



# **Sustaining Quality and Warranty**

Module 7A Key Quality Needs to Measure and Assess

# Motivation

*Why is this module important?*



- If the product doesn't conform to quality standards (from the customer's point of view), the customer may be unsatisfied, and you may not be able to sell the product.
- The company selling the product is responsible for maintaining 100% control of the product's quality. While the supplier's quality control is critical, the supplier is not responsible for the final product sold.
- There are many sources of variability that impact the product's quality.

# Module Outline



- Learning objectives
- Acceptance sampling strategies
  - Benefits and risks
  - Sampling plans
  - Operating characteristics curve
- Statistical process control (SPC)
  - Understanding types and importance of variations
  - Variable and attribute control charts
  - Control chart examples

# Learning Objectives



- LO1. Determine quality needs of incoming parts
- LO2. Develop quality monitoring control plan

# What This Module Addresses



- How to implement and conduct efficient acceptance-sampling schemes to decide whether to accept/reject product batches
- What sources of variability impact product quality
- How to use control charts to keep the manufacturing process within statistical control

# Acceptance Sampling

*What it is and how it works*



- **Acceptance sampling** verifies that parts coming from a vendor comply with quality requirements

## **How it works:**

- A company receives a shipment from its vendor (the company randomly selects a sample from the lot and inspects it)
- Based on the results of inspection, the company accepts or rejects the lot—also known as **lot sentencing**

*Note:* Acceptance sampling plans do not provide a methodology for quality control. They are used only to accept or reject the lots. For real quality control, use SPC tools such as control charts.

# Acceptance Sampling

*When is it needed?*

- When dealing with an unproven supplier
- During start-ups and when manufacturing new products
- During destructive testing
- When the cost of 100% inspection is very high, or doing so is not technically feasible
- When products are likely to suffer damage during shipment



# Acceptance Sampling

## *Key benefits*

- Motivates suppliers to get it right the first time
- Costs less because it requires less inspection-related work (reduces labor and instrument costs)
- Minimizes handling of parts—meaning less likelihood of damaging a part during the inspection
- Can reduce the amount of inspection error significantly
- Applies to destructive testing





# Acceptance Sampling

## *Disadvantages*

- Creates risk of accepting bad lots and rejecting good lots
- Does not provide adequate information about the product or manufacturing process
- Requires significant planning and documentation to conduct acceptance sampling compared to 100% inspection

*Note:* 100% inspection will be considerably more expensive than this



# Types Of Sampling Plans

## *Single and double*



- **Single sampling plan:** Make a lot-sentencing decision based on one sample of  $n$  units
- **Double sampling plan:** Make a lot-sentencing decision based on two samples
  - If you decide to reject the lot due to the first sample, you may take a second sample
  - If the total number of defective items in both samples is less than a predetermined number of defectives, then accept the lot; otherwise, reject it

# Types Of Sampling Plans

*Multiple and sequential*



- **Multiple sampling plan:** This plan is similar to the double-sampling technique, but you take more than two samples before making a lot-sentencing decision

*Note:* In multiple sampling, the sample sizes,  $n$ , are usually smaller than they are in single or double sampling

- **Sequential sampling plan:** You select units from the lot one at a time. Based on inspection of each unit, accept a lot, reject it, or select another one.

# Formation of Lots for Inspection

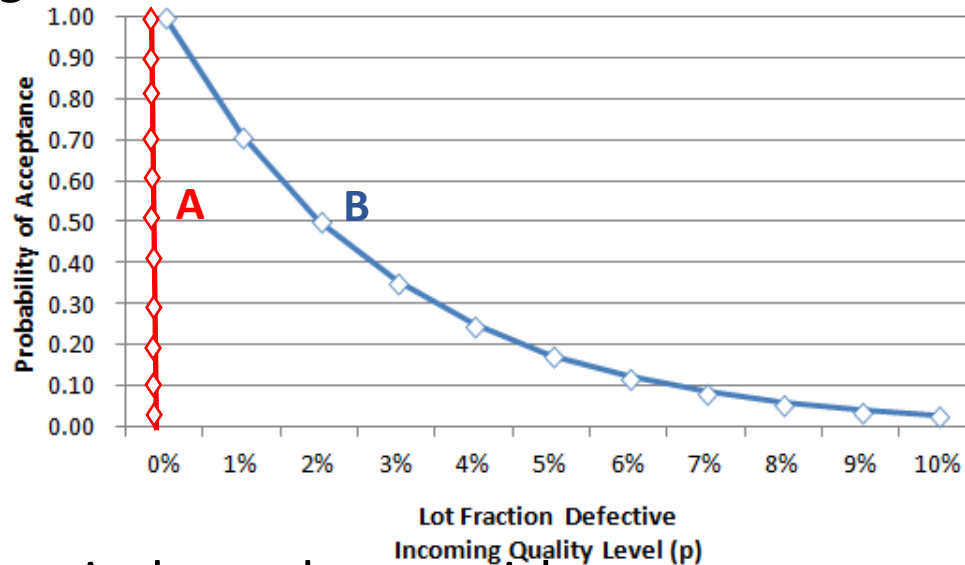
## *Key considerations*

- It is more economical to inspect a larger lot than a smaller lot
- Lots should be homogeneous
- Lots should be formed so that they conform to the materials-handling systems used in both the supplier's and consumer's facilities



# Acceptance Sampling

## Operating characteristic curves



- This curve is dependent on risk acceptance to part malfunction
- Line “A” suggests a part that has high risk associated with faultiness—i.e., a part is expensive to replace, and a defect will cause catastrophic failure, like a part in a pacemaker
- Line “B” suggests a part that has low risk associated with faultiness—i.e., a part is easy to replace, and a defect will not cause catastrophic issue, an easy-to-replace filter

# Statistical Process Control

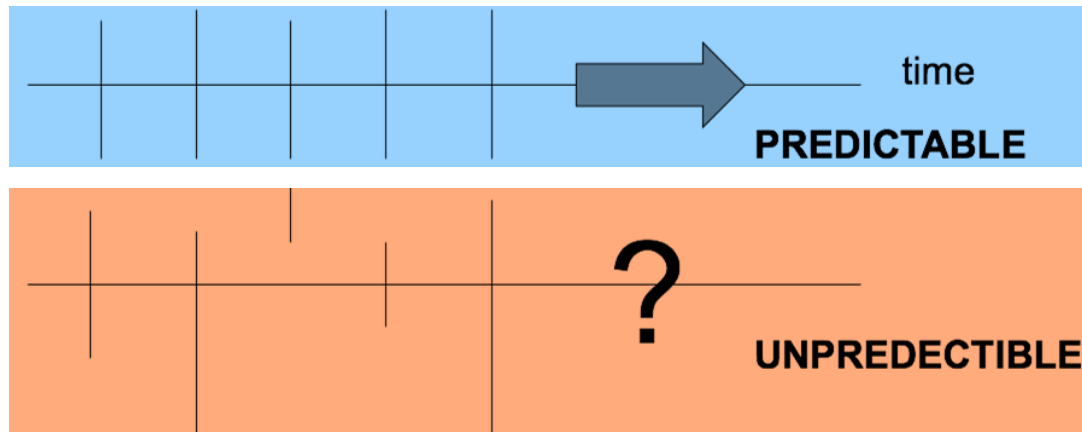
*What it is and how to account for it*

- Understanding types and importance of variations
- Sources of variations
- Variable and attribute control charts
- Control chart examples



# Understanding Variation

*Why it's important*

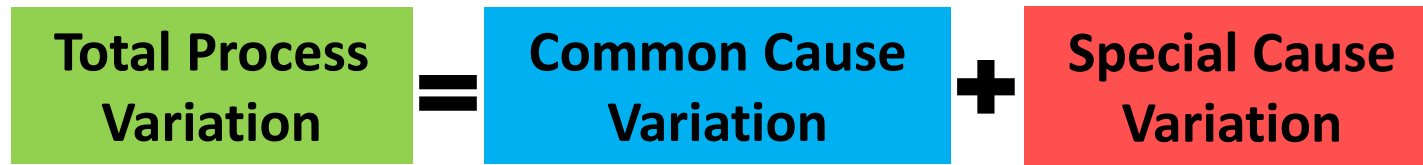


- When a process is stable, we consider it **predictable**
- A process affected by special causes is unstable and therefore considered **unpredictable**
- Unpredictable processes need evaluations for acceptability

*While a fast food burger should have consistency—not unpredictability—the placement of the pickle may vary, but the burger will still be acceptable.*

# Sources Of Variation

*Total process variation*



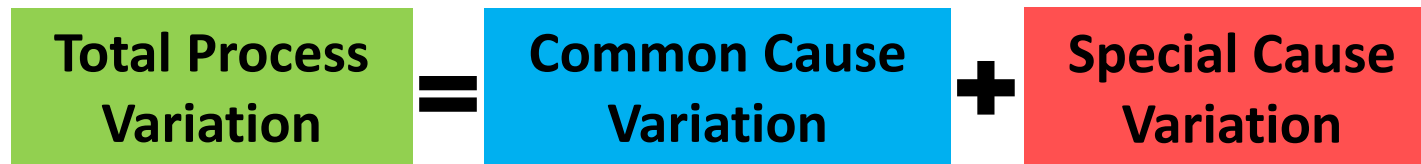
**Variation is often due to differences in:**

- People
- Machines
- Materials
- Methods
- Measurement
- Environment



# Sources Of Variation

*Common cause variation*



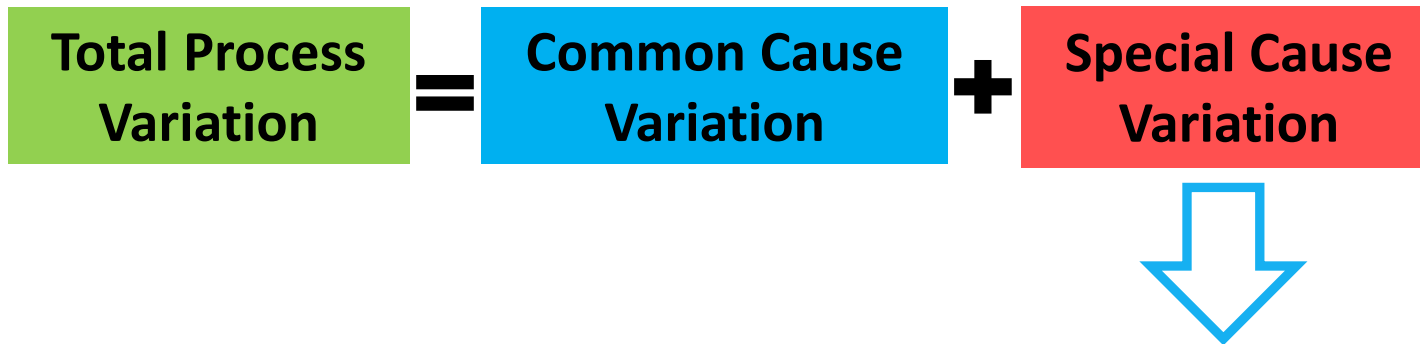
**Variation is often due to:**

- Naturally occurring and expected events
- The result of normal variation in materials, tools, machines, operators, and the environment

*"Common cause" variation in our burger example would be the placement of pickles on different sides of the buns. We expect that level of variation, and it is acceptable.*

# Sources Of Variation

*Special cause variation*



**Variation is often due to:**

- Abnormal or unexpected variation
- An assignable cause
- Variation beyond what is considered inherent to the process  
*"Special cause" variation would be realizing many burgers had 10 pickles instead of 1. Clearly, something unusual is causing unacceptable variation and it needs addressing!*

# Statistical Process Control

*What it is and how to account for it*



- **SPC** monitors a process to identify special causes of variation and the need for corrective action when appropriate
- SPC relies on **control charts**

## Control Charts:

- Establish a state of statistical control
- Monitor a process and signal when it's out of control
- Determine process capability



Key Quality Needs to Measure and Assess

U.S. DEPARTMENT OF ENERGY • OFFICE OF ENERGY EFFICIENCY & RENEWABLE ENERGY

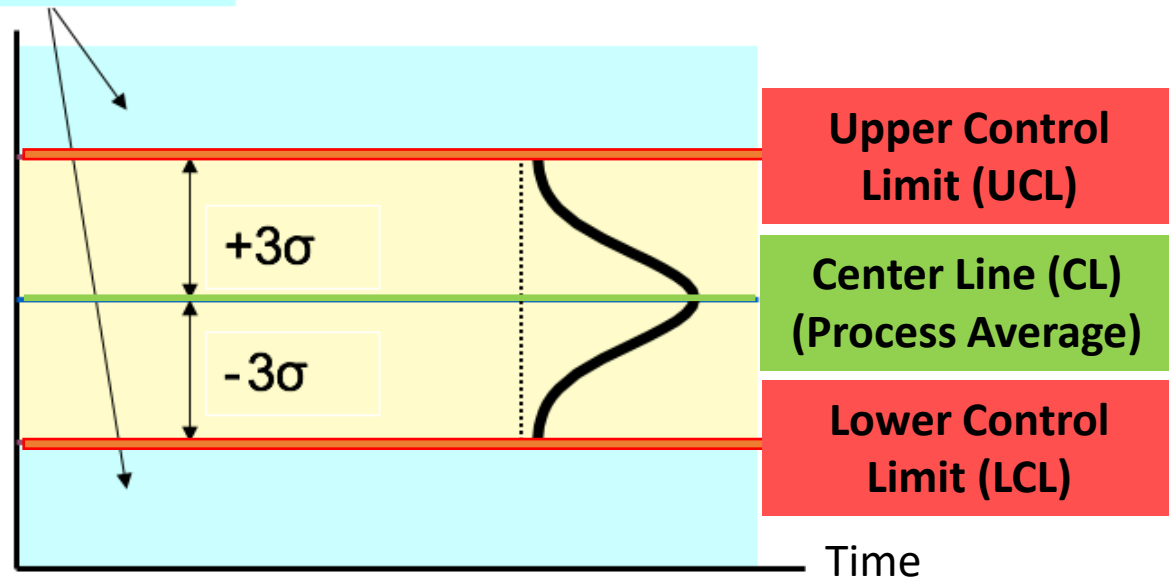
# Control Charts

## Basics



Special Cause Variation:  
Range of **unexpected** variability

Common Cause  
Variation: Range of  
**expected** variability



$UCL = \text{Process Average} + 3 \text{ Standard Deviations}$

$LCL = \text{Process Average} - 3 \text{ Standard Deviations}$

# Control Charts

## *Commonly used types*



### **Variables data (used for measured numeric data):**

- The standard charts for variables data, **X-bar** and **Range charts** (R-charts) help determine if a process is stable and predictable
- They use actual numeric data from a process
- X-bar and s-charts (an s-chart is more accurate than an R-chart but needs more data)
- Charts for individuals (x-charts)

*Definitions in the following slides*

# Control Charts

*Commonly used types (cont.)*



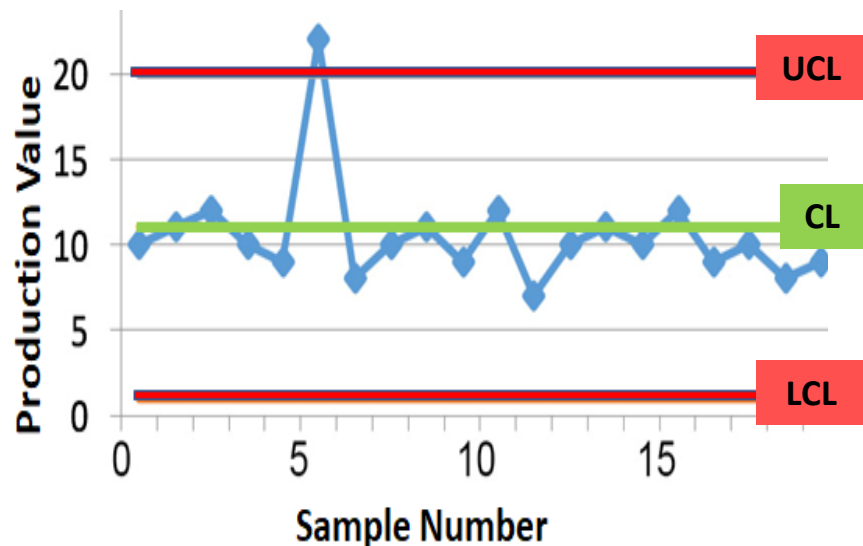
## Attribute data (used when variable data is not available):

- Attribute data is qualitative data
- For “proportion defectives” (p-chart)
- For “number of defects per item” (c-chart), **attribute data** is not acceptable for production part submissions unless you can obtain **variable data**. Examples of use include the presence or absence of a required label, or the installation of all required fasteners.

# X-Bar and R-Chart

*What they are and when to use them (cont.)*

- The **R-chart** monitors variables data from regular sample collections. An R-chart shows how the range of the subgroups changes over time.
- Processes that have a subgroup size of two or more use it
- It starts with at least 20 subgroups of observed values
- The chart tells when to correct the process and when to leave it alone
- It distinguishes between normal and abnormal variation



Key Quality Needs to Measure and Assess

U.S. DEPARTMENT OF ENERGY • OFFICE OF ENERGY EFFICIENCY & RENEWABLE ENERGY

# Computing Control Limits



- The value  $A_2R$  is used to estimate  $3\sigma$  (where  $A_2$  is known as the Shewhart Factor) and it depends on the subgroup size  $n$
- The upper and lower control limits are as follows:

$$\text{Upper Control Limit (UCL)} = \bar{\bar{X}} + A_2(\bar{R})$$

$$\text{Lower Control Limit (LCL)} = \bar{\bar{X}} - A_2(\bar{R})$$



# R-chart Control Limits



- The upper and lower control limits for an R-chart are as follows:

$$\text{Upper Control Limit (UCL)} = D_4(\bar{R})$$

$$\text{Lower Control Limit (LCL)} = D_3(\bar{R})$$

Where  $D_4$  and  $D_3$  are taken from the **Shewhart table** for subgroup size =  $n$

# X-Bar and R-Charts

## Example



## Process measurements

Sample Collected on Day Number	Individual Measurements				Subgroup Measurements	
					Mean, X-Bar	Range, R
1	15	17	15	11	14.5	6
2	12	16	9	15	13.0	7
3	17	21	18	20	19.0	4
4	14	18	13	17	15.5	4
					Grand Mean equals $\bar{X}$	Mean Range equals $\bar{R}$

# X-Bar and R-Charts

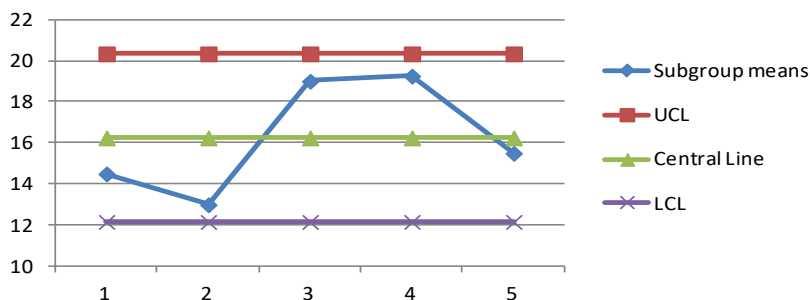
## Solution



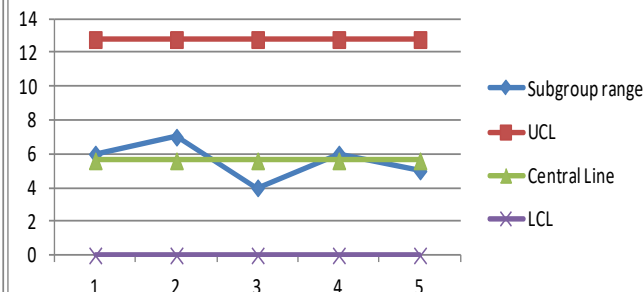
Sample Collected on day	Individual measurements				Subgroup measures	
					Mean, $\bar{x}$	Range, R
1	15	17	15	11	14.5	6
2	12	16	9	15	13	7
3	17	21	18	20	19	4
4	16	19	20	22	19.25	6
5	14	18	13	17	15.5	5
Grand mean					16.25	5.6

Control Limit calculations	x-bar Chart	R-Chart	For n= 4, from Appendix 2 of your textbook		
LCL	12.1676	0	A2	D3	D4
Grand Mean (Center Line)	16.25	5.6	0.729	0	2.282
UCL	20.3324	12.7792			

**x-bar chart**



**R-Chart**



Key Quality Needs to Measure and Assess

# Process Stability



- Process variation is random variation (i.e., has a common cause) and not nonrandom variation (i.e., has no special or assignable causes)
- The use of process control charts determines process stability
- Generally, if at least one point plots beyond the control limits or much farther away from the process mean than is expected, the process is out of control
- Additionally, if the data points behave in a systematic or nonrandom manner, then the process may be out of control

# Attribute Charts



- Attribute charts can assist in monitoring processes providing goods or services
- These charts can be used whenever counts or percentages of nonconformities can be obtained
- **p chart:** fraction nonconforming
- **np chart:** number nonconforming
- **Percent nonconforming charts**
- **c charts:** counts of nonconformities
- **u charts:** nonconformities per unit

# Selecting Attribute Control Charts

## *C-Charts*



**When studying the proportion of products rendered unusable by nonconformities, one the following charts should be used:**

- Fraction nonconforming (p) chart
- Number nonconforming (np) chart
- Percent nonconforming chart should be used

**When tracking the number of nonconformities, one of the following charts should be used:**

- Number of nonconformities (c) chart
- Number of nonconformities per unit (u) chart

# C-Charts Fraction Nonconforming

## *Development*



The sample fraction nonconforming is given as

$$\hat{p} = \frac{D}{n}$$

where  $\hat{p}$  is a random variable with mean and variance

$$\mu = p \quad \sigma^2 = \frac{p(1-p)}{n}$$

# C-Charts Fraction Nonconforming

*Standard given*



- If a standard value of  $p$  is given, then the control limits for the fraction nonconforming are as follows:

$$\text{Upper Control Limit (UCL)} = p + 3\sqrt{\frac{p(1-p)}{n}}$$

$$\text{Center Line (CL)} = p$$

$$\text{Lower Control Limit (LCL)} = p - 3\sqrt{\frac{p(1-p)}{n}}$$



# C-Charts Fraction Nonconforming

*No standard given*

- If no standard value of  $p$  is given, then the control limits for the fraction nonconforming are as follows:

Upper Control  
Limit (UCL)

$$= \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Center Line (CL) =  $p$

Lower Control  
Limit (LCL)

$$= \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

where

$$\bar{p} = \frac{\sum_{i=1}^m D_i}{mn} = \frac{\sum_{i=1}^m \hat{p}_i}{m}$$

# C-Charts Fraction Nonconforming

## Example 1



- A process that produces bearing housings is investigated. Select 10 samples a lot size of 100.

Sample #	1	2	3	4	5	6	7	8	9	10
# Nonconf.	5	2	3	8	4	1	2	6	3	4

- Is this process operating in statistical control?

# C-Charts Fraction Nonconforming

*Example 1 (cont.)*



$n = 100, m = 10$

Sample #	1	2	3	4	5	6	7	8	9	10
# Nonconf.	5	2	3	8	4	1	2	6	3	4
Fraction Nonconf. ( $\hat{p}_i$ )	0.05	0.02	0.03	0.08	0.04	0.01	0.02	0.06	0.03	0.04

$$\bar{p} = \frac{\sum_{i=1}^m \hat{p}_i}{m} = 0.038$$

# C-Charts Fraction Nonconforming

*Example 1 (cont.)*



Control Limits are:

**Upper Control  
Limit (UCL)**

$$= 0.038 + 3\sqrt{\frac{0.038(1-0.038)}{100}} = 0.095$$

**Center Line (CL)**

$$= 0.038$$

**Lower Control  
Limit (LCL)**

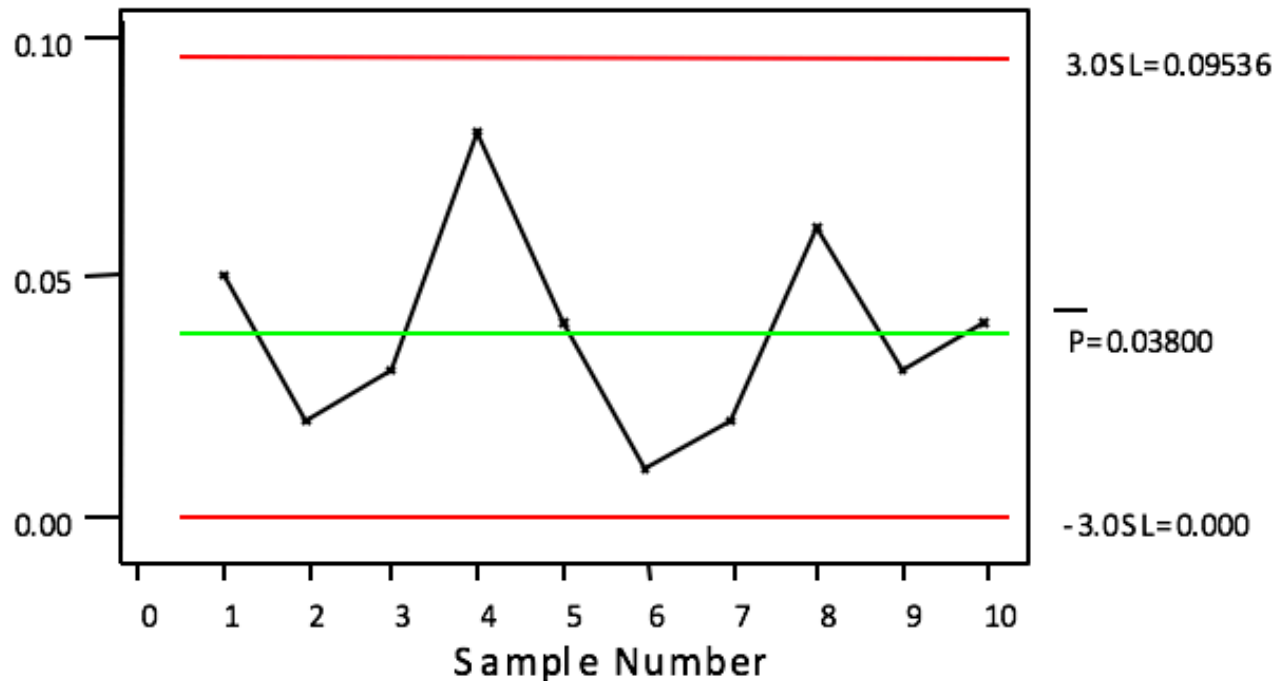
$$= 0.038 + 3\sqrt{\frac{0.038(1-0.038)}{100}} = -0.02 \rightarrow 0$$

# C-Charts Fraction Nonconforming

Example 1 (cont.)



P Chart for C1



Upper Control  
Limit (UCL)

Center Line (CL)

Lower Control  
Limit (LCL)

- Variation is acceptable when results stay within the upper and lower control limits

# Nonconformities or Defects



- There are many instances where an item will contain nonconformities, but the item itself is not classified as nonconforming.
- It is often important to construct control charts (c-charts) for the total number of nonconformities or the average number of nonconformities for a given “area of opportunity.” (The inspection must be the same for each unit.)

# Constant Sample Size

## *Procedures*



### **C-chart**

**Standard Given:**

$$UCL = c + 3\sqrt{c}$$

$$CL = c$$

$$LCL = c - 3\sqrt{c}$$

**No Standard Given:**

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$

$$CL = \bar{c}$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

# C-Charts

## *Example 2 – Pure & White*



- Pure & White, a manufacturer of copy-machine paper, monitors their production using a c-chart. Pure & White produces paper rolls that are 12 feet long and 6 feet in diameter
  - A sample is taken from each completed roll,  $n=1$ , and the lab checks it for nonconformities
- Nonconformities include discolorations, inconsistent paper thickness, flecks of dirt in the paper, moisture content, and ability to take ink. All of these nonconformities have the same weight on the c-chart
- A sample may come from anywhere in the roll so that the area of opportunity for these nonconformities is large, while the overall quality of the paper creates only a very small chance of a specific nonconformity occurring at any one location



# C-Charts

## Example 2 – Pure & White (cont.)



- Create a c-chart for the Pure & White company by using the data given in the table

Center line	11
$UCL_c$	20
$LCL_c$	1

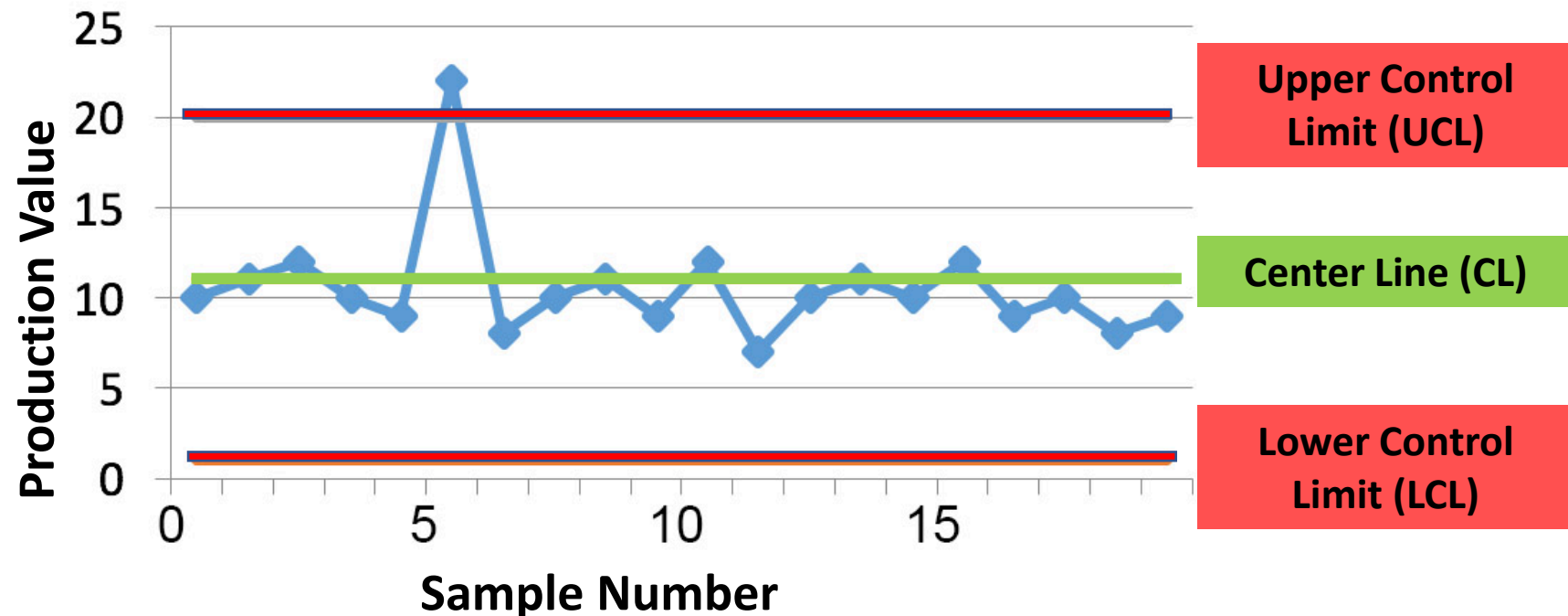
Sample Number	Production Value
8	10
9	11
10	9
11	12
12	7
13	10

# C-Charts

## Example 2 – Pure & White (cont.)

□ Is this process under statistical control? **No.**

*Note:* The plot points represent the count of nonconformities (c-value) in each sample (per prior slide values)



# List Of Terms

## *In glossary*



- [Acceptance sampling](#) uses statistical sampling to determine whether to accept or reject a production lot of material.
- [Single sampling plan](#) a single sampling plan for attributes is a statistical method by which the lot is accepted or rejected on the basis of one sample.
- [Double sampling plan](#) is performed after the first sample is tested. There are then three outcome possibilities: 1) accept lot, 2) reject lot, 3) no decision. If the outcome is (3), and a second sample is taken then the procedure is to combine the results of both samples and make a final deciding on that information.
- [Multiple sampling plans](#) are the extension of the double sampling plans when more than two samples are needed to reach a conclusion. The advantage of multiple sampling is a smaller sample size.
- [Sequential sampling](#) is the ultimate extension of the multiple samples where items are selected from a lot one at a time and after inspection of each a decision is made to accept the lot or select a both sample to review.
- [Predictable Variation](#) also referred to as **Expected Variation** are processes for which process variation is the result of random causes. The more stable processes produce products that adhere to specifications to a greater degree than those produced by unstable processes. The more variation in measured product characteristics, the less the product adheres to specifications,
- [Unpredictable Variation](#) also referred to as **Unexpected Variation** falls outside the upper and lower acceptable limits established for a product specification, which can lead to product discard or salvage. Therefore, a key manufacturing performance objective is the establishment of stable and predictable processes that limits variation to what can be described as random, minimum variation around target values.
- [Statistical Process Control \(SPC\)](#) is a method of [quality control](#) which employs [statistical methods](#) to monitor and control a process. This helps ensure the process operates efficiently, producing more specification-conforming product with less waste (rework or [scrap](#)).
- [Control Charts](#) also known as Shewhart charts (after [Walter A. Shewhart](#)) or process-behavior charts, are a [statistical process control](#) tool used to determine if a [manufacturing](#) or [business process](#) is in a state of [control](#).

# List Of Terms

## *In glossary (cont.)*



- **X and R Chart**  $\bar{x}$   $\{\bar{x}\}$   $Xx$   $xxxx$  and **x and** is a type of control chart used to monitor variables data when samples are collected at regular intervals from a business or industrial process.
- **Attribute Data** is data that can't fit into a continuous scale but instead is chunked into distinct *buckets*, like small/medium/large, pass/fail, acceptable/not acceptable, and so on.
- **Variable Data** is data, such as length or pressure, in a time-ordered sequence. In contrast, attribute control charts plot count data, such as the number of defects or defective units.
- **X-bar chart** - is a type of **Shewhart** control chart that is used to monitor the **arithmetic means** of successive samples of constant size,  $n$ . This type of control chart is used for characteristics that can be measured on a continuous scale, such as weight, temperature, thickness etc.
- **R-Chart** or Range Chart plots average **R** value, or **R-bar** to determine the upper control limit.
- **Shewhart Individuals Control Chart or Table** is used in statistical quality *control*, the individual/moving-range chart is a type of *control* chart used to monitor variables data from a business or industrial process for which it is impractical to use rational subgroups.
- **P-Chart** - is a type of **control chart** used to monitor the proportion of **nonconforming units** in a **sample**, where the sample proportion nonconforming is defined as the ratio of the number of nonconforming units to the sample size,  $n$ .
- **NP Chart** - is a type of **control chart** used to monitor the number of **nonconforming units** in a **sample**.
- **Percent Nonconforming Charts** is a statistical quality control chart used to monitor the proportion of nonconforming units in a sample, where the sample proportion nonconforming is defined as the ratio of the number of nonconforming units to the sample size.
- **C Charts** is a type of **control chart** used to monitor "count"-type data, typically total number of nonconformities per unit. It is also occasionally used to monitor the total number of events occurring in a given unit of time.
- **U Charts** is a type of **control chart** used to monitor "count"-type data where the sample size is greater than one, typically the average number of nonconformities per unit.