02	0.030	0.030	0.000	0.00%
4	5	0.02	0.240	0.248	0.008	3.18%
6	7	0.25	0.250	0.467	0.217	86.63%
6	7	0	0.250	0.232	-0.018	-7.14%
7	5	0.06	0.150	0.143	-0.007	-4.47%

7 Bus Unreduced Equivalent

Optimal dc Reactance Nominal Load Conditions

Branch data p.u.				Branch flows (MW)			Flow errors			
Line from	Line to	R	X	ac (from)	Classic dc	Optima l dc	Classic dc (MW)	%	Optimal dc (MW)	%
1	3	0.02	0.24	21.27	20.48	17.59	0.79	3.70%	3.67	17.27%
1	2	0.06	0.15	-61.27	-61.10	-57.59	0.17	0.27%	3.67	6.00%
2	6	0.005	0.06	- 152.48	-160.61	-149.49	8.13	5.33%	2.99	1.96%
2	5	0.01	0.12	20.25	25.21	21.22	4.96	24.48%	0.97	4.77%
2	4	0.015	0.18	72.96	72.67	75.57	0.29	0.40%	2.61	3.57%
2	3	0	0.18	75.18	78.22	75.11	3.04	4.04%	0.07	0.09%
3	4	0.015	0.03	-33.66	-33.32	-37.29	0.34	1.01%	3.63	10.80%
4	5	0.02	0.24	-41.92	-41.90	-41.72	0.02	0.05%	0.19	0.46%
6	7	0.25	0.25	-35.19	-56.86	-33.06	21.67	61.56%	2.14	6.07%
6	7	0	0.25	-68.59	-56.86	-66.43	11.73	17.11%	2.16	3.15%
7	5	0.06	0.15	184.53	179.18	180.51	5.36	2.90%	4.02	2.18%

7 Bus Unreduced Equivalent Optimal dc Reactance Nominal Load Conditions



7 Bus Unreduced Equivalent Optimal dc Reactance Nominal Load Conditions

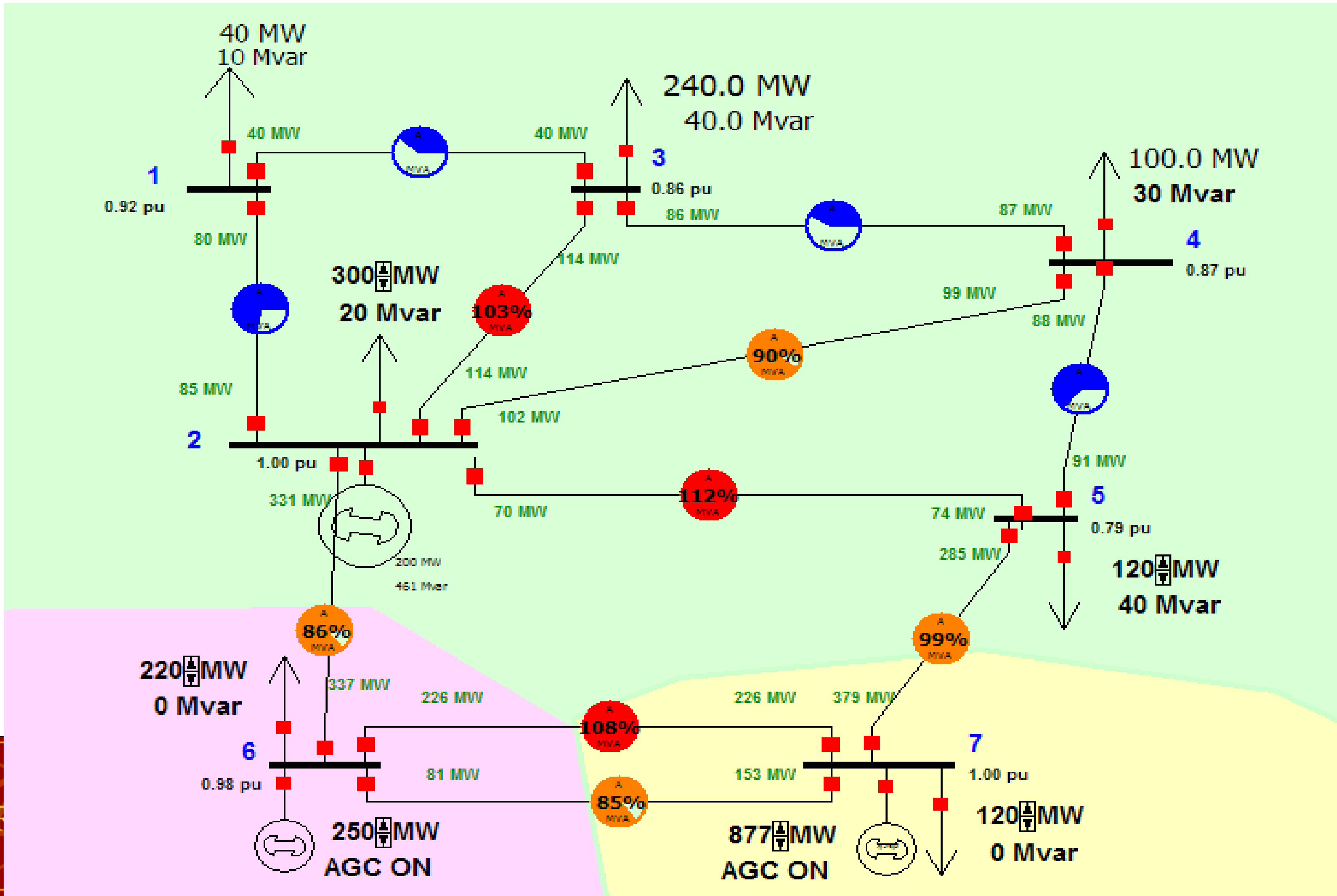
Nominal Load Case-Branch Flow Errors Aggregate results*

	Classic dc	Optimal dc
Maximum Error (MW)	21.67	4.02
Average Error (MW)	5.64	2.39
Average Error (%)	10.30%	3.81%
Maximum Error (%)	61.56%	10.80%

*Neglecting small branch flows

7 Bus Unreduced Equivalent

Optimal dc Reactance Heavily Loaded Conditions



7 Bus Unreduced Equivalent
Optimal dc Reactance Heavily Loaded Conditions

Optimal dc Reactance under Heavily Loaded Conditions

Line from	Line to	R	X	Optimal dc reactance	Difference in reactance (p.u.)	Difference in reactance (%)
1	3	0.02	0.240	0.231	-0.009	-3.54%
1	2	0.06	0.150	0.150	0.000	-0.08%
2	6	0.01	0.060	0.063	0.003	4.72%
2	5	0.01	0.120	0.125	0.005	4.00%
2	4	0.02	0.180	0.178	-0.002	-1.36%
2	3	0	0.180	0.185	0.005	2.95%
3	4	0.02	0.030	0.030	0.000	0.00%
4	5	0.02	0.240	0.285	0.045	18.86%
6	7	0.25	0.250	0.690	0.440	176.17%
6	7	0	0.250	0.318	0.068	27.13%
7	5	0.06	0.150	0.201	0.051	34.04%

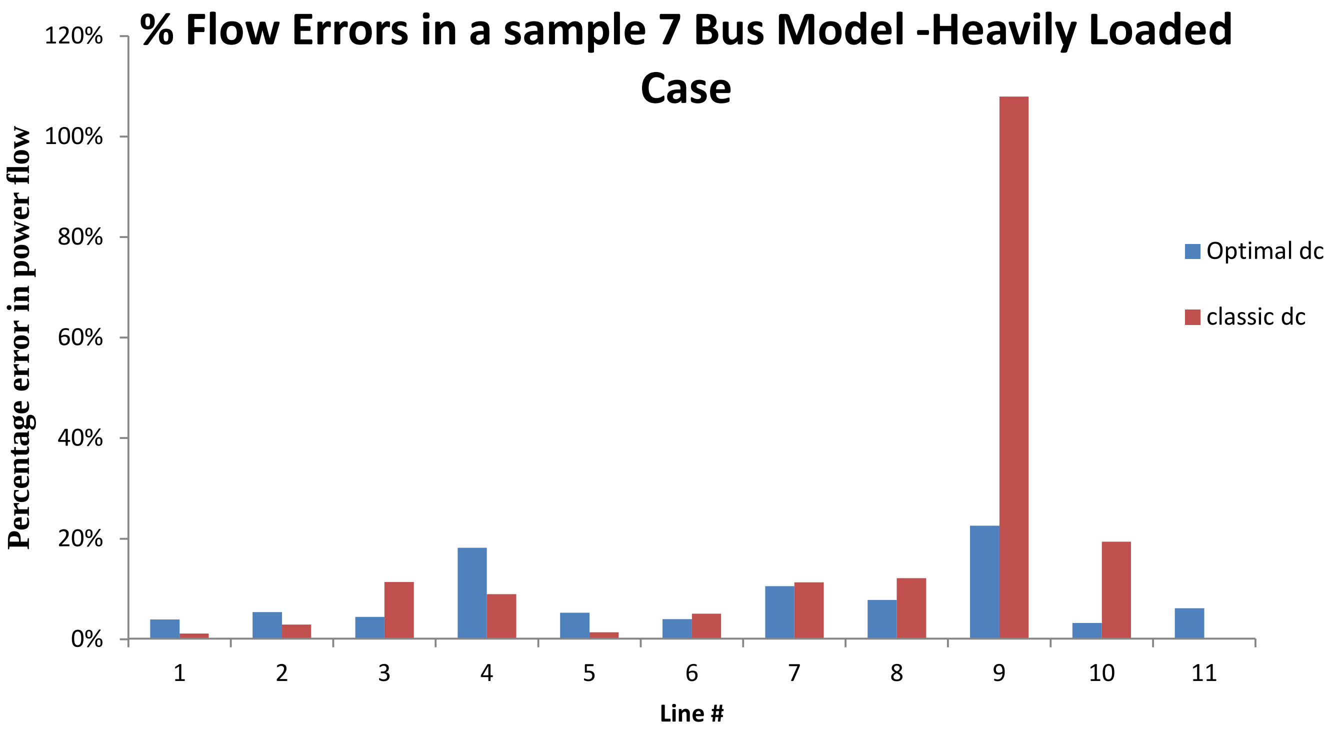
7 Bus Unreduced Equivalent
Optimal dc Reactance Heavily Loaded Conditions

Heavily Loaded Conditions-Comparison between Classic dc and Optimal dc Flow Errors

Branch data p.u.				Branch flows (MW)			Flow errors			
							Classic dc		Optimal dc	
Line from	Line to	R	X	ac (from)	Classic dc	Optimal dc	(MW)	%	(MW)	%
1	3	0.02	0.24	36.95	36.52	38.40	0.43	1.17%	1.45	3.92%
1	2	0.06	0.15	-76.95	-79.22	-81.10	2.27	2.95%	4.15	5.39%
2	6	0.005	0.06	-294.73	-328.25	-307.88	33.52	11.37%	13.15	4.46%
2	5	0.01	0.12	-91.31	-83.13	-107.94	8.18	8.96%	16.63	18.21%
2	4	0.015	0.18	95.86	97.22	100.92	1.36	1.42%	5.07	5.29%
2	3	0	0.18	109.18	114.71	113.57	5.53	5.07%	4.39	4.02%
3	4	0.015	0.03	-94.26	-104.95	-104.22	10.70	11.35%	9.96	10.57%
4	5	0.02	0.24	-102.06	-114.48	-110.04	12.42	12.17%	7.98	7.81%
6	**7**	**0.25**	**0.25**	**-75.27**	**-156.55**	**-92.27**	**81.27**	**107.97%**	**17.00**	**22.58%**
6	**7**	**0**	**0.25**	**-194.17**	**-156.55**	**-200.45**	**37.63**	**19.38%**	**6.27**	**3.23%**
7	5	0.06	0.15	325.89	325.70	346.08	0.19	0.06%	20.18	6.19%

7 Bus Unreduced Equivalent Optimal dc Reactance Heavily Loaded Conditions

Heavily Loaded Case-Percentage Branch Flow Errors



7 Bus Unreduced Equivalent
Optimal dc Reactance Heavily Loaded Conditions

Heavily Loaded Case-Branch Flow Errors Aggregate Results

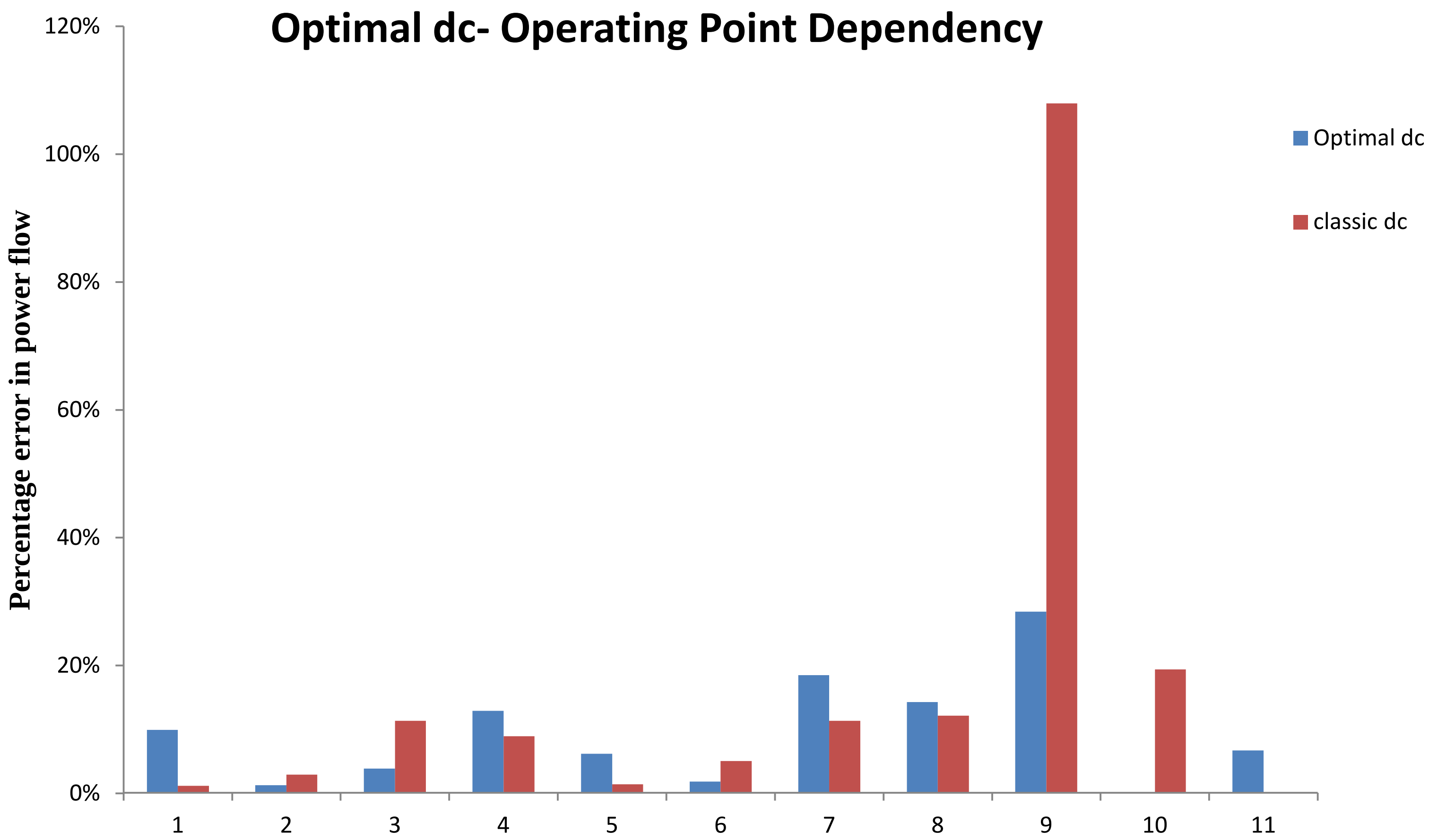
	Classic dc	Optimal dc
Maximum Error (MW)	81.27	20.18
Average Error (MW)	17.59	9.66
Maximum Error (%)	107.97%	22.58%
Average Error (%)	16.53%	8.33%

7 Bus Unreduced Equivalent

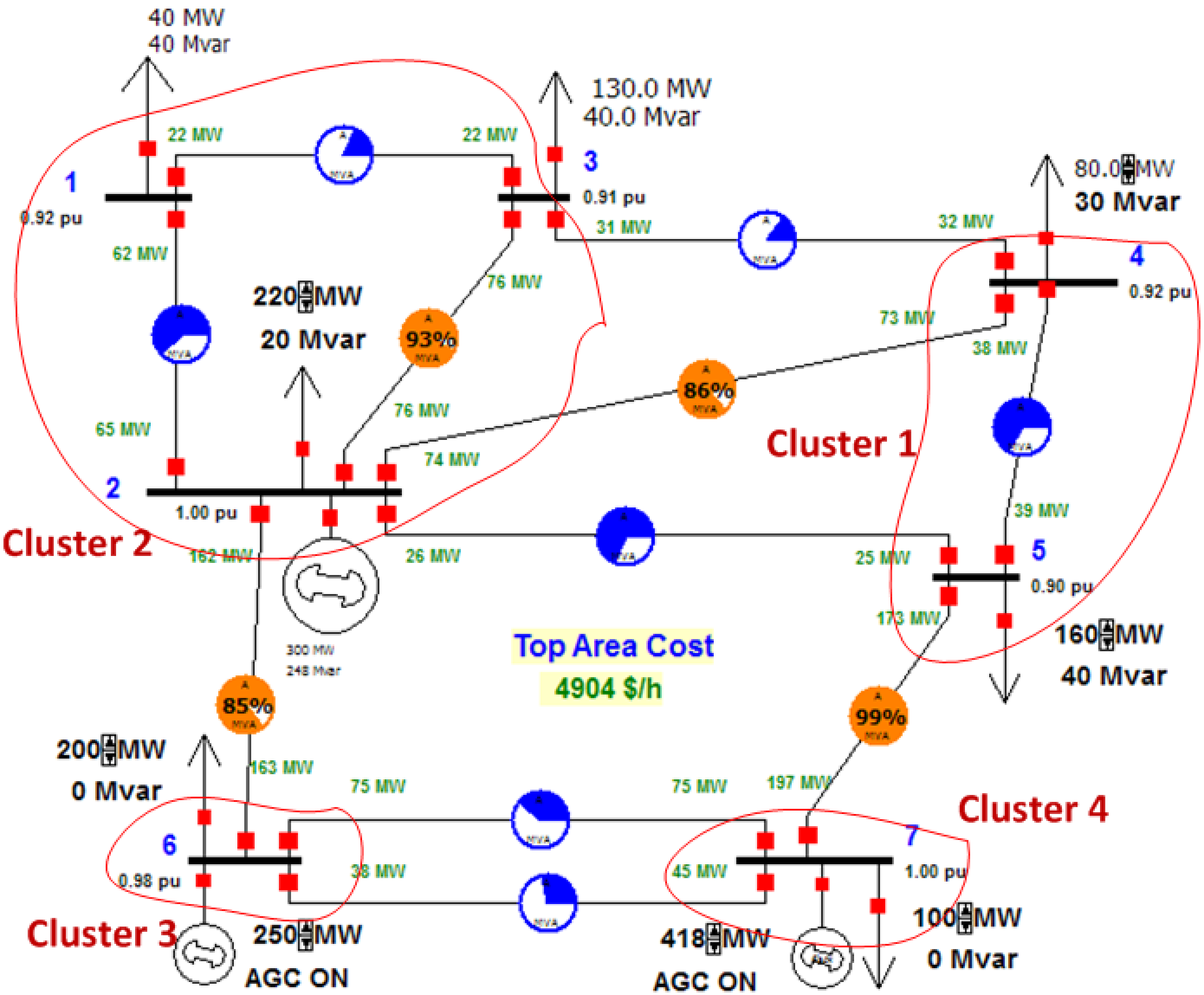
Optimal dc Reactance Nominal v Heavily Loaded Conditions

Line from	Line to	R	X	Opt. dc Reactance (p.u.)		Difference in Reactance (p.u.)
				Nom. Load	Heavy Load	
1	3	0.02	0.240	0.221	0.231	-13%
1	2	0.06	0.150	0.173	0.150	17%
2	6	0.01	0.060	0.054	0.063	10%
2	5	0.01	0.120	0.114	0.125	5%
2	4	0.02	0.180	0.169	0.178	0%
2	3	0	0.180	0.185	0.185	0%
3	4	0.02	0.030	0.030	0.030	15%
4	5	0.02	0.240	0.248	0.285	48%
6	7	0.25	0.250	0.467	0.690	37%
6	7	0	0.250	0.232	0.318	41%
7	5	0.06	0.150	0.143	0.201	-13%

7 Bus Unreduced Equivalent Operating Point Dependence Nominal Model with Heavily Loaded Conditions



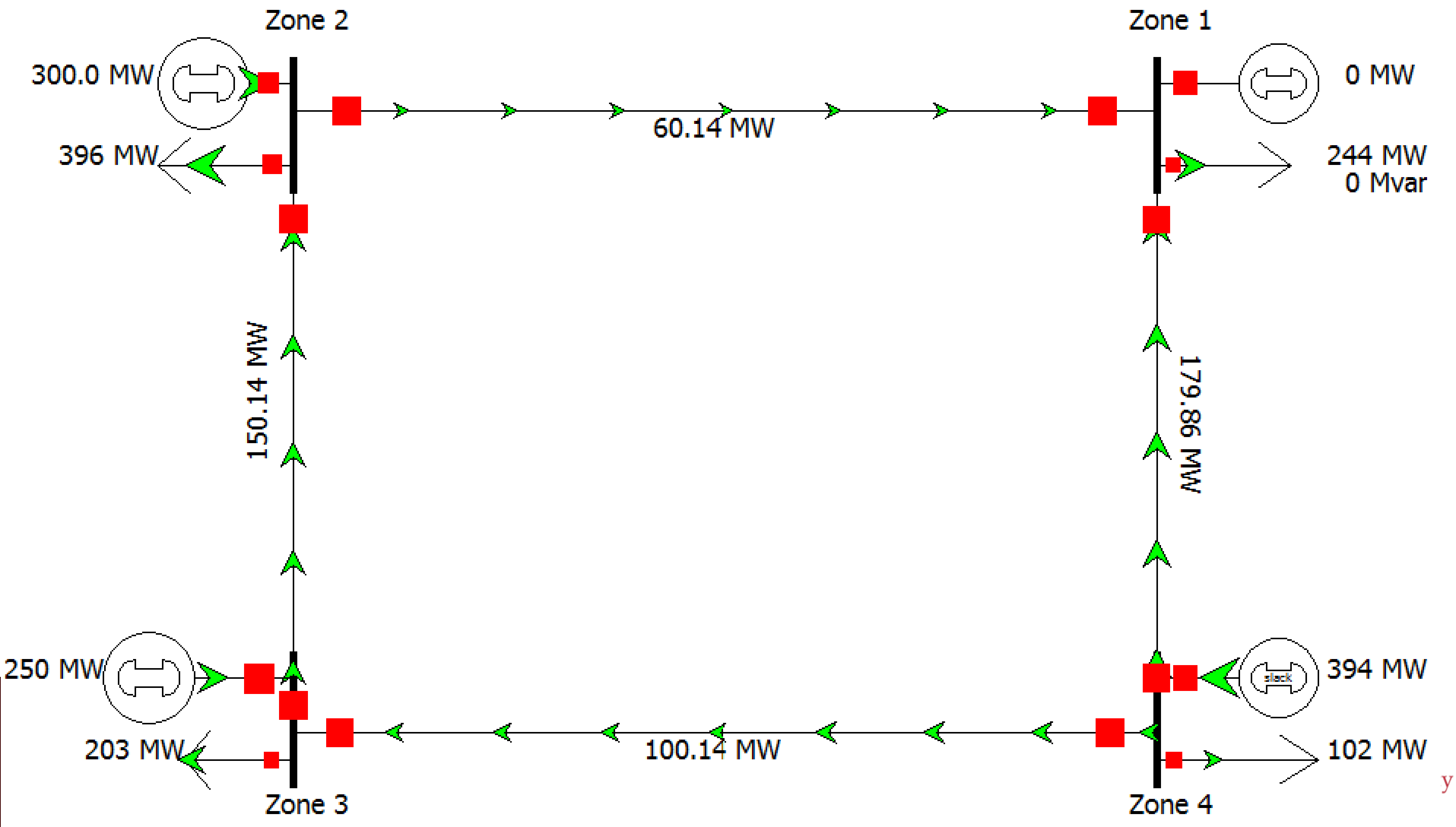
7→ 4 Bus *Reduced* Equivalent
Optimal dc Reactance Heavily Loaded Conditions



Cornell University

7→ 4 Bus *Reduced* Equivalent

Optimal dc Reactance Heavily Loaded Conditions



7→ 4 Bus Reduced Equivalent
Optimal dc Reactance Heavily Loaded Conditions

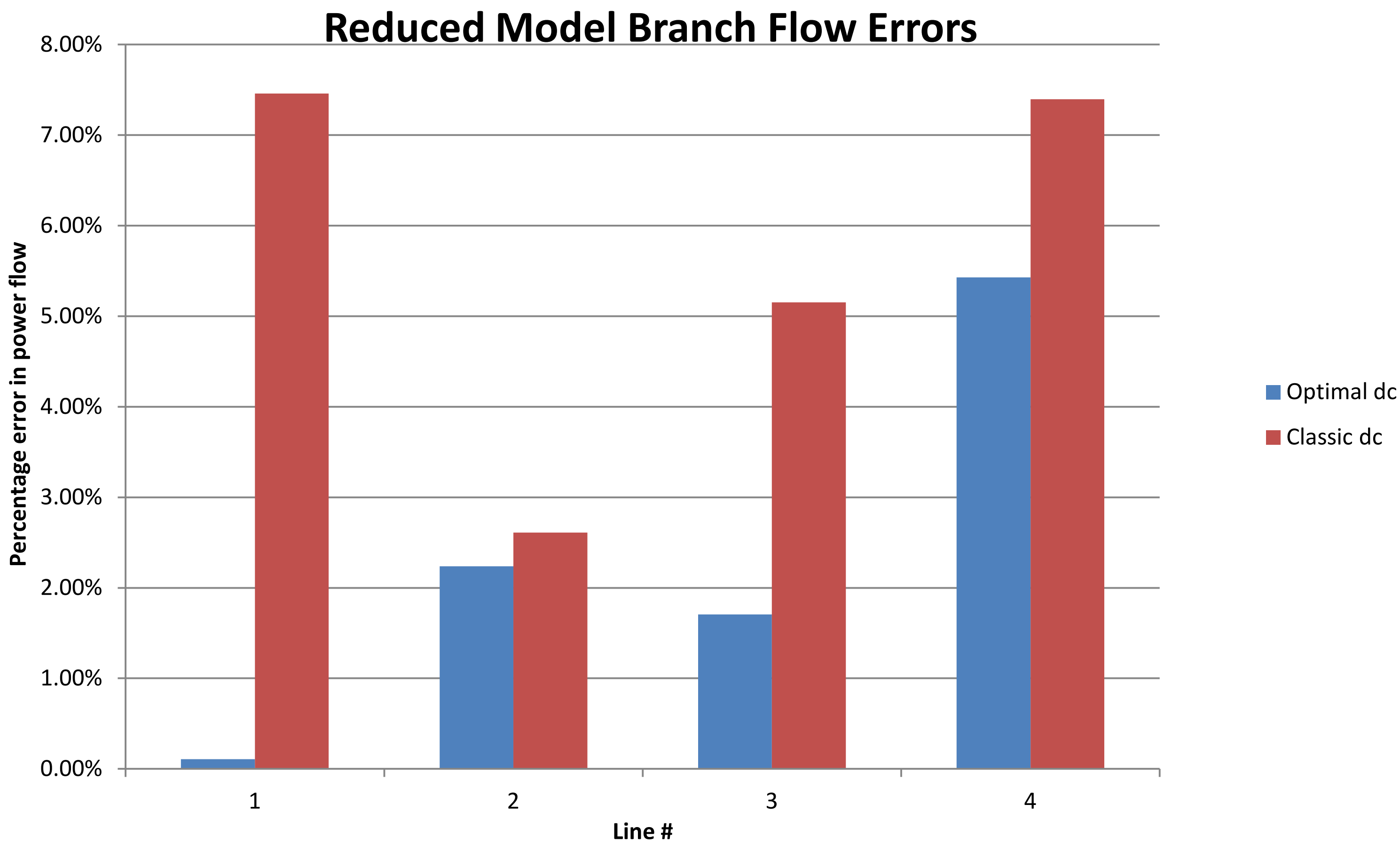
7→4 Bus Reduced Model Comparison of Power Flow Errors between Classic dc and Optimal dc

Branch data p.u.		Branch flows (MW)			Flow errors			
Line from	Line to	ac (from)	Classic dc	Optimal dc	Classic dc (MW)	Classic dc %	Optimal dc (MW)	Optimal dc %
1	2	-60.071	-64.553	-60.1361809	4.48	7.46%	0.06	0.11%
1	4	-183.98	-179.175	-179.8637931	4.80	2.61%	4.12	2.24%
2	3	-152.74	-160.611	-150.1362059	7.87	5.15%	2.60	1.71%
3	4	-105.89	-113.718	-100.1362059	7.83	7.40%	5.75	5.43%

7→4 Bus Reduced Model Comparison of Power Flow Errors Aggregated Results

	Classic dc	Optimal dc
Maximum Error (MW)	7.87	4.11
Average Error (MW)	6.34	3.36
Maximum Error (%)	7.40%	5.43%
Average Error (%)	5.66%	2.37%

7→ 4 Bus Reduced Equivalent Optimal dc Reactance Heavily Loaded Conditions



IEEE 18-Bus Unreduced Equivalent

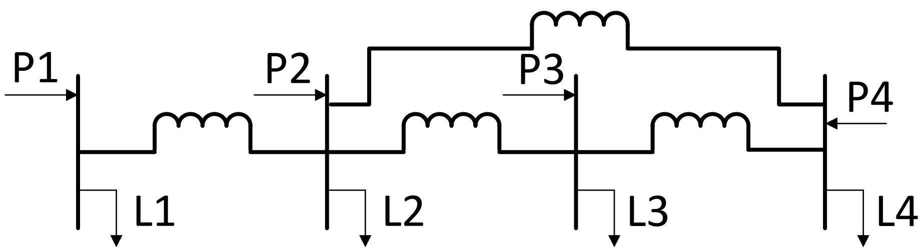
- 118 bus network example (IEEE network)
  - – Classical dc model
  - – Optimum dc model
  - – Flow errors

Some Subtle Points

- Overdetermined matrix equation for reactances.

$$\Lambda_{(N\bullet L\times L)}\,[1/x_R]=[0]$$

- Λ is singular
- dc model is current divider network

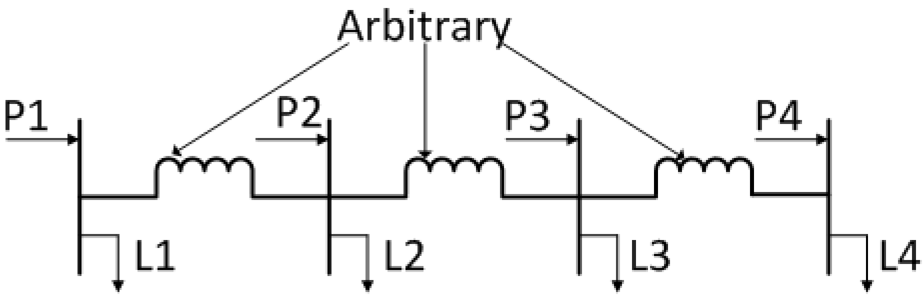


- All reactances scaled by same multiplier give same current division.
- Rank deficient is (L-1)

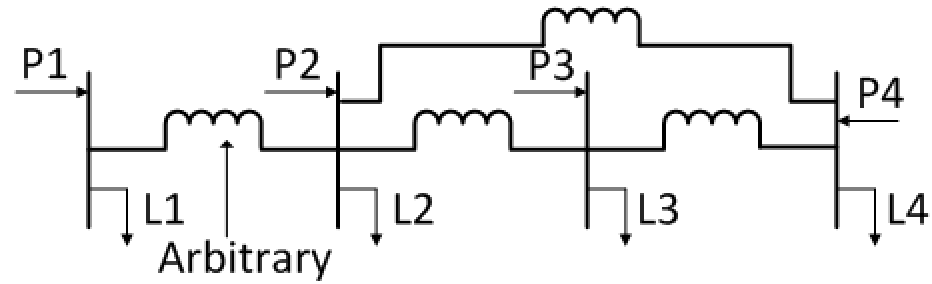
$$\Lambda^{*}[1/x_R]=\begin{bmatrix} P_{i->j}/\theta^{*}_{ij} \\ 0 \end{bmatrix} \qquad 1/x_R=\left[(\Lambda^{*})^{T}\Lambda^{*}\right]^{-1}\left(\Lambda^{*}\right)^{T}\begin{bmatrix} P_{i->j}/\theta^{*}_{ij} \\ 0 \end{bmatrix}$$

Some Subtle Points

- Topology may reduce rank of Λ further.



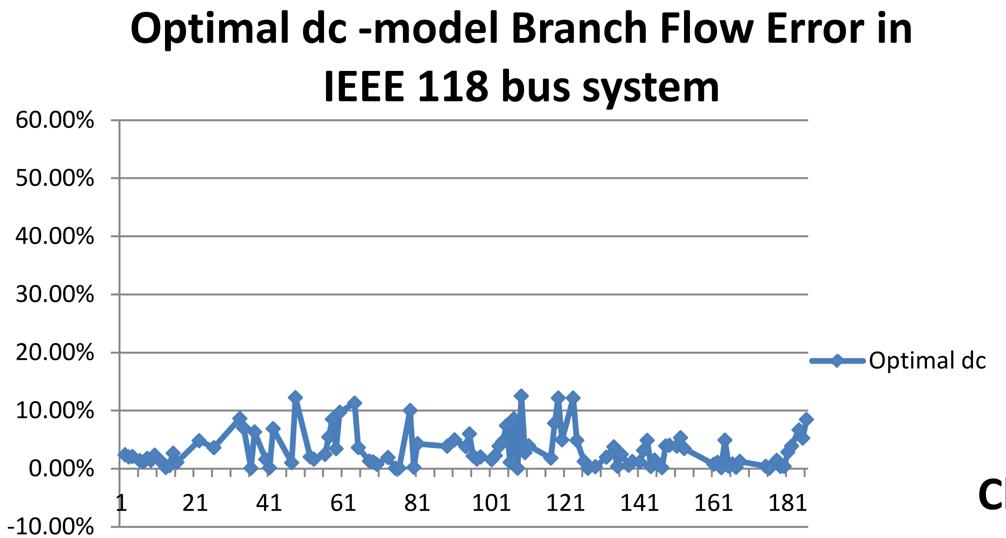
Rank= 3-3=0



Rank=4-2=2

- Must add arbitrary constraints for:
  - All elements in a radial branches.
  - One other element.

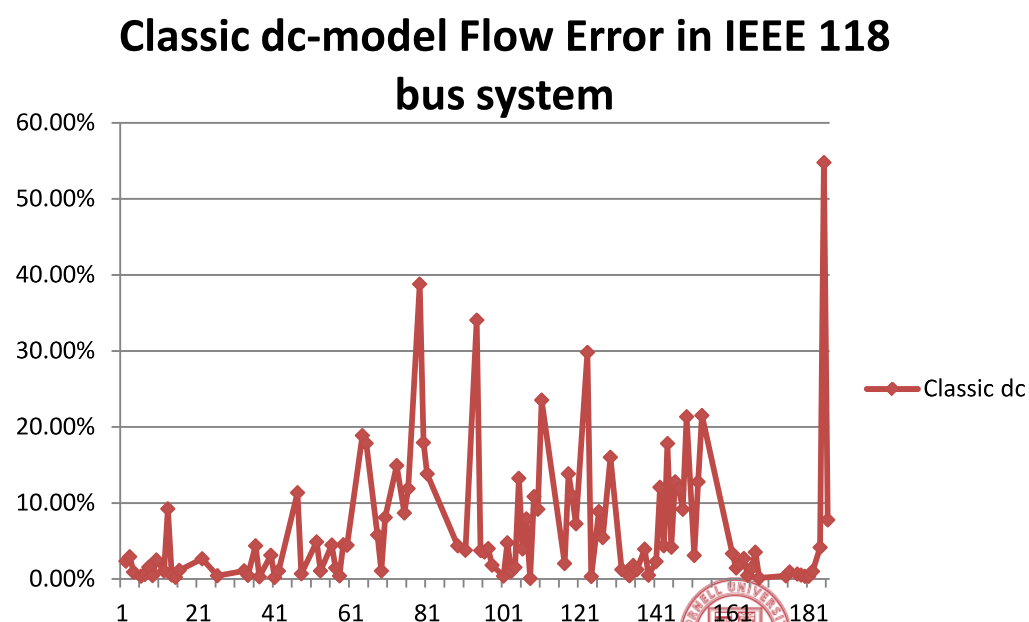
118 Bus Unreduced Equivalent Optimal v. Classical dc-Model Reactance Nominal Load



Nominal Load Case-Branch Flow Errors Aggregate results*

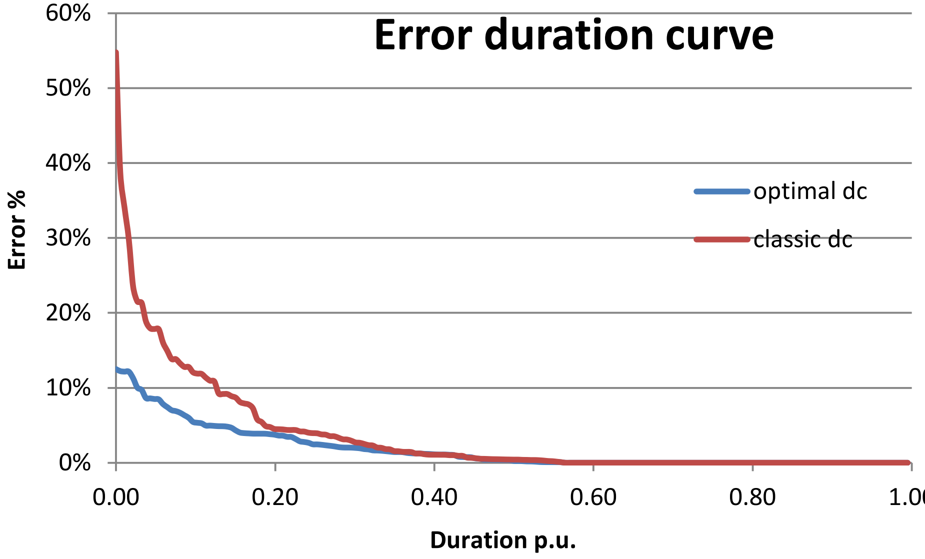
	Optimal dc	Classical dc
Maximum Error (MW)	13.15	17.37
Average Error (MW)	1.26	2.18
Maximum Error (%)	12.51%	54.77%
Average Error (%)	1.84%	3.68%

*Neglecting small branch flows

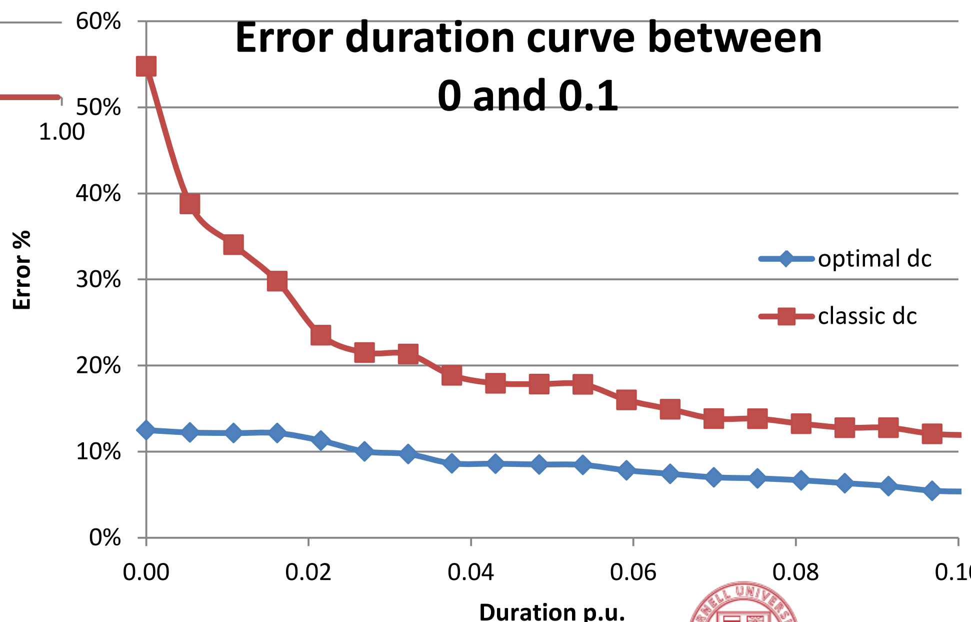


118 Bus Unreduced Equiv alent Optimal v. Classical dc-Model Nominal Load

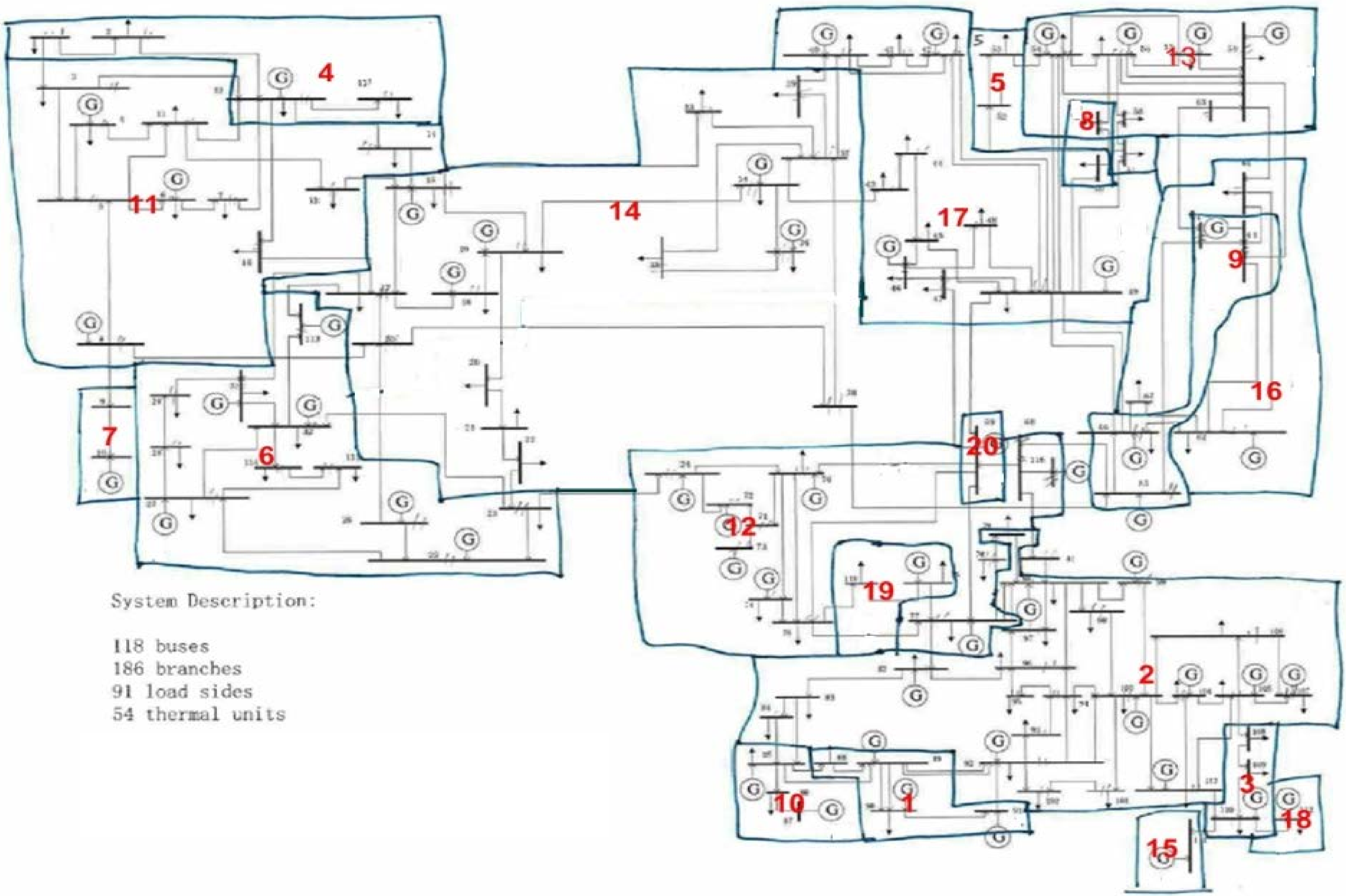
Error duration curves for Power Flow Comparison-Nominal case



Error duration curves between 0 and 0.1 for Power Flow Comparison- Nominal case 118 bus system



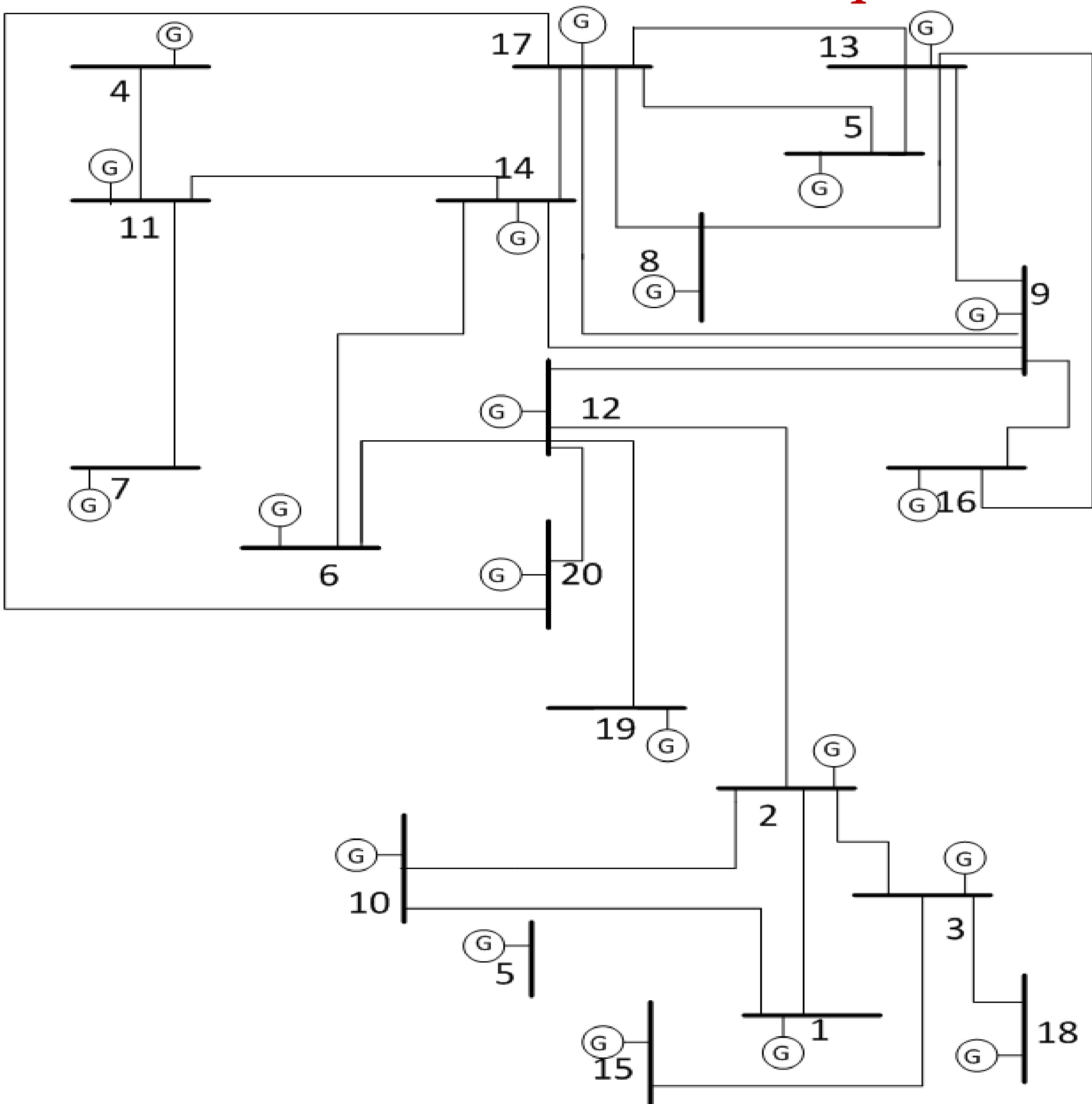
118→20 Bus Reduced Equivalent



System Description:

118 buses
186 branches
91 load sides
54 thermal units

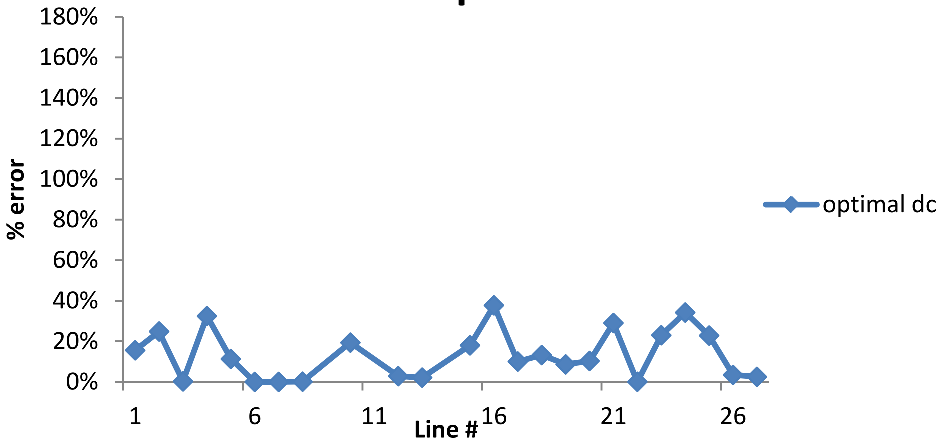
118→20 Bus Reduced Equivalent



118→20 Bus Reduced Equivalent Optimal v. Classical dc-Model Nominal Load

Reduced 118→20 Bus Mode

Reduced 118 Bus Model Branch Flow Errors Optimal dc

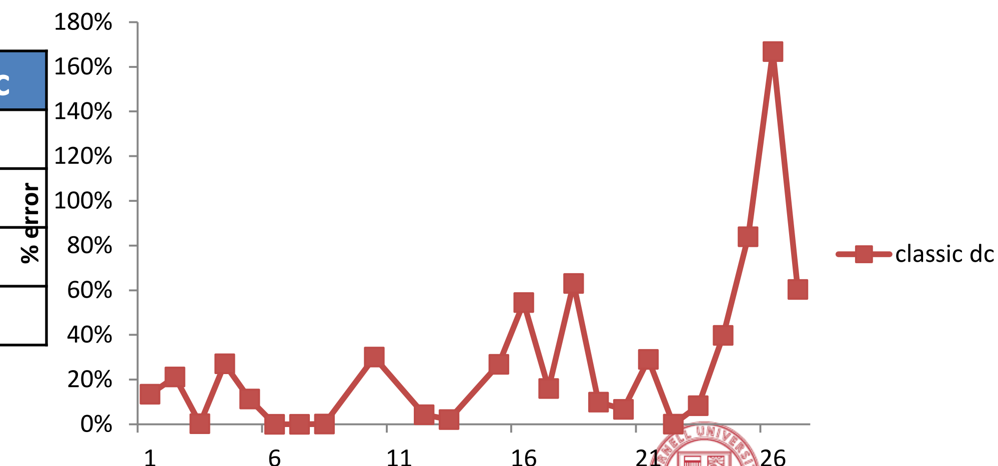


118→20 Bus Reduced Model Comparison of Power Flow Errors Aggregated Results

	Classic dc	Optimal dc
Maximum Error (MW)	159.46	68.48
Average Error (MW)	28.10	15.03
Maximum Error (%)	166.80%	37.75 %
Average Error (%)	25.01%	11.93%

Reduced 118→20 Bus Model

Reduced 118 Bus Model Branch Flow Errors classic dc



Conclusions

- For bus aggregation based network equivalencing it was found that:
  - By reducing small elements in the Λ matrix:
    - Significant speed up in computation time.
    - Problem size may be reduced to fit on a PC
  - By calculation optimum branch reactance evaluation:
    - Unreduced dc network flows errors are significantly decreased.
    - Reduced of flow errors in the reduced network equivalent are also significantly decreased.

Questions?