Hybrid Neural Network Modeling for Multiple Intersections along Signalized Arterials - Current Situation and Some New Results

Wan Li, Chieh Wang, Yunli Shao, Hong Wang
National Transportation Research Center
Oak Ridge National Laboratory,
Oak Ridge: TN37934, USA
E-mail: {w5i; cwang; shao; wangh6}@ornl.gov

Guohui Zhang, Tianwei Ma
Department of Civil and Environmental Engineering
University of Hawaii at Manoa
2540 Dole Street, Honolulu, Hawaii 96822, USA
E-mail: {guohui; tianwei}@hawaii.edu

Jon Ringler
Econolite Systems
1250 N. Tustin Ave.
Anaheim, CA 92807, USA
E-mail: JRingler@econolite.com

Danielle Chou
Vehicle Technologies Office
US Department of Energy
1000 Independence Ave SW
Washington, DC 20585, USA
Email: danielle.chou@ee.doe.gov

Abstract—Traffic flow along signalized arterials is a dynamic, nonlinear, and stochastic system in which the relationship between the signal timing plan and traffic delays is too complicated to be modeled using first principles approaches. With advances in sensing technologies, various data sets are available, allowing effective data-driven modeling to be conducted for further controller design. In this keynote paper, a Hybrid Neural Network (HNN) is proposed to model the multiple intersections along a signalized arterial in Honolulu, in which both modeling structure and the relevant training algorithms have been developed. HNN modeling using real data has shown a set of promising results, with dynamic model performance assessed using model error Probability Density Function (PDF). A simple HNN model can easily be used as a starting point for an artificial intelligence-based closed-loop control design that controls the signal timing to reduce the traffic delay.

Keywords - signalized intersections; modeling; neural networks; performance analysis; signalized arterials simulation.

I. INTRODUCTION

The nature of the traffic flow system in signalized arterials can be represented as a dynamic and stochastic system \cite{2} – \cite{5} for which the inputs are the traffic demand and signal timing at each intersection, and the outputs are the traffic flow status (e.g., travel delays, queue length, and traffic flow speed) and energy consumed when vehicles pass through the arterial. Since the traffic demand and traffic flows (number of vehicles and their compositions) are random, the system is stochastic in nature. This is a Multi-Input and Multi-Output (MIMO) stochastic dynamic system. If it is in the continuous-time domain, its solution is obtained using partial differential equations induced from the well-known Itô stochastic differential equations with random boundary conditions. The solution for such a complicated model is quite difficult to obtain, and it frequently must be solved using high-performance computing, which generally cannot be used for real-time control design and implementation. Therefore, data-driven modeling methods— in particular, those widely used in Artificial Intelligence (AI) technology—are regarded as effective ways to establish simple dynamic models between signal control and traffic flows so that system performance can be controlled and optimized in real time. The advantage of using AI-based models is that these models can be adaptively learned using evolving real-time data. As a result, the use of neural network modeling has been a subject of study for many years.

Indeed, advances in wireless-driven vehicular communications have greatly facilitated modeling exercises, and emerging cooperative intelligent transportation control system operations have enabled many smart traffic control and management applications to improve traffic safety and operational efficiency \cite{1}. Vehicle-to-Everything (V2X) communications allow vehicles to communicate with other vehicles (vehicle-to-vehicle); infrastructure (vehicle-to-infrastructure); pedestrians, bicyclists, and devices (vehicle-to-device); and internet through cellular networks and/or dedicated short-range communication technologies. The information exchanges supported by V2X communication systems can be used to effectively balance traffic demand distribution among traffic networks and facilitate traffic flow progression. With these new data available in a real-time format, it is now possible to further enhance AI-based modeling, and ultimately control, to optimally coordinate signal controls for traffic flow systems along arterials.

In addition, for stochastic modeling of traffic flow systems, one of the important criteria is the reliability of and confidence in the obtained models for control and optimization. Thus, not only do the models need to be built using real-time input and output data, but also there is a need to ensure that the model so obtained is reliable and has a high level of confidence interval for users. In this context, the use of modeling error entropy, or its Probability Density Function (PDF), should be considered as the modeling objective function to be minimized. Ideally, a narrowly distributed modeling error...
PDF centered at zero mean would indicate that the models obtained have high reliability and confidence intervals. This is exactly its novelty compared with existing AI-based models for transportation systems, in which only sum-squares-error has been used to judge whether the obtained model is good or not. The method of using modeling error entropy and PDF to perform online adaptive learning was established several years ago [1], and this approach can be applied in combination with the existing AI modeling tools to establish reliable and robust AI-based models for the traffic flow system.

Based upon the above analysis, it can be seen that the following challenges remain in terms of AI-based modeling and control for signalized intersections along arterials and the urban grid road network:

- Although the theory of AI-based modeling and control for signal control is maturing, the field testing and closed-loop control implementation for a large number of intersections is still limited because of the insufficient real-time data for fast feedback control realization.
- The existing AI-based modeling for transportation systems cannot yet capture the nonlinear and dynamic stochastic nature with high reliability and robustness.
- Guaranteed control performance for energy minimization is still lacking.

In this effort, neural network modeling was studied for signalized intersections along an arterial in Honolulu using the real-time data from the system. A Hybrid Neural Network (HNN) model, which is a subset of neural networks, was constructed, and its learning algorithm was established. A comprehensive assessment of the modeling effort was conducted using least squares and gradient approaches.

The rest of this paper is organized as follows. Section II summarizes the literature review on traffic signal control problems with neural network models. Section III describes the system structure and the forms of dynamic models that represent the relationship between the traffic delays and signal timing plans. Section IV presents the linear modeling using recursive least squares to show the nonlinearity of the system. Section V addresses the formulation of HNN and defines its inputs and outputs together with the formulation of training algorithms for both linear and nonlinear parts. The modeling results and modeling performance analysis for an arterial with seven signalized intersections are also discussed in this section. The conclusions and acknowledgement close the article.

II. RELATED WORK

Traffic system modeling aims to establish linear or nonlinear relationships between traffic states—e.g., traffic volume, travel time (travel delay), and travel speed—given spatiotemporal traffic information. Most studies leverage a single data source. For example, the objective is to predict near-term traffic flow given historical traffic flow data. Other studies using multiple data sources need to capture dominant dependencies between different features. For example, Ke et al. [6] developed a model to predict lane-based traffic speed based on speed and traffic volume data. Transportation system modeling techniques can be divided into two categories: non-learning based and learning based methods [7]. For example, classical non-learning-based methods include autoregressive integrated moving average [8] and K-nearest neighbors [9]. These models are usually more interpretable but cannot capture the spatial correlations of traffic states. Moreover, they are not appropriate for nonstationary data. Traditional learning-based methods include regression [10], Kalman filter [11], and support vector machine [12]. These methods are generally more effective than non-learning-based models. However, they usually fail to capture the nonlinear spatiotemporal correlations of traffic data. Nowadays, we have more data sources and increasing computational power, so more advanced learning-based methods, e.g., different types of neural networks, have shown promising performance. The most commonly used neural networks for transportation system modeling include Artificial Neural Networks (ANN) [13], Long Short-Term Memory (LSTM) [14], Convolutional Neural Networks (CNNs) [6], and Graph-based Neural Networks (GNN) [15]. Compared with ANNs, CNNs and LSTMs have advantages in capturing nonlinear spatial and temporal dependencies of traffic features. However, their limitations become obvious when the transportation network is very large. GNNs are proved to be powerful tools for large-scale traffic signal control systems. GNNs can extract features from graph-structured data and predict future traffic states in an efficient and effective manner.

With the established dynamic stochastic models for transportation system, the next step is to develop real-time optimal control strategies to reduce travel delay and energy consumptions. Conventional traffic control methods for multiple intersections in a network, such as SCOOT [16], GreenWave [17], SOTL [18], Max-pressure [19], and SCATS [20], usually assumed a simplified traffic conditions with complete traffic information available, e.g., pre-defined traffic flows and driving behaviors. Hence, they are not applicable for real-world traffic control for multiple intersections.

For a large-scale traffic system, it is usually a difficult task to predict the effects of modifying signal timing parameters due to the nonlinear and stochastic nature in a traffic network. Comparing to the conventional signal control methods, Neural Network (NN)-based signal control methods can address the challenges on traffic system modeling and traffic signal optimization. The studies from [21][22] tested a NN-based controller for single intersections. Both studies applied the concept of fuzzy logic and their NNs are five-layer type, e.g., input, fuzzification, inference, consequence, and defuzzification. They used number of vehicles passing the intersection and number of vehicles waiting in the queues as inputs and the outputs are the traffic signal plans. In [22], reinforcement learning and gradient descent method were applied to update the shape of fuzzy membership functions by computing the weights of the NN. The advantages of NN models are more obvious in a larger network. Srinivasan et al. [23] developed a distributed unsupervised traffic responsive
signal control method for traffic signal control and coordination. Each agent is a local traffic controller for one intersection. They integrated the simultaneous perturbation stochastic approximation theorem in fuzzy NN. Stochastic approximation is a commonly used technique in stochastic optimization for online weight updates in NN. It is usually preferable when the gradient of the loss function is not readily available. The proposed model was tested in a traffic network with 25 intersection in Singapore. The results demonstrated that the model could be used to obtain the controller that reduces significant amount of travel delay. Choy and Srinivasan [24] further improved the study [23] by developing a HNN model with multistage online learning process to solve the distributed traffic signal control problem with an infinite horizon. It is challenging to calculate the analytical optimal solution for the distributed control problems. This study applied an approximation technique, receding-horizon limited-memory, for to approximate optimal solution. Each local signal controller was made up of a five-layered fuzzy NN that aimed at computing the optimal signal plans. Experiment results suggested that HNN model was effective and efficient in solving the large-scale traffic control problem in a distributed control manner. There are several limitations in NN-based traffic signal control algorithms. First, as mentioned by [22], NN learning is not efficient under complex continuous system because of lack of stochastic exploration. Second, learning process is usually too long to be implemented in real time in the field.

Recently, Reinforcement Learning (RL) models have been studied extensively and made impressive progress in traffic control domains. RL can learn from observed data and adapt to real-time changes of traffic demands. RL is a trail-and-error learning process without making any unrealistic assumptions on traffic system modeling. There are four key components in Decentralized Reinforcement Learning (DRL): agent, environment, state, and reward. In transportation system, environment is often defined as traffic conditions and state is a feature representation of the environment. DRL will have an agent for each intersection to learn a model and predict whether current signal phase should be changed or not. The decision will be implemented in the environment and the reward (travel delay, vehicle throughput, or energy efficiency) is sent back to the agent to help it improve the decision-making process. The key challenges in RL are (i) how to describe the environment quantitatively, (ii) how to model the relationships between decision (signal timings) and reward (traffic states) due to its exponentially expanding complexities; and (iii) how to implement coordination and information sharing between multiple agents/intersections. There are generally two categories of RL: model-free and model-based RL. To successively apply model-based approach, the transition function (predict next state given current action) must be known. However, it is usually difficult to obtain it in real-world. Model-free RL directly estimate the reward given state-action pairs and select the optimal action accordingly [32]. Hence, model-free RL, e.g., Q-learning and SARSA, are commonly used in traffic signal control problems. For model-free RL, exploration is required to gain knowledge by sampling. Model-free RL can be categorized as value-based and policy-based methods [33]. Value-based RL learning the value function (or a generalization called the Q-function) and policy-based methods directly learn the optimal policy or approximate optimal policy. Comparing to the traditional reinforcement learning approach whose states need to be discretized and low-dimensional, DRL can handle high dimensional input data, e.g., image, and learn functions to extract useful information and approximate policy from input states. By combining deep learning with reinforcement learning, it addresses the “curse of dimensionality” issue, helps to improve the model scalability, and reduce learning time. Li et al. [25] set up a Deep Neural Network (DNN) to learn the Q-function of DRL from the sampled traffic states (inputs) and the corresponding traffic conditions (outputs). The objective is formulated as a Q-function which aims to maximize the future rewards given the current state and action. Instead of relying on a conventional Q-table, they used the deep Stacked Autoencoders (SAE) neural network to estimate Q-function. Comparing to the conventional reinforcement learning approaches, their DRL can reduce delay by 14% and largely reduce number of vehicles stops at intersections. Wei et al. [26] developed a DRL model for traffic signal control with real-world large data set. In their method, traffic condition is extracted from an image. The image is directly used as an input for a CNN model to supplement other hand-crafted traffic features (queue length, waiting time, and number of vehicles) of environment. They applied an offline model to test different signal timing plans and collect data samples of signal timings and traffic conditions. After that, an online model will determine the optimal action to take (change signal status or not). Their model was tested on a large-scale real traffic dataset from surveillance cameras. Motivated by Max Pressure (MP) control, Wei et al. [27] developed a reinforcement learning approach for large-scale road network. In RL, the objective is to maximize the long-term rewards by trial-and-error search while MP aims at minimizing pressure by greedy algorithm. This study set the reward function in RL the same as the objective of ML so that they can achieve the same result as MP to maximize vehicle throughput. As claimed by the authors, this is the first study that applies individual RL model and achieves coordination without any prior knowledge. Chen et al. [28] designed a DLR model for a city-level network with more a thousand intersections. DRL for multi-intersection control and coordination is quite a difficult problem due to the scalability and data feasibility. They incorporated DRL agent design with pressure, e.g., different of queue length at downstream and upstream intersections. The DRL agent aims at balancing the distribution of vehicles in the traffic network and maximize the system throughput. They tested their proposed model with Manhattan dataset containing signalized 2510 traffic lights. Comparing to other state-of-the-art signal control methods including fixed time, max pressure, and different variations of reinforcement learning methods, their proposed model was proved to generate shorter travel time and larger vehicle throughput.
Based on the literature, most studies used average queue length, average waiting time, average speed, and vehicle throughput as reward to evaluate an action in RL. There are various kinds of measures to describe environment states, e.g., queue length, waiting time, speed, and signal phases for each lane or for a road segment. Traditional RL use a tabular or linear model to approximate the state function to improve efficiency [31]. However, the state space in real world is usually very large which limit the capability of traditional RL. With the development of deep learning, DRL models can handle the large state space. For example, recent studies use images as state where vehicle trajectories and queue length can be extracted [26][27] for state representation. The action in RL relates to signal phases changes. It can be the ratio of signal phase duration over the total cycle length [31] or an indicator to decide if an different signal phase should be activated to green [26]. Most of the traffic signal control studies with RL use value-based methods which usually requires discrete actions. The model takes the state presentation as input and parameterized by neural networks.

Although DRL model improves traffic signal control in the complex transportation systems, it treats neighboring intersections as the same and fail to model the spatial dependencies of traffic flows. Different intersections should be modeled carefully in realistic transportation network. To address the issue mentioned above, graph neural networks are proved to be an effective tool to capture the traffic dynamics in large-scale transportation network. The transportation system can be model by a graph consisting of nodes and edges. GNN can handle inputs given on general graphs. Wei et al. [29] proposed a model, CoLight, to control traffic signals on a large-scale road network with hundreds of intersections. They applied a graph attentional network to facilitate communication between intersections and consider the temporal and spatial influences of neighboring intersections. The model leverages the attention mechanism to model the influence of upstream and downstream intersections on the target intersection by learning different weights for different intersections. Extensive experiments have been conducted using synthetic and real-world data. Their proposed model outperformed other state-of-the-art methods in terms of reducing average travel time. Zhong et al. [30] developed a probabilistic graph neural network for traffic signal control and cooperation. They used decentralized reinforcement learning to model the transportation system. A graph attention module was then applied to learn dependencies of neighboring intersections. Finally, a graph inference model was proposed to learn the latent representations of adjacent intersections by considering traffic uncertainties. Their model can characterize the posterior with respect to latent variables and allow Bayesian inference. The rationality of model design can be explained by transportation theory.

Coordination is essential for large-scale transportation system with multiple intersections. Wei et al. [34] categorized traffic signal control and coordination problems into three categories: joint action learners, independent (distributed) learners without communication and distributed learner with communication. Joint learners use a single centralized agent to control all intersections [34]. This approach could lead to the curse of dimensionality that the state-action space will grow exponentially as the number of intersections increases. Unlike joint agent, each distributed agent control one intersection. If communication does not exist between distrusted agent, each agent observes its own local environment. This method usually does not perform well when the environment becomes complicated. Distributed learning with communication allows agent to share information on their observations. Graph-based NN model for traffic signal control problems can learn the communication from the message passing on the graph. TABLE I summarizes the representative NN-based traffic signal control studies based on a few characteristics we discussed above.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Method</th>
<th>Traffic features</th>
<th>Coordination</th>
<th>Road Network</th>
<th># of Intersections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wei and Zhang [21]</td>
<td>Fuzzy neural network</td>
<td># of vehicles; queue length</td>
<td>No communication</td>
<td>Synthetic</td>
<td>1</td>
</tr>
<tr>
<td>Bingham [22]</td>
<td>Neurofuzzy traffic controller</td>
<td># of vehicles; queue length</td>
<td>No communication</td>
<td>Synthetic</td>
<td>1</td>
</tr>
<tr>
<td>Srinivasan et al. [23]</td>
<td>Fuzzy neural network with stochastic approximation theorem</td>
<td>Traffic flow; occupancy</td>
<td>Distributed control with communication</td>
<td>Real (CBD Singapore)</td>
<td>25</td>
</tr>
<tr>
<td>Choy et al. [24]</td>
<td>HNN with reinforcement learning and evolutionary algorithm</td>
<td>Traffic flow; occupancy</td>
<td>Distributed control with communication</td>
<td>Real (CBD Singapore)</td>
<td>25</td>
</tr>
<tr>
<td>Li et al. [25]</td>
<td>Value-based reinforcement learning</td>
<td>Queue length</td>
<td>No communication</td>
<td>Synthetic</td>
<td>1</td>
</tr>
<tr>
<td>Wei et al. [26]</td>
<td>Value-based reinforcement learning</td>
<td>Queue length; # of vehicles; waiting time; image</td>
<td>No communication</td>
<td>Synthetic</td>
<td>1</td>
</tr>
<tr>
<td>Wei et al. [27]</td>
<td>Value-based reinforcement learning with max pressure control</td>
<td># of vehicles</td>
<td>No communication</td>
<td>Real (New York City)</td>
<td>16</td>
</tr>
<tr>
<td>Chen et al. [28]</td>
<td>Value-based reinforcement learning</td>
<td># of vehicles</td>
<td>No communication</td>
<td>Real (New York City)</td>
<td>2510</td>
</tr>
<tr>
<td>Wei et al. [29]</td>
<td>Graph attention network for cooperation</td>
<td>Queue length</td>
<td>With communication</td>
<td>Real (New York City)</td>
<td>196</td>
</tr>
<tr>
<td>Zhong et al. [30]</td>
<td>Probabilistic graph neural network</td>
<td>Queue length, # of vehicles</td>
<td>With communication</td>
<td>Real (Hangzhou)</td>
<td>16</td>
</tr>
</tbody>
</table>
In addition, the research team at the University of Hawaii has extensively developed machine learning-based approaches to address various traffic data analysis and formulation issues. For example, they estimate vehicle classification volumes based on single-loop detector outputs [36]. The proposed ANN has three layers with back-propagation architecture. Vehicle classification categories employed by this study were consistent with the four-bin classification system currently used by the Washington State Department of Transportation (WSDOT) dual-loop detection system. To achieve the best bin volume estimates, a specific neural network is designed and configured for each vehicle category. The proposed ANN is trained and tested using data collected from loop detector stations on I-5 in the greater Seattle area. Our test results indicate that the proposed ANN method worked stably and effectively for the studied stations. The estimated bin volumes were reasonably accurate and can be applied to transportation practice. The temporal and spatial transferability tests showed that the proposed ANN is robust and can be applied to estimate bin volumes during different time periods and at different loop stations on I-5 without introducing significant errors.

Work in [37] conducted a study to develop a Deep Learning (DL) framework to predict the taxi-passerger demand while the spatial, the temporal, and external dependencies were considered simultaneously. The proposed DL framework combined a modified density-based spatial clustering algorithm with noise (DBSTCAN) and a conditional generative adversarial network (CGAN) model. More specifically, the modified DBSCAN model was applied to produce a number of sub-networks considering the spatial correlation of taxi pick-up events in the road network. And the CGAN model, fed with the historical taxi passenger demand and other conditional information, was capable to predict the taxi-passerger demands. The proposed CGAN model was composed of two LSTM neural networks, which are termed as the generative network G and the discriminative network D, respectively. Adversarial training process was conducted to the two LSTMs. In the numerical experiment, different model layouts were compared. It was found that different network layouts provided reasonable accuracy. With limited training data, more LSTM layers in the generator network resulted in not only higher accuracy, but also more difficulties in training. Comparisons were also conducted between the proposed prediction model and four typical approaches, including the moving average method, the autoregressive integrated moving method, the neural network model, and the LSTM neural network model. The comparison results showed that the proposed model outperformed all the other methods.

Another research effort undertaken in [38] is to investigate how the integration of clustering models and deep learning approaches can learn and extract the network-wide taxi hotspots in both temporal and spatial dimensions. A Density Based Spatiotemporal Clustering Algorithm with Noise (DBSTCAN) was established to extract the historical taxi hotspots, which changed with time. A conditional generative adversarial network with Long Short-term Memory Structure (LSTM-CGAN) model was proposed for taxi hotspot prediction, which is capable of capturing the spatial and temporal variations of taxi hotspots simultaneously. Specifically, the DBSTCAN was applied to process the large-scaled geo-coded taxi pickup data into time-varying historical hotspot information. The proposed LSTM-CGAN model was then trained by the network-wide hotspot data. As illustrated in the numerical tests, it was found that the proposed LSTM-CGAN model provided comparable results with different model layouts and model with 4 LSTM layers in both generator and discriminator performed best. The comparison results indicated that the proposed LSTM-CGAN model outperformed all these benchmark methods and demonstrated great potential to enable many shared mobility applications.

Work in [39] reported a novel multi-agent reinforcement learning method, named as Knowledge Sharing Deep Deterministic Policy Gradient (KS-DDPG) to achieve optimal control by enhancing the cooperation between traffic signals. By introducing the knowledge-sharing enabled communication protocol, each agent can access to the collective representation of the traffic environment collected by all agents. The proposed method is evaluated through two experiments respectively using synthetic and real-world datasets. The comparison with state-of-the-art reinforcement learning-based and conventional transportation methods demonstrates the proposed KS-DDPG has significant efficiency in controlling large-scale transportation networks and coping with fluctuations in traffic flow.

Based upon the above analysis, it can be seen that there are still following challenges on NN based modeling and control strategies for networked signal-timing control:

1) The modeling using data driven requires a good set of data in real-time;
2) In terms of control strategies, there is a need to structure the control model so that it can be easily implemented in real-time. A affine type of dynamic model would be a choice. This will be described in the following sections;
3) Most studies have been focussed on simulations and real-time 24/7 implementation is lacking.

These challenges constitute research questions to be answered and therefore in the following sections, a novel modeling and control, namely the HNN modeling and control developed by the authors, will be described that summarizes the authors recent work on multiple signalized intersection control.
III. TRAFFIC FLOW SYSTEM DESCRIPTION

Figure 1 shows the signalized arterials to be modeled, where the input is the signal timing plan at each intersection and the output is the traffic delays of different phases (left turns, right turns and straight movements).

![Figure 1. The signalized arterial in Honolulu.](image)

The objective is to build up dynamic models that reflect the dynamics of the system; the data used were collected from Econolite systems.

Taking \( u(k) \) as the input and \( y(k) \) as the output vector that is composed of the signal timing plan (i.e., green light time duration under fixed cycle length) and the traffic delays for each phase (i.e., through movements, left turns, and right turns) at an intersection respectively, the dynamics of the system can be generally modeled as follows

\[
y(k + 1) = f(y(k), u(k), \omega(k)) \tag{1}
\]

where \( f(\ldots) \) is the nonlinear vector function representing the system dynamics, \( \omega(k) \) is the random noise term, and \( k \) is the sample number, which can be a multiplication of cycle duration in signal timing control.

IV. LINEAR MODELING

To perform the required data-driven modeling, it was imperative to first check whether the system could be truly represented as a nonlinear system. To answer that question, we performed linear modeling initially. Indeed, if the system was linear, then the modeling error should have a Gaussian-like distribution. Otherwise, the system should be regarded as a nonlinear system in which neural network modeling and other nonlinear system modeling need to be considered to build reliable models for the system.

A. Modeling structure

When the system is linear, the following simple model can be assumed for each intersection

\[
y(k + 1) = ay(k) + bu(k) + \omega(k) \tag{2}
\]

where \([a, b]\) are unknown parameters to be estimated, \( \omega(k) \) is noise, and the modeling exercise is to use available data \([u(k), y(k)]\) to estimate the parameters \([a, b]\). This is a standard application of least squares estimation. For this purpose, denote

\[
\theta = \begin{bmatrix} a \\ b \end{bmatrix}, \varphi(k) = \begin{bmatrix} y(k) \\ u(k) \end{bmatrix}
\]

Then, the following recursive least squares algorithm is used to estimate \([a, b]\) using the data collected from the Econolite/University of Hawaii platform

\[
\begin{align*}
\theta(k + 1) &= \theta(k) + P(k)\varphi(k)\epsilon(k) \\
q\epsilon(k) &= y(k + 1) - \theta^T(k)\varphi(k) \\
P^{-1}(k + 1) &= P^{-1}(k) + \varphi(k)\varphi(k)^T
\end{align*} \tag{4}
\]

where \( \theta(k) \) is the estimate of \( \theta \) at sample time \( k \) (of every five cycles), \( P(k) \) is the variance matrix, with the initial conditions being \( \theta(0) = 0, P(0) = 100I_{2 \times 2} \).

It can be seen that (4) is a typical recursive least squares algorithm with the maximum forgetting factor as the linear modeling here is to just test whether the system in nonlinear or time varying so as to justify the use of nonlinear system model. A “less-than-one” forgetting factor can also be used in order to track time-varying feature of the system. This allows the estimation algorithm to be adaptive and robust with respect to changes of the system such as operational environmental changes or system parameter changes. In this case, standard modification is needed for the above algorithm.

B. Modeling results showing the nonlinear feature

The modeling results are shown in Figures 2–5. The original data are normalized between zero and one as shown in Figure 2, the estimated parameters are given in Figure 3, the modeling error is displayed in Figure 4, and the corresponding PDF of the modeling error is illustrated in Figure 5. It can be seen that the system is clearly not linear, as the modeling error PDF is not Gaussian.

![Figure 2. Original data—normalized to [0, 1].](image)
Figure 3. Estimated a and b.

Figure 4. Modeling error—RLS residual signal

Max{error} = 30%

Figure 5. Modeling error PDF.

V. HYBRID NEURAL NETWORK

As the system is nonlinear and non-Gaussian, HNN modeling is described in this section. In this context, a dynamic model was considered that reflected the relationship between the input and the output. Moreover, to improve the model, traffic volume was also considered as an extra input. Thus, the system had two input vectors (i.e., signal time plan and traffic volume) and one output vector, traffic delays.

The system model was therefore assumed as follows:

\[ y(k + 1) = Ay(k) + Bu(k) + f(y(k), u(k - 1), v(k)) \]  \hspace{1cm} (5)

where \( y(k) \) and \( u(k) \) denote average delay per vehicle and green time for multiple intersections at time index \( k \). \( f(\ldots) \) is an unknown nonlinear vector function to be learnt and \( o(k) \) is noise. \( \{A,B\} \) are the weight matrices to be identified simultaneously with the estimate for the unknown nonlinear dynamics.

Let NN be used to approximate \( f(y(k), u(k - 1), v(k)) \) by \( \hat{f}(y(k), u(k - 1), v(k), \pi) \), where \( v(k) \) denotes traffic volume; \( \pi \) groups all NN weights and biases. Then the training of the NN as well as the two matrices was to obtain accurate and reliable models for the traffic flow system. In this case, we considered seven intersections of an arterial all together, as indicated in the red box in Figure 1.

The objective of training was to minimize the following performance function:

\[ \min J = \frac{1}{2} (\hat{y}(k + 1) - y(k + 1))^2 \]  \hspace{1cm} (6)

which is basically a minimum variance error criteria, where it has been defined that

\[ \hat{y}(k + 1) = Ay(k) + Bu(k) + \hat{f}(y(k), u(k - 1), v(k), \pi) \]  \hspace{1cm} (7)

and \( \{A, B, \pi\} \) are parameters to be trained. In the above equation, vectors \( \hat{y}(k) \) and \( \hat{f}(\ldots) \) are the estimates of \( y(k) \).
and \( f(\ldots) \), respectively using the real-time data from Econolite Systems.

### A. Gradient rule for training

Using gradient optimization, the following recursive estimation and training algorithm can be readily obtained to read

\[
\begin{align*}
\hat{A}(k + 1) &= \hat{A}(k) - \lambda_1 \frac{\partial J}{\partial A} |_{\hat{A}(k), \hat{B}(k), \hat{R}(k)} \\
\hat{B}(k + 1) &= \hat{B}(k) - \lambda_2 \frac{\partial J}{\partial B} |_{\hat{A}(k), \hat{B}(k), \hat{R}(k)} \\
\hat{\pi}(k + 1) &= \hat{\pi}(k) - \lambda_3 \frac{\partial J}{\partial \pi} |_{\hat{A}(k), \hat{B}(k), \hat{R}(k)}
\end{align*}
\]

(8)

(9)

(10)

where \( \lambda_1, \lambda_2 \) and \( \lambda_3 \) are pre-specified positive learning rates which are typically selected to be less than 1.0, and the gradients are calculated from

\[
\frac{\partial J}{\partial A} |_{\hat{A}(k), \hat{B}(k), \hat{R}(k)} = \left( \hat{y}(k + 1) - y(k + 1) \right) \frac{\partial \hat{y}}{\partial A} |_{\hat{A}(k), \hat{B}(k), \hat{R}(k)}
\]

(11)

\[
\frac{\partial J}{\partial B} |_{\hat{A}(k), \hat{B}(k), \hat{R}(k)} = \left( \hat{y}(k + 1) - y(k + 1) \right) \frac{\partial \hat{y}}{\partial B} |_{\hat{A}(k), \hat{B}(k), \hat{R}(k)}
\]

(12)

\[
\frac{\partial J}{\partial \pi} |_{\hat{A}(k), \hat{B}(k), \hat{R}(k)} = \left( \hat{y}(k + 1) - y(k + 1) \right) \frac{\partial \hat{y}}{\partial \pi} |_{\hat{A}(k), \hat{B}(k), \hat{R}(k)}
\]

(13)

where \( y(k + 1) \) is the measured real-time data from the Econolite systems.

The selection of the learning rates are also critical here in order to ensure a good balance between the responsiveness of the learning and its stability in providing convergent neural network training. Using the second-order derivative analysis such as Jacobian Matrices measure one can obtain the ranges for these learning rates.

The training algorithm described in (8) – (13) provides a set of simultaneous estimates for both linear parameters and neural network weights. Also, as the control input \( u(k) \) to be designed is linearly involved in the model, the controller design using AI-techniques can be easily implemented as a direct inverse calculation so long as the matrix \( B \) is of a full column rank. This approach effectively facilitates the real-time implementation for the whole system.

### B. Data and their processing

To model the system in (5), relevant data from the seven intersections were collected along the arterial as shown in Figure 1. In this context, the details of the data collected are as summarized in the Table II.

<table>
<thead>
<tr>
<th>Study area</th>
<th>Intersection 1-7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date collected</td>
<td>March 3–5, 8–12, 15–19, 22–26, 29–31, April 1–2 (23 weekdays) in 2021</td>
</tr>
<tr>
<td>Time duration</td>
<td>4 pm – 7 pm</td>
</tr>
<tr>
<td>Signal timing</td>
<td>All phases of major and minor streets</td>
</tr>
<tr>
<td>Traffic volume</td>
<td>All movements</td>
</tr>
<tr>
<td>Traffic delay</td>
<td>All movements</td>
</tr>
<tr>
<td>Sampling index</td>
<td>Every five signal cycles (each cycle = 180 s)</td>
</tr>
</tbody>
</table>

### C. Modeling results

Before the HNN model was trained, the raw data needed to be preprocessed to remove or reduce volatility, as shown in Figure 9. For traffic signal and traffic volume data, normalization was conducted to scale data between zero and one. For traffic delay data, after normalization, simple exponential smoothing was applied to further filter the data to remove noise, as shown in (14), where \( I(k) \) is the filtered delay, \( y(k) \) is normalized delay, and \( \alpha \) is the smoothing factor between zero and one. As alpha decreases, the observation of delay at \( k \) has less impact on the output \( I(k) \), indicating that the randomness of the delay measurements is reduced. After training of the HNN model, inverse normalization and inverse smoothing were applied to generate actual model output. This process is shown in Figure 8.

\[
I(k) = \alpha y(k) + (1 - \alpha) I(k - 1)
\]

(14)

The HNN model was trained by 78% of the total data points and was tested with data from March 22–26 (22% of total data). Figure 9 illustrates the HNN model structure applied in this study.

The modeling results were evaluated by mean absolute percentage error (MAPE), root mean square error (RMSE), and mean absolute error (MAE) as in (15)–(17), where \( y_n(k) \) is the true delay at time \( k \) of phase \( n \) and \( \hat{y}_n(k) \) is the predicted delay at time \( k \) of phase \( n \).

\[
\text{MAPE} = \frac{1}{NK} \sum_{k=1}^{N} \sum_{n=1}^{M} \left| \frac{y_n(k) - \hat{y}_n(k)}{y_n(k)} \right|
\]

(15)

\[
\text{RMSE} = \frac{1}{NK} \sum_{k=1}^{N} \sum_{n=1}^{M} \sqrt{(y_n(k) - \hat{y}_n(k))^2}
\]

(16)

\[
\text{MAE} = \frac{1}{NK} \sum_{k=1}^{N} \sum_{n=1}^{M} |y_n(k) - \hat{y}_n(k)|
\]

(17)

Table III and Table IV show the prediction results for all phases of all seven intersections, the phases of main streets and side streets, and the phase of each intersection. Note that delay prediction at main streets is more accurate than at side streets.
streets. The reason is that traffic volumes at side streets are much lower and more stochastic compared with main streets.

![HNN model structure](image)

**Figure 9.** HNN model structure.

<table>
<thead>
<tr>
<th>TABLE III. TRAINING AND TESTING RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training (all)</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>MAPE</td>
</tr>
<tr>
<td>RMSE</td>
</tr>
<tr>
<td>MAE</td>
</tr>
</tbody>
</table>

**Table IV. Testing Results at Each Intersection**

<table>
<thead>
<tr>
<th>Intersection</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAPE</td>
<td>4.0%</td>
<td>5.0%</td>
<td>5.7%</td>
<td>7.7%</td>
</tr>
<tr>
<td>RMSE</td>
<td>3.7 s</td>
<td>5.7 s</td>
<td>10.7 s</td>
<td>11.0 s</td>
</tr>
<tr>
<td>MAE</td>
<td>2.2 s</td>
<td>4.3 s</td>
<td>6.6 s</td>
<td>8.7 s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Intersection</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAPE</td>
<td>7.7%</td>
<td>6.7%</td>
<td>6.1%</td>
</tr>
<tr>
<td>RMSE</td>
<td>12.6 s</td>
<td>8.8 s</td>
<td>10.3 s</td>
</tr>
<tr>
<td>MAE</td>
<td>9.1 s</td>
<td>6.2 s</td>
<td>7.6 s</td>
</tr>
</tbody>
</table>

**Figure 10 and Figure 11 show the distribution and PDF of training errors. Training errors are roughly symmetrically distributed along the horizontal axis.**

**Figure 12 and Figure 13 show distribution and PDF of testing errors.**

**Figure 14 shows comparisons of predicted delay from the HNN model and the true delay of each phase at intersection 1. There are four phases at intersection 1. Figure 14 (a-d) shows the delay comparisons of each phase, respectively.**

<table>
<thead>
<tr>
<th>Comparisons - Testing data - Intersection #1 Phase #1</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Comparison chart" /> (a) Phase 1: Westbound left turning movement</td>
</tr>
</tbody>
</table>
When the model is ready, we will develop an AI-based optimal traffic control system based on the model to minimize entire system costs, including travel delay and energy consumption.

Once a reliable system model is obtained, AI-based control design is required to establish a real-time closed-loop feedback control system that uses the traffic flow state as feedback [40][41]. This approach controls the signal timing intelligently at intersections so that the resulting traffic flow can be made smoother with minimized energy consumption. This control method requires controller design using AI techniques. Because of the random nature of traffic flow systems, stochastic optimal control in a multi-objective Bayesian framework will be investigated in the future.

ACKNOWLEDGMENT

The work performed is funded by the Vehicle Technologies Office of the US Department of Energy under the AI for Mobility program.

This manuscript has been authored by UT-Battelle, LLC, under contract DE-AC05-00OR22725 with the US Department of Energy (DOE). The US government retains and the publisher, by accepting the article for publication, acknowledges that the US government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this manuscript, or allow others to do so, for US government purposes. DOE will provide public access to these results of federally sponsored research in accordance with the DOE Public Access Plan (http://energy.gov/downloads/doe-public-access-plan).

REFERENCES