Tensor Computation:
Application to Power System Analysis

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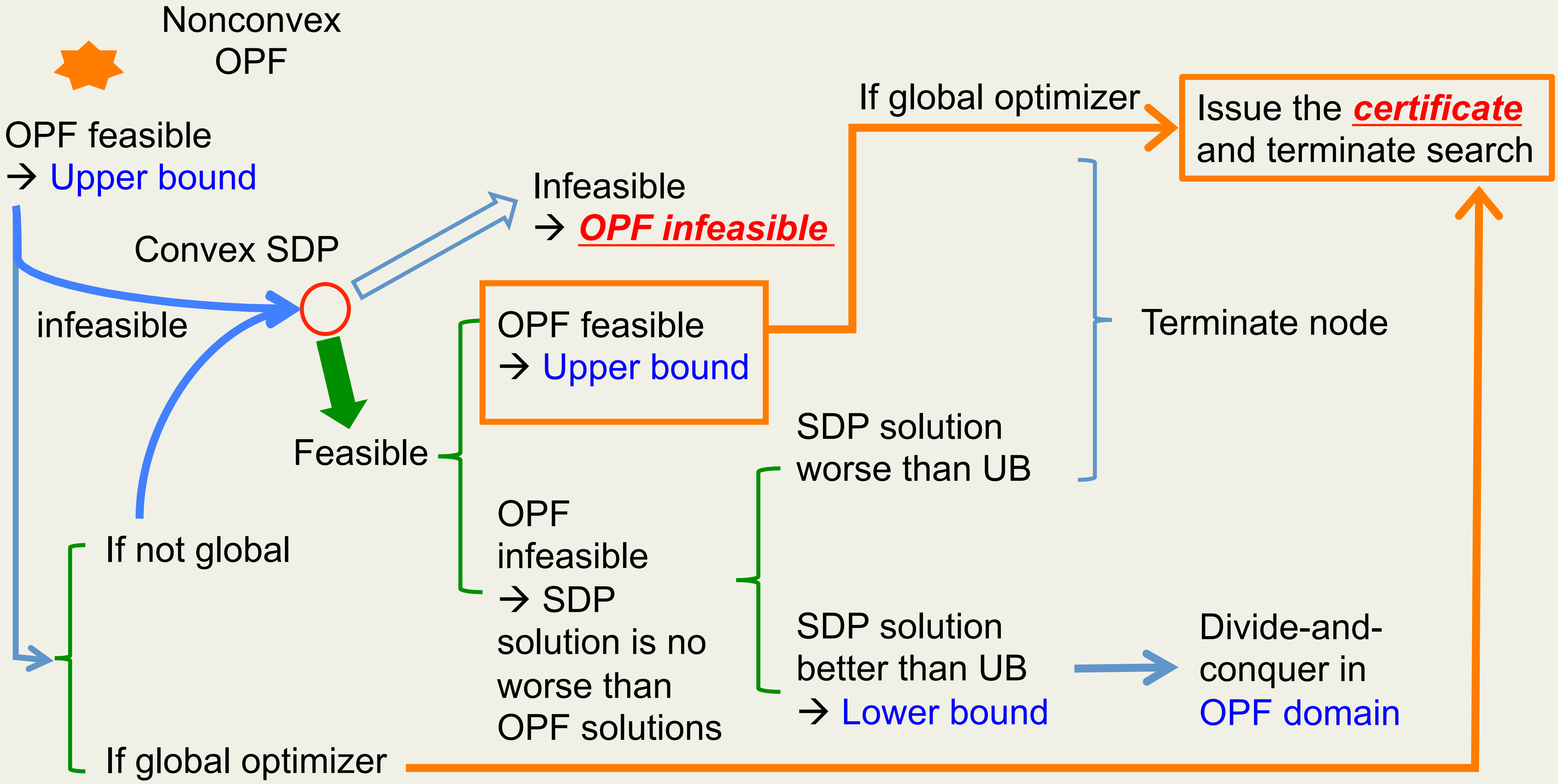
Summary of the Achievements during 2014-15 I

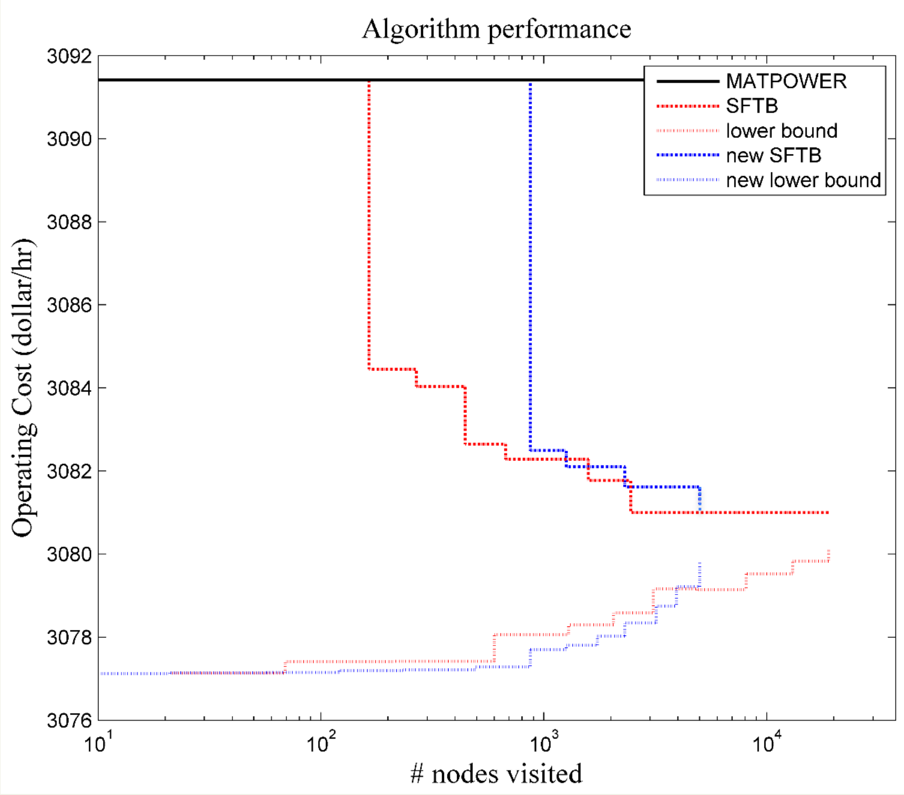


Summary of the Achievements during 2014-15 II

- Divide-and-conquer
  - Angular cut
    - Linear constraints in the SDP framework
  - Voltage magnitude cut
    - Linear constraints in the SDP framework
- Clique decomposition and merging
  - Control of the sizes of cliques for efficient computation
- Certificates of original AC OPF problems
  - Infeasibility
  - Global solution using the trust-region method
    - 0.1-2% of cost saving

Summary of the Achievements during 2014-15 II

- 14-bus case
  - Global solution found
  - ~3,000 nodes visited
- Identified problem
  - Same SDP solution found at parent and child nodes
  - Inappropriate cuts
  - Branching is based upon SDP solution → Wrong voltage estimates



Big Question:
How do we find voltages to represent an SDP solution?

Big Question

- Angular and the voltage magnitude cuts
  - SDP solution is not feasible in the power flow domain
  - Projection of the SDP solution to the power flow domain
- Current approach
  - Rank-1 approximation from $W \rightarrow v$
  - Projection of *v* on the feasible power flow domain
  - Projected voltage vector does not meet the power injections and flows from the SDP solution
- New approach
  - Voltage is not well defined in SDP for a multiple rank solution, but the power injections and flows are
  - Two approaches to extract voltage vector from
    - Power balance equations → Power Flow problem
    - All available information → State estimation

Physically Not-Meaningful SDP Solution

$W = \lambda_1 u_1 u_1^T + \lambda_2 u_2 u_2^T + \ldots$ where $\lambda_1 >> \lambda_2 \geq \lambda_3 \geq \ldots \geq 0$

- Problems that feasibility is marginally relaxed
  - 1st term approximation of *W*, i.e., $W \cong \lambda_1 u_1 u_1^T \rightarrow v = \sqrt{\lambda_1} u_1$
  - Shortcoming: Unacceptable voltage magnitudes
- Power flow problem: power injections at all buses are known
  - *2N* Equations and *2N-1* Unknowns
  - Use *2N-1* equations: *N-1* Real power and *N* reactive power
  - Slack bus compensate system-wide power mismatch
- State Estimation
  - Power injections and voltage magnitudes: *3N* information
  - Power flows: *4L* information
  - Over-determined problem

1^{st} Term Approximation

- 14-bus case
- Eigenvalue decomposition
  - 22 non-zero eigenvalues
  - λ_1 (= 14.26) >> λ_2 (= 0.0722) > … > λ_{22} (= 0.0011) > 10^{-3}
  - 1^{st} term approximation → v_1
  - $\|W - v_1 v_1^T\|_2 / \|W\|_2$ = 0.005 ≈ λ_2/λ_1
- Measurements, *y*
  - Voltage magnitudes from *W*
  - Real and reactive power injections
  - Real and reactive power flows
- Error in the measurements
  - True value $\hat{y}_1$ is estimated based on v_1
  - $\|y - \hat{y}_1\|_2 / \|y\|_2$ = 0.237

$$v_1 = \begin{pmatrix} 1.048 \\ 1.042 - j0.026 \\ 1.010 - j0.112 \\ 1.003 - j0.068 \\ 1.008 - j0.051 \\ 1.028 - j0.045 \\ 1.006 - j0.057 \\ 1.030 + j0.007 \\ 0.989 - j0.091 \\ 0.982 - j0.087 \\ 0.995 - j0.068 \\ 0.997 - j0.064 \\ 0.992 - j0.067 \\ 0.962 - j0.097 \end{pmatrix}$$

Power Flow Problems

- Power flow problem
  - Each node has
    - 4 variables: 2 voltages and 2 power injections
    - 2 power balance equations
  - Solve two unknowns when 2 variables are known
    - Slack bus: 2 voltages
    - PV bus: 1 voltage + 1 power injection
    - PQ bus: 2 power injections
- Current Algorithms
  - Newton-Raphson method
  - Gauss-Seidel method
  - Decoupled power flow method
  - Holomorphic Embedding
  - All the power mismatches are assigned to the slack bus
    → No mismatch is allowed

Conventional Power Flow Algorithms

- NR, GS, DPF, and HELM include one slack bus
  - Voltages are known at that bus
- Slack bus in PF problems
  - System-wide power mismatch is assigned
    - Losses over a grid are compensated by the generator at the slack bus
    - Reference bus: voltage angle is fixed, usually zero
  - Voltage magnitude is known
- All the buses except one slack bus are converted to PQ buses, then PF problem is solved as a function of
  - The choice of the slack bus
  - Voltage magnitude at the slack bus

Power Flow Problems: Numerical Results

- Covert all PV and PQ buses to PQ buses
- Conventional method, v_C
  - $\|W - v_C v_C^T\|_2 / \|W\|_2 = 0.006$
  - Mismatches at the slack bus = 0.02MW and 0.04MVar
  - Losses = 4.69MW
  - $\|y - \hat{y}_C\|_2 / \|y\|_2 = 0.228$
- Comparison
  - $\|W - v_1 v_1^T\|_2 / \|W\|_2 = 0.005$
  - $\|y - \hat{y}_1\|_2 / \|y\|_2 = 0.237$
- Marginally better result

$$v_C^{PF} = \begin{pmatrix} 1.051 \\ 1.044 - j0.026 \\ 1.012 - j0.112 \\ 1.006 - j0.068 \\ 1.010 - j0.051 \\ 1.031 - j0.045 \\ 1.009 - j0.057 \\ 1.033 + j0.007 \\ 0.992 - j0.091 \\ 0.985 - j0.087 \\ 0.998 - j0.068 \\ 1.000 - j0.063 \\ 0.995 - j0.066 \\ 0.965 - j0.097 \end{pmatrix}$$

State Estimation

- SDP solution yields
  - Voltage magnitudes, power injections, and flows
  - Over-determined problem
- Conventional state estimator
  - Power flow equations: $y = f(v) \rightarrow \Delta v = -\left[(\nabla_v f)^T W (\nabla_v f)\right]^{-1} (\nabla_v f)^T W \left[y - f(v)\right]$
  - Find voltages and errors: $\delta v = -\left[(\nabla_v f)^T W (\nabla_v f)\right]^{-1} (\nabla_v f)^T W \left[y - f(\hat{v})\right]$
  - Bad data detection and elimination
- Problems
  - Measurements *y* is not consistent
    - Average Jacobian $\overline{\nabla_v f}$ should be used instead
  - Gain matrix $(\nabla_v f)^T W (\nabla_v f)$ can be ill-conditioned

State Estimation: Numerical Results

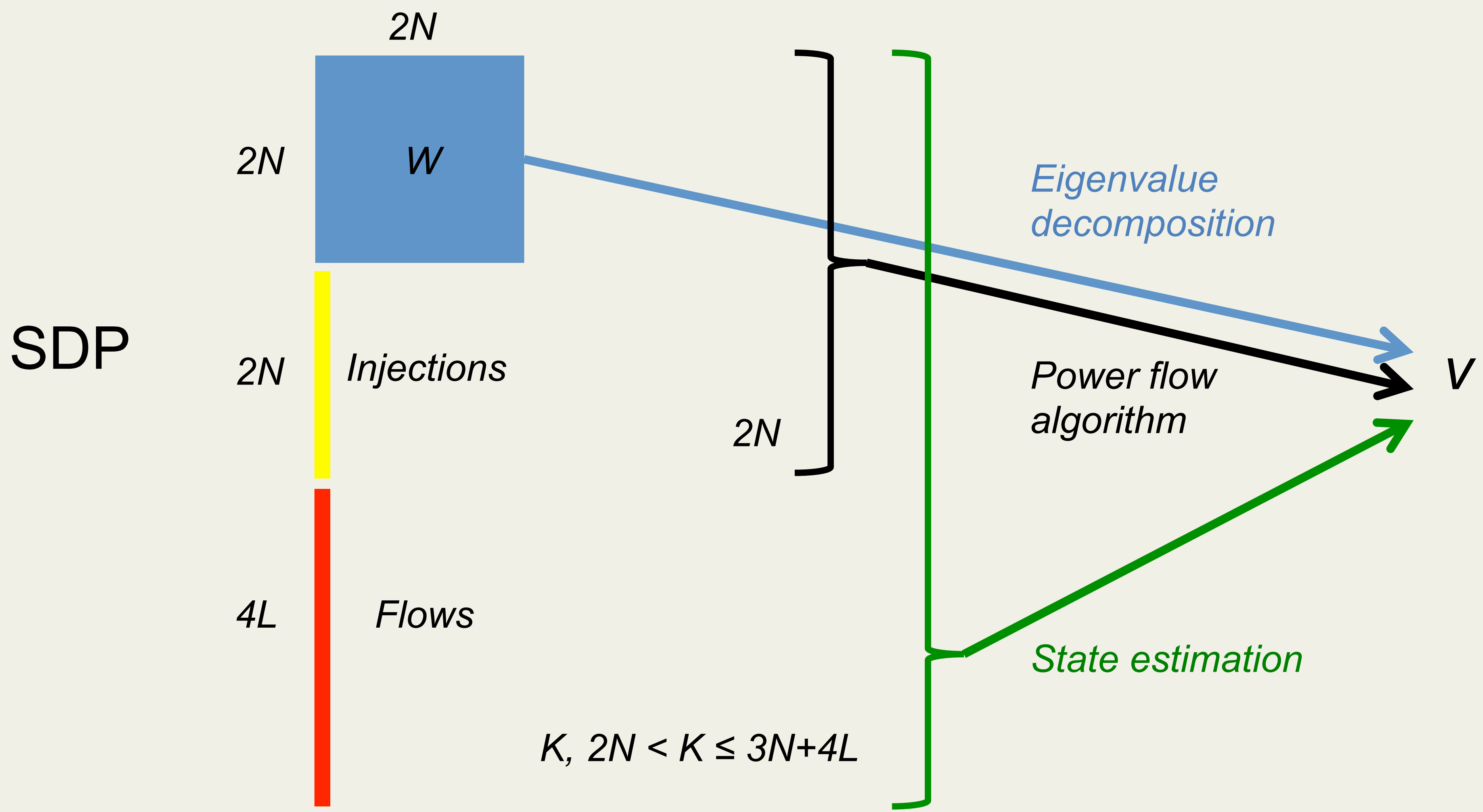
- Pass the Chi-square test (k = 122 – 27 = 95)
- Weight factor
  - Injections at PQ bus = 1
  - Everything else = 0.8
- Conventional method, v_C
  - $\left\|D_{\sqrt{w}}(y-\hat{y}_C)\right\|_2 / \left\|D_{\sqrt{w}}y\right\|_2$ = 0.208
  - $\left\|W - v_C v_C^T\right\|_2 / \left\|W\right\|_2$ = 0.129
  - $\left\|\delta v_C\right\|_2 / \left\|\hat{v}_C\right\|_2$ = 0.038
  - $v_C = E\xi$, ξ has 22 non-zero elements
    - ξ_1 (= 3.554) >> ξ_2 (=0.160) > … > ξ_{22} (= 0.001) > 10^{-3}

$$v_C^{SE} = \begin{pmatrix} 0.987 \\ 0.975 - j0.040 \\ 0.923 - j0.161 \\ 0.938 - j0.101 \\ 0.948 - j0.076 \\ 0.974 - j0.083 \\ 0.941 - j0.092 \\ 0.953 + j0.008 \\ 0.929 - j0.146 \\ 0.926 - j0.147 \\ 0.942 - j0.119 \\ 0.946 - j0.108 \\ 0.939 - j0.116 \\ 0.902 - j0.168 \end{pmatrix}$$

Summary: Conventional Algorithms

$$v_1 = \begin{pmatrix} 1.048 \\ 1.042 - j0.026 \\ 1.010 - j0.112 \\ 1.003 - j0.068 \\ 1.008 - j0.051 \\ 1.028 - j0.045 \\ 1.006 - j0.057 \\ 1.030 + j0.007 \\ 0.989 - j0.091 \\ 0.982 - j0.087 \\ 0.995 - j0.068 \\ 0.997 - j0.064 \\ 0.992 - j0.067 \\ 0.962 - j0.097 \end{pmatrix} \quad v_C^{PF} = \begin{pmatrix} 1.051 \\ 1.044 - j0.026 \\ 1.012 - j0.112 \\ 1.006 - j0.068 \\ 1.010 - j0.051 \\ 1.031 - j0.045 \\ 1.009 - j0.057 \\ 1.033 + j0.007 \\ 0.992 - j0.091 \\ 0.985 - j0.087 \\ 0.998 - j0.068 \\ 1.000 - j0.063 \\ 0.995 - j0.066 \\ 0.965 - j0.097 \end{pmatrix} \quad v_C^{SE} = \begin{pmatrix} 0.987 \\ 0.975 - j0.040 \\ 0.923 - j0.161 \\ 0.938 - j0.101 \\ 0.948 - j0.076 \\ 0.974 - j0.083 \\ 0.941 - j0.092 \\ 0.953 + j0.008 \\ 0.929 - j0.146 \\ 0.926 - j0.147 \\ 0.942 - j0.119 \\ 0.946 - j0.108 \\ 0.939 - j0.116 \\ 0.902 - j0.168 \end{pmatrix} \rightarrow \begin{cases} \|y - \hat{y}_1\|_2 / \|y\|_2 = 0.237 \\ \|y - \hat{y}_{PF}\|_2 / \|y\|_2 = 0.228 \\ \|D_{\sqrt{w}}(y - \hat{y}_{SE})\|_2 / \|D_{\sqrt{w}} y\|_2 = 0.208 \end{cases}$$

Summary II



2N
2N
W
Eigenvalue decomposition
SDP
2N
Injections
Power flow algorithm
V
2N
4L
Flows
State estimation
K, 2N < K ≤ 3N+4L

Desired Capabilities in PF and SE Tools

- Operation of modern power systems
  - Uncertainty
- Technologies are ready
  - Renewables, smart grid technologies
  - Voltage control capability
- Current tools compute voltages
  - Using *K* individual equations $y_j = f_j(v) \;\; j = 1, 2, \ldots, K$ collectively solve for voltages
    - Difficult to integrate: Uncertainties and Errors
  - All the data must be
    - Either consistent with very small errors or outliers for SE
    - Exact without errors for PF

Conventional Power Flow Algorithms in CCS

- Power balance equations in the Cartesian coordinate system
  - Y_k: Y_{bus} matrix at Bus k $= G_k + jB_k$
  - Real (p_k) and reactive (q_k) power injections at Bus k
  - Voltage in CCS, v: $v^T G_k v = p_k, \; v^T B_k v = q_k$
- Voltage at Bus k
  - e_k is the k^{th} column vector of the identity matrix I_{2N}
  - Voltage magnitude at PV buses

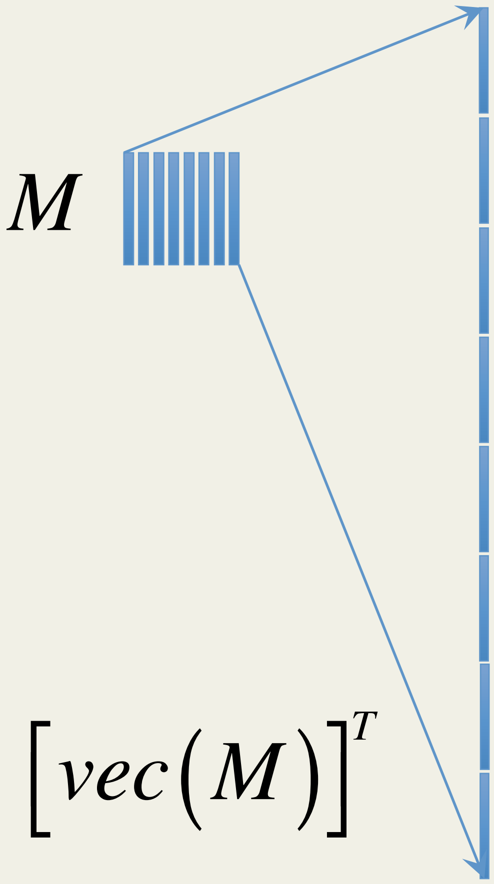
    $$v^T E_k E_k^T v = |v_k|^2 \quad \text{where } E_k = \begin{bmatrix} e_k & e_{k+N} \end{bmatrix}$$
  - Voltage angle at the slack bus

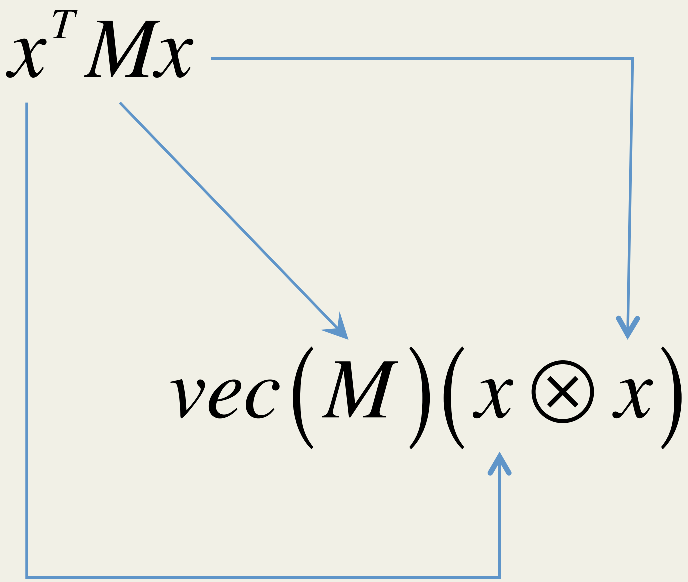
    $$v^T e_{ref+N} e_{ref+N}^T v = \theta_{ref} = 0$$
- Generalized form with a symmetric matrix M:

$$v^T M v = c$$

Kronecker Product

- Two dimensional matrix → one dimensional vector
- Matrix-vector multiplication → vector-vector multiplication
- Kronecker product converts all the equations in a linear form





Tensor-based Power System Analysis

- Kronecker product $\otimes$
  - Bridge between tensor and matrix computations
  - Power balance equations

$$vec(G_k)(v \otimes v) = p_k, \;\; vec(B_k)(v \otimes v) = q_k$$

  - Power flows

$$vec(G_{km})(v \otimes v) = p_{km}, \;\; vec(B_{km})(v \otimes v) = q_{km}$$

  - Voltage magnitudes: $vec\left(E_k E_k^T\right)(v \otimes v) = |v_k|^2$
  - Voltage angles: $vec\left(e_{ref+N} e_{ref+N}^T\right)(v \otimes v) = \theta_{ref} = 0$
- Generalized equation: $vec(A_k)(v \otimes v) = b_k$

Application to Power Flow Algorithm

- Exact case
  - PF has a just-determined system
    - *2N* unknowns and *2N* equations
  - Power flow equations become
    - Bi-linear in *v*: $\begin{bmatrix} vec(A_1) \\ \vdots \\ vec(A_{2N}) \end{bmatrix} (v \otimes v) = \begin{pmatrix} b_1 \\ \vdots \\ b_{2N} \end{pmatrix}$
- Inexact case but just-determined system
  - Slack bus: only angle is known → $|v|_{ref}$ unknown
    - Same as other buses
    - Real power constraint
  - Uncertainty will be associated with constraints as *w*
  - Weighted least square problem: $\begin{bmatrix} w_1 vec(A_1) \\ \vdots \\ w_{2N} vec(A_{2N}) \end{bmatrix} (v \otimes v) = \begin{pmatrix} w_1 b_1 \\ \vdots \\ w_{2N} b_{2N} \end{pmatrix}$

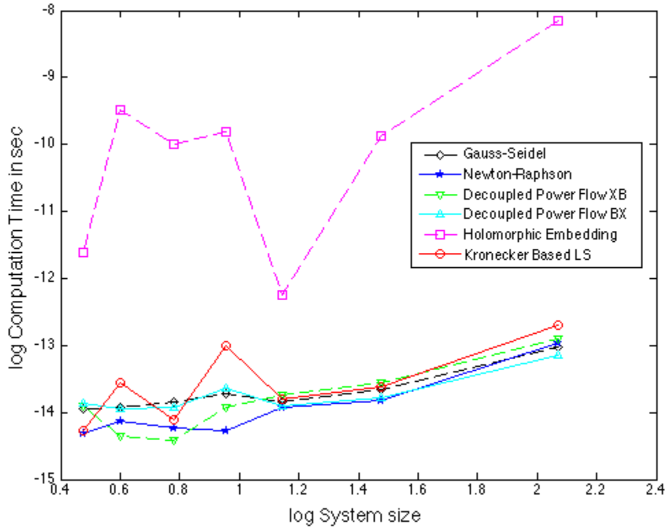
Power Flow Problems: Numerical Results

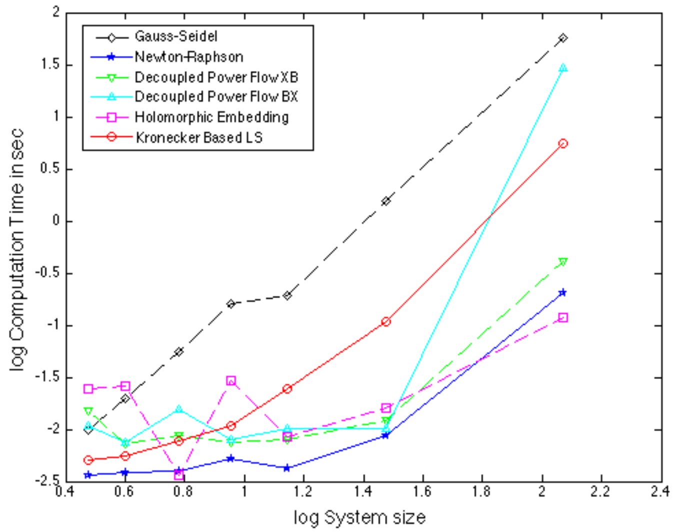
- Covert all PV and PQ buses to PQ buses
- Kronecker product, v_K
  - $\|W - v_K v_K^T\|_2 / \|W\|_2$ = 0.06
  - Mismatches are distributed over all buses (≤ 10^{-3})
  - Losses = 4.36MW and -26.34MVar
  - $\|y - \hat{y}_K\|_2 / \|y\|_2$ = 0.224
- Conventional method, v_C
  - $\|W - v_C v_C^T\|_2 / \|W\|_2$ = 0.06
  - Mismatches at the slack bus: 0.02MW and 0.04MVar
  - Losses = 4.57MW and 18.34MVar
  - $\|y - \hat{y}_C\|_2 / \|y\|_2$ = 0.228

$$v_C^{PF} = \begin{pmatrix} 1.048 \\ 1.042 - j0.026 \\ 1.010 - j0.112 \\ 1.003 - j0.068 \\ 1.008 - j0.051 \\ 1.028 - j0.045 \\ 1.006 - j0.057 \\ 1.030 + j0.007 \\ 0.989 - j0.091 \\ 0.982 - j0.087 \\ 0.995 - j0.068 \\ 0.997 - j0.064 \\ 0.992 - j0.067 \\ 0.962 - j0.097 \end{pmatrix}$$

Comparison I: Exact Solution

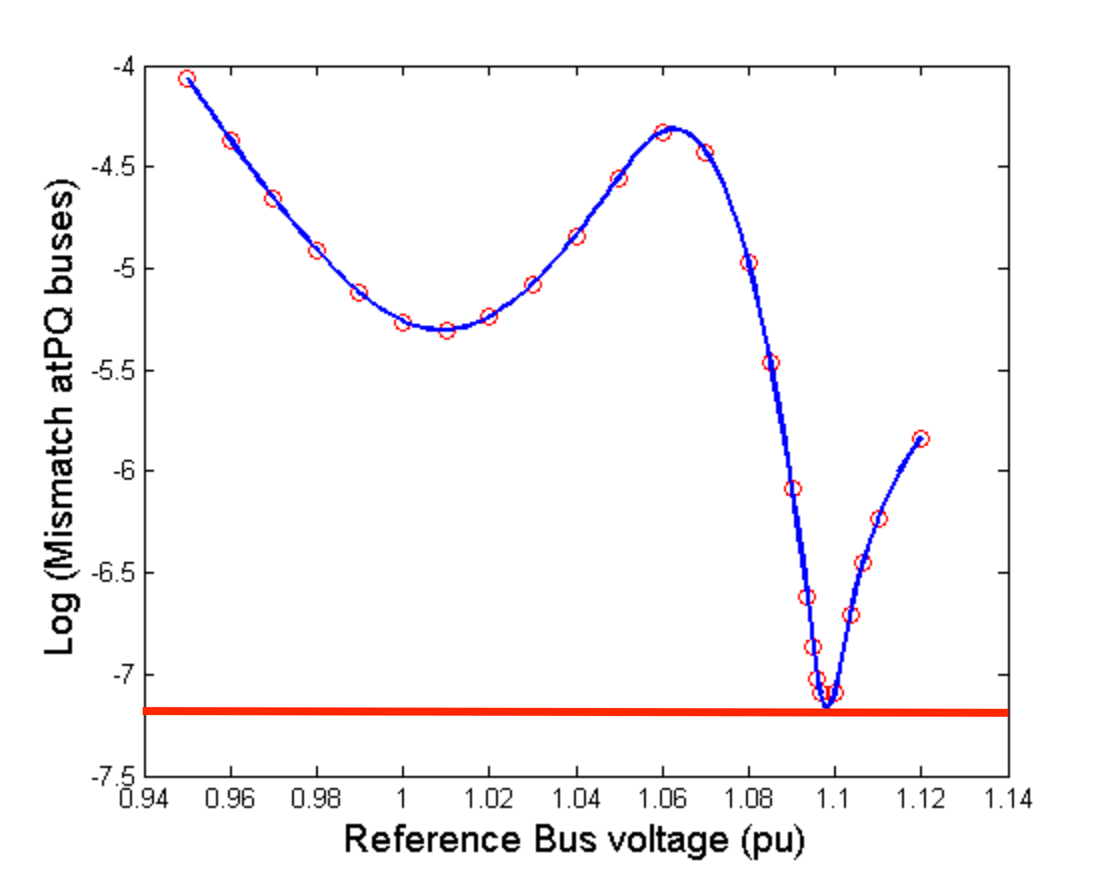
- Conventional power flow problem
  - PV, PQ, and slack buses
- Computation errors
  - Difference between estimated and target values
  - HELM: difficult to reduce the error
  - Proposed method yields similar error range
- Computation cost with the size of the system
  - FDPF, GS, NR, and proposed method: linearly increases
  - HELM: sub-linear increase if # terms is known for Pade approxs.





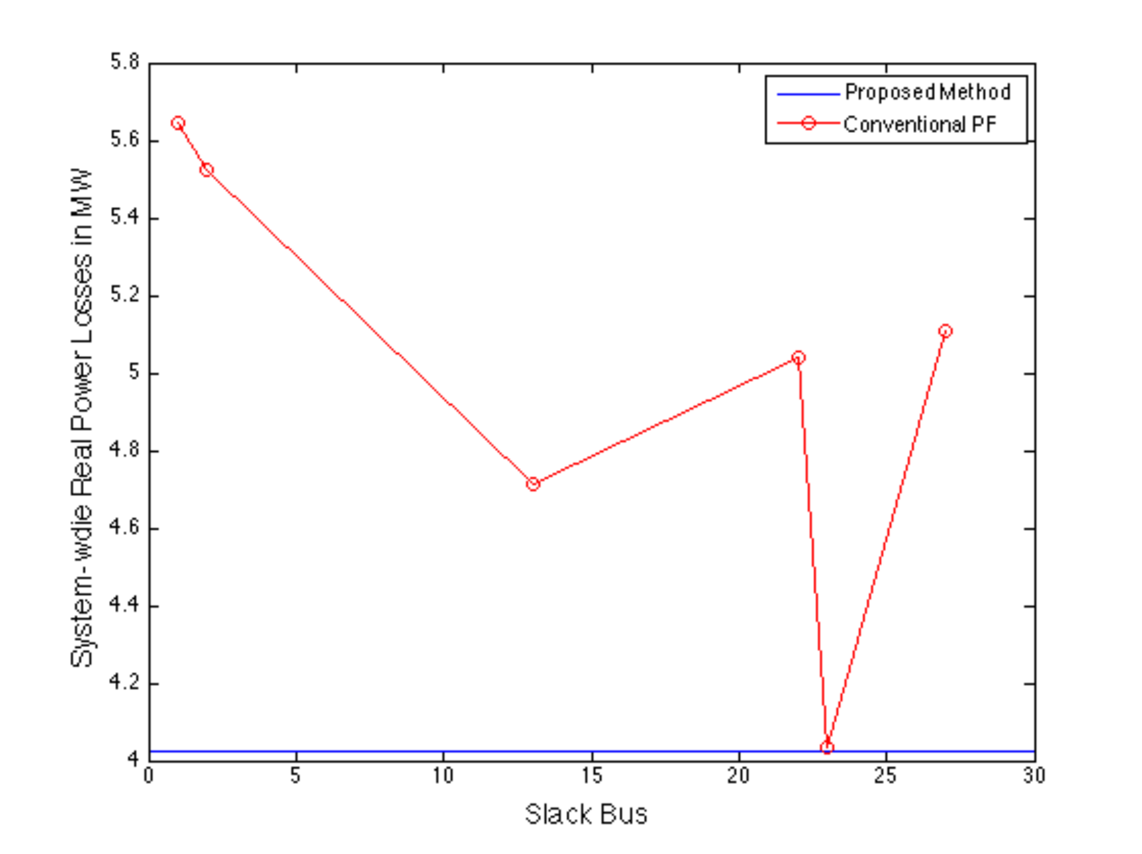
Comparison II: Just-determined System

- Conventional methods
  - Slack bus is fixed
  - All other buses are PQ buses
  - Losses wrt voltage magnitude at the slack bus
- Proposed method
  - Add additional equation to make losses small
  - No predetermined slack bus



Comparison III: Just-determined System

- IEEE 30-bus case
  - 1 slack bus
  - 5 PV buses
  - 24 PQ buses
- Voltage magnitudes are fixed at PV buses
- Choose 1 slack bus and change $|v|_{slack}$ to minimize losses
- High computation time
- Proposed method finds proper voltages with no selection of slack bus



New State Estimation Algorithm

- Kronecker product $\otimes$
  - Measurements *y* become
    - Bi-linear in *v*:

$$D_{\sqrt{w}}\begin{bmatrix} vec(A_1) \\ \vdots \\ vec(A_Y) \end{bmatrix}(v \otimes v) = D_{\sqrt{w}}\begin{pmatrix} y_1 \\ \vdots \\ y_Y \end{pmatrix} \rightarrow \hat{v}$$

    - Error in *v*:

$$D_{\sqrt{w}}\begin{bmatrix} \hat{v}^T A_1 \\ \vdots \\ \hat{v}^T A_Y \end{bmatrix}\delta v = D_{\sqrt{w}}\begin{pmatrix} y_1 - \hat{v}^T A_1 \hat{v} \\ \vdots \\ y_Y - \hat{v}^T A_Y \hat{v} \end{pmatrix} \rightarrow \delta v$$

    - Bad data detection and elimination

State Estimation: Numerical Results

- Kronecker product, v_K
  - $\|D_{\sqrt{w}}(y-\hat{y}_K)\|_2 / \|D_{\sqrt{w}}y\|_2$ = 0.022
  - $\|W - v_K v_K^T\|_2 / \|W\|_2$ = 0.088
  - $\|\delta v_K\|_2 / \|\hat{v}_K\|_2$ = 0.017
  - $v_K = E\zeta$, ζ has 14 non-zero elements
    - ζ_1 (=3.606) >> ζ_2 (=0.024) > … > ζ_{14} (=0.001) > 10^{-3}
- Conventional method, v_C
  - $\|D_{\sqrt{w}}(y-\hat{y}_C)\|_2 / \|D_{\sqrt{w}}y\|_2$ = 0.208
  - $\|W - v_C v_C^T\|_2 / \|W\|_2$ = 0.129
  - $\|\delta v_K\|_2 / \|\hat{v}_K\|_2$ = 0.038
  - $v_C = E\xi$, ξ has 22 non-zero elements
    - ξ_1 (= 3.554) >> ξ_2 (=0.160) > … > ξ_{22} (= 0.001) > 10^{-3}

$$v_K^{SE} = \begin{pmatrix} 1.050 \\ 1.044 - j0.023 \\ 1.017 - j0.099 \\ 1.008 - j0.062 \\ 1.012 - j0.046 \\ 1.037 - j0.040 \\ 1.011 - j0.052 \\ 1.026 + j0.004 \\ 0.999 - j0.080 \\ 0.995 - j0.076 \\ 1.008 - j0.059 \\ 1.010 - j0.057 \\ 1.006 - j0.059 \\ 0.980 - j0.083 \end{pmatrix}$$

Proposed Probabilistic Power Flow Algorithm

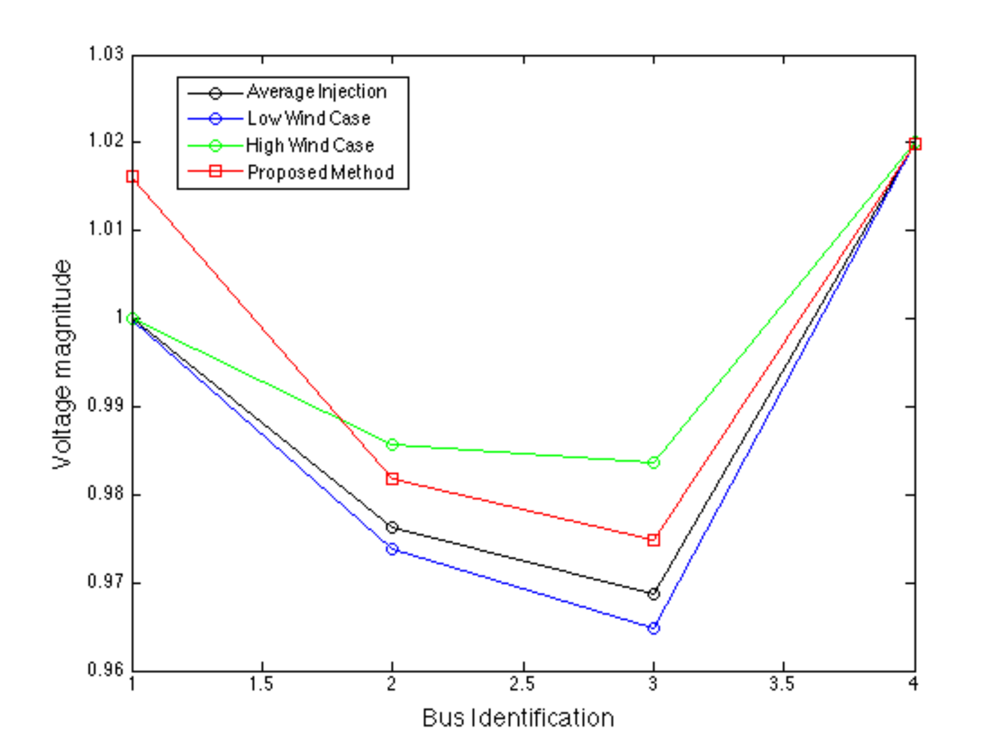
- Accommodates
  - Uncertainties on power injections
  - Capabilities of the improved control on voltage magnitudes
  - Mismatches on injection
- A new PPF algorithm
  - Includes the uncertainties, capabilities, and mismatches
  - Estimates the best fitting voltages

$$
\begin{bmatrix} w_1 vec(A_1) \\ \vdots \\ \left\{ \begin{matrix} w_{k_1} vec(A_k) \\ \vdots \\ w_{k_K} vec(A_k) \end{matrix} \right\} \\ \vdots \\ w_{2N} vec(A_{2N}) \end{bmatrix} (v \otimes v) = \begin{pmatrix} w_1 b_1 \\ \vdots \\ \left\{ \begin{matrix} w_{k_1} b_{k_1} \\ \vdots \\ w_{k_K} b_{k_K} \end{matrix} \right\} \\ \vdots \\ w_{2N} b_{2N} \end{pmatrix}
$$

Same matrix but different realizations

Numerical Results

- 4-bus system
  - Slack bus: inflexible $|v_4|$
  - 1 PV bus 1: Flexible $|v_1|$
  - 2 PQ buses
    - Wind at Bus 3
    - Low/High scenario
- Numerical results
  - Black: PF on average
  - Red: proposed method
  - Injection error at Bus 3
    - Black: -10MW – +50MW
    - Red: -7MW – +42MW
  - Losses
    - 8.4MW vs. 7.7MW



Conclusions

- Three methods to extract voltages from a multiple-rank SDP solution
  - Rank-1 approximation from *W*
  - Power flow analysis from injections
  - State estimation using *W*, injections, and power flows

    → State estimation yields a reasonable estimate of voltages
- A new power flow algorithm and state estimation technique are proposed using Kronecker product
  - Incorporate the capability of a generator in voltage control and the uncertainty of injections
  - Tend to minimize real power losses