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# Using Linear Programming to Create Optimal Budget Scenarios

Office of Programming, Analysis, and Evaluation (PA&E)

August 3rd, 2022

# Agenda

- PRWG Introduction
- Optimization Overview
- PRWG & Optimization
- Challenges
- Results
- Conclusion

# Introduction

## Cadence Doyle

Cadence Doyle is a lead analyst for Technomics, providing data science solutions for DHS and DOE clients. She supports the PRWG through PA&E. She is a Professional Cost Estimator/Analyst (PCEA) and a Certified Scrum Master (CSM).

Cadence graduated with an M.S. in Mathematics and Statistics from Georgetown University and B.S. in Mathematics and Latin from Dickinson College

## Joshua Gonzalez

Joshua Gonzalez is a lead analyst for Technomics, providing data analytics and visualizations for DOD & DOE clients, and currently supports PA&E through the PRWG.

Joshua graduated with an M.S. in Geospatial Information Systems from the University of Maryland and B.S. in Geographic Information Systems from the University of Maryland

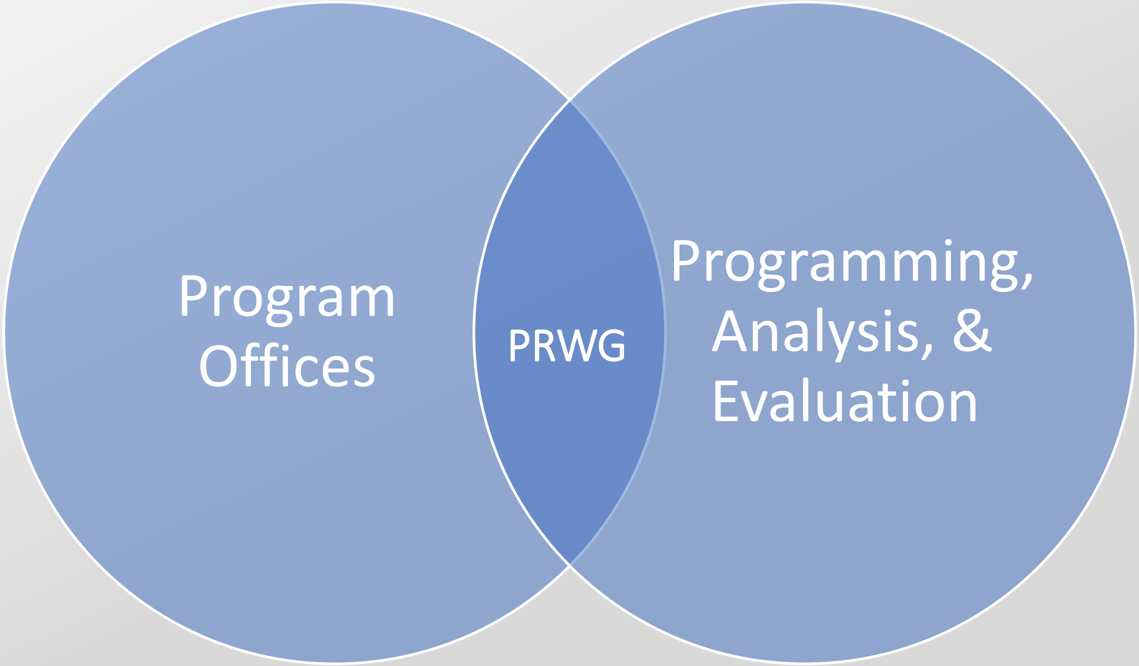
## Jenna Vandervort

Jenna Vandervort is a senior associate at Technomics and has supported PA&E since the spring of 2021, using data analytics and statistics to support modeling and cost estimating capabilities.

Jenna graduated with a B.S. in Industrial Engineering from Worcester Polytechnic Institute.

# Programmatic Recapitalization Working Group (PRWG)

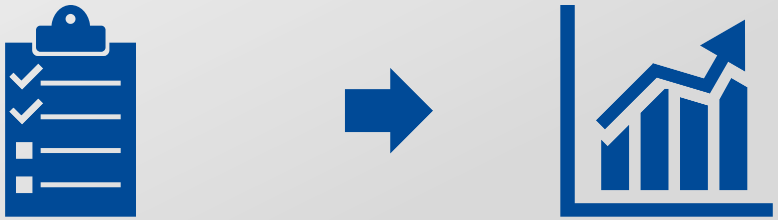
- The PRWG consists of different program offices responsible for the recapitalization of programmatic equipment
  - Ensures effective use of funds
  - Discusses any potential funding overlaps
- The PRWG charter specified the need for data collection, **data analytics**, and dashboards, necessitating PA&E's involvement
- Recent activities on the PRWG have focused on **prioritizing** investments



# Optimization Overview

# Introduction to Optimization

- Most use optimization tools everyday, like using GPS technology to get to work in the fastest way possible
- Optimization treats business decisions as a mathematical model to optimize an objective function
- Optimization can mean either minimization or maximization



# Example Optimization Problem

A stool sells for $4, takes one hour to make, and uses 6 planks of wood. A chair sells for $5, takes two hours to make, and uses 6 planks of wood. You have 36 planks and 10 hours. How many chairs and stools should you make to maximize revenue?

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$$\text{Number of Stools} = x$$
$$\text{Number of Chairs} = y$$

Maximize: $Z = 4x + 5\,y$

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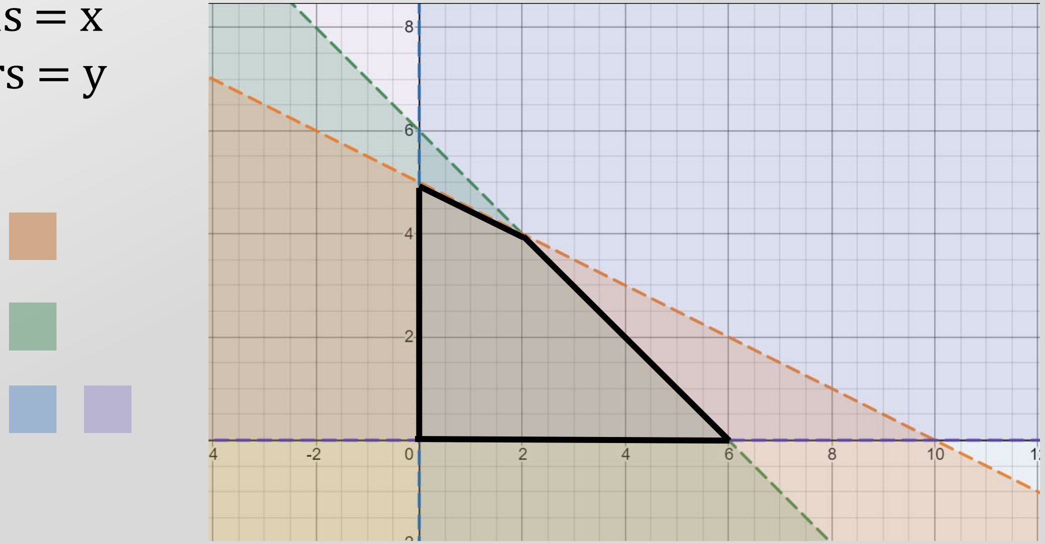

# Example Optimization Problem

- We can list out the set of all possible solutions to this integer programming problem

$$\{ (0,0), (0,1), (0,2), (0,3), (0,4), (0,5), (1,1), (1,2), (1,3), (1,4), (2,0), \ldots, (5,1), (6,0) \}$$

- We can use brute force to test all integers to satisfy the constraints and pick the variable combination that maximizes $Z$

For $x = 3, y = 3,\ Z = 27$

For $x = 2, y = 4,\ Z = 28$

Maximize: $Z = 4x + 5\,y$

Subject to:
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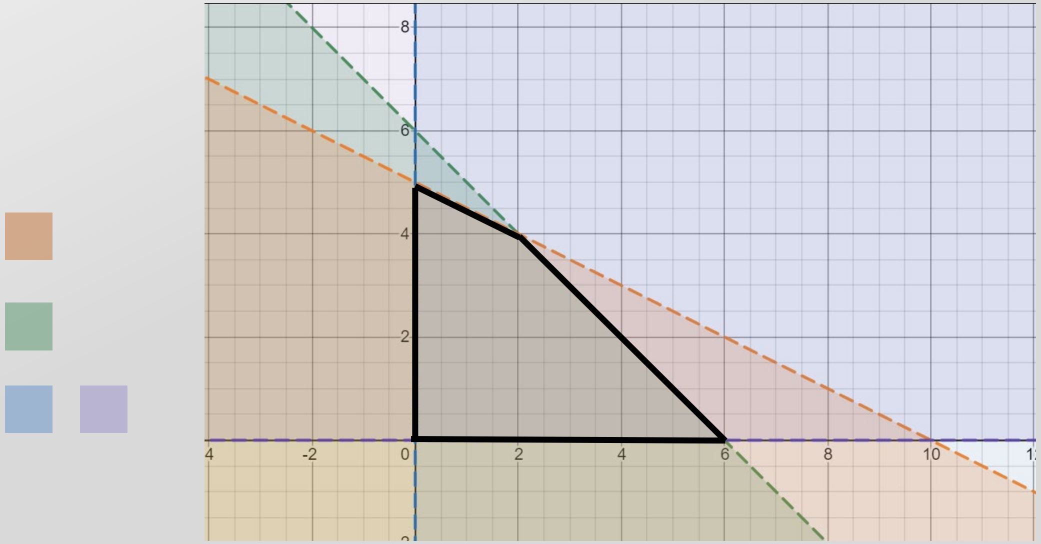

# Example Optimization Problem

- Mathematicians have developed various algorithms to find an optimal solution, one being the simplex method
- The simplex method is an iterative method, generating a sequence of solutions within the feasible region before stopping at the optimal
- The simplex method essentially acts as a search function, moving from one feasible solution to another

Maximize: $Z = 4x + 5\,y$

Subject to:
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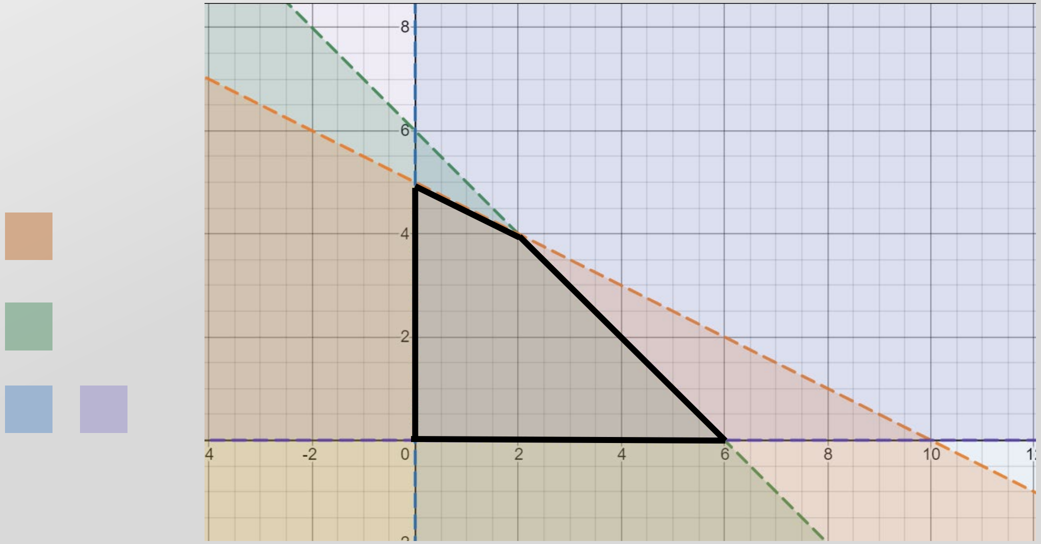

# PRWG Problem Statement

When recapitalizing programmatic equipment, how can we buy down the greatest amount of risk while staying within the confines of our budget?

# Linear Programming with PRWG

- The PRWG asks the sites to identify the risk to mission associated with each piece of equipment
    - Risk to Mission (Impact)
    - Risk to Mission (Probability)

- We also collect the procurement cost (TY$) for each piece of equipment

**Risk Matrix - Threats**

| | | Impact | | | | |
|---|---|---|---|---|---|---|
| Performance | | Minimal or no impact | Acceptable with some reduction in margin. | Acceptable with significant reduction in margin. | Acceptable; no remaining margin. | Unacceptable. Will not meet requirements. |
| Schedule | | Minimal or no impact | Additional resources required; able to meet need dates. | Minor slip in key milestones; not able to meet need date. | Major slip in key milestone or critical path impacted. | Can't achieve key team or major program milestones. |
| | | Minimal (1) | Minor (2) | Moderate (3) | Significant (4) | Severe (5) |
| Probability | Nearly Certain >90% (5) | 5 | 10 | 15 | 20 | 25 |
| | Highly Likely >= 75% to 90% (4) | 4 | 8 | 12 | 16 | 20 |
| | Likely >=25% to 75% (3) | 3 | 6 | 9 | 12 | 15 |
| | Unlikely >=10% to 25% (2) | 2 | 4 | 6 | 8 | 10 |
| | Very Unlikely <10% (1) | 1 | 2 | 3 | 4 | 5 |

# PRWG Data into Linear Programming

## Program Inputs

Budget for FY22:
$16,000,000

## PRWG Inputs

| Equipment Identifier | Procurement Cost | Risk to Mission (Impact) | Risk to Mission (Probability) |
|---|---|---|---|
| a | $ 20,000,000.00 | 1 | 2 |
| b | $ 2,255,000.00 | 2 | 2 |
| c | $ 8,000,000.00 | 2 | 1 |
| d | $ 13,250,000.00 | 4 | 4 |
| e | $ 2,000,000.00 | 5 | 2 |
| f | $ 3,000,000.00 | 1 | 1 |

Maximize: $Z = 1a + 2b + 2c + 4d + 5e + 1f$

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Subject to:
$$\begin{cases} 20{,}000{,}000a + 2{,}255{,}000b + 8{,}000{,}000c + 13{,}250{,}000d + 2{,}000{,}000e + 3{,}000{,}000f \leq 16{,}000{,}000 \\ a, b, c, d, e, f \in \{0,1\} \end{cases}$$

Maximize: $Z = 2a + 2b + 1c + 4d + 2e + 1f$

Subject to:
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# Difference in Prioritization Schema

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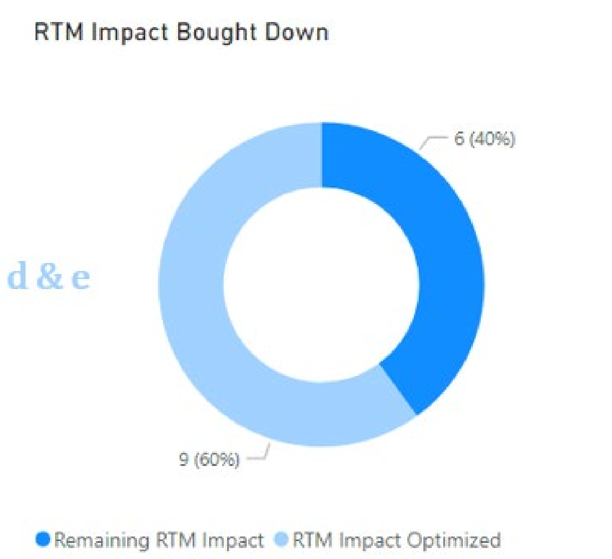


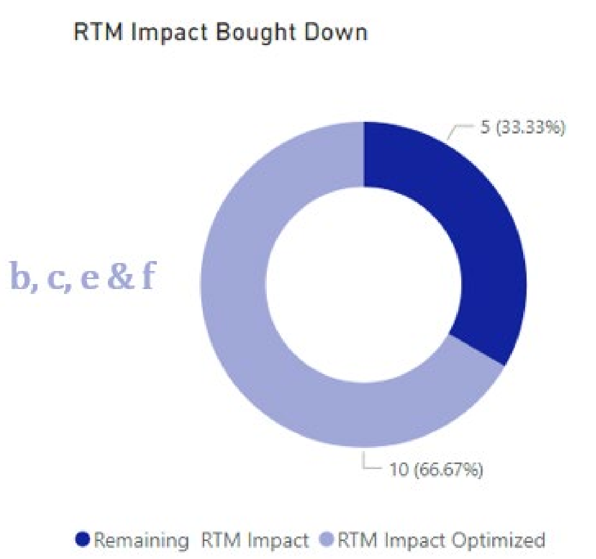


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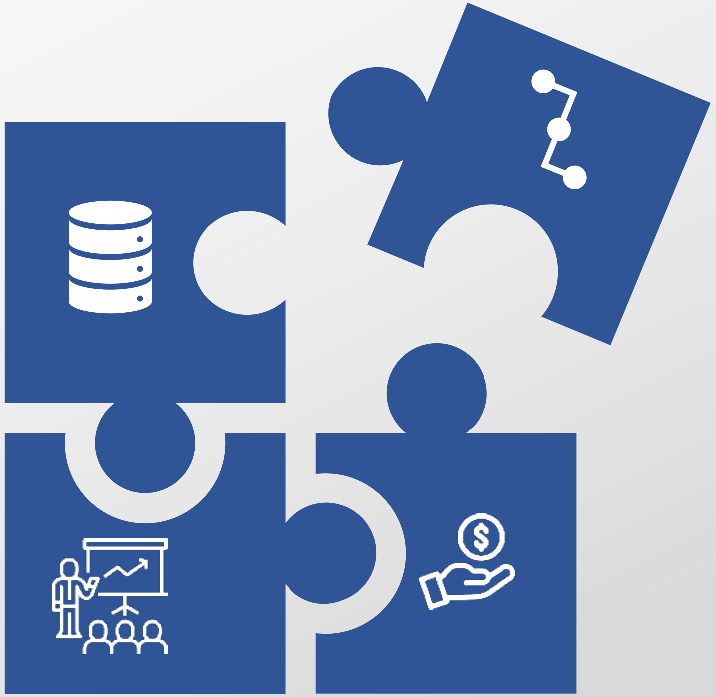
# Challenges

## Data Quality

Capturing a **complete and accurate risk assessment** for programmatic equipment ensures at-risk equipment is not overlooked during optimization

## Communication & Results

Optimization results can be hard to communicate or visualize and are agnostic from the political nature of **management decisions**



## Making Connections

Understanding **how equipment relates to other pieces** of equipment when recapitalizing and how that impacts budget scenarios must be incorporated into future modeling efforts

## Additional Costs

Current data tracks and optimizes based on procurement costs, however **overall acquisition costs can be much larger** and are often phased out over time

# Results

- Program offices can use our outputs to compare the optimal budget scenarios with current plans, helping to **remove subjectivity** from PPBE

- Program managers can use linear programming to **form budget proposals**, using graphics from dashboards to illustrate how much more risk they can buy down with additional funding

# Conclusion

- Mathematical optimization requires analysts to **understand the system** they are modeling and the **mechanics of implementation**

- Model outputs require **human oversight** and **qualitative discussions** to make the best decision

- Linear programming offers program managers a credible, defensible approach to
  - Initiate data-driven discussions with sites about **competing priorities**
  - Communicate how their **funding impacts** the amount of risk bought down
  - Make **informed decisions** across their portfolio