Random Topology Power Grid Modeling and the Simulation Platform

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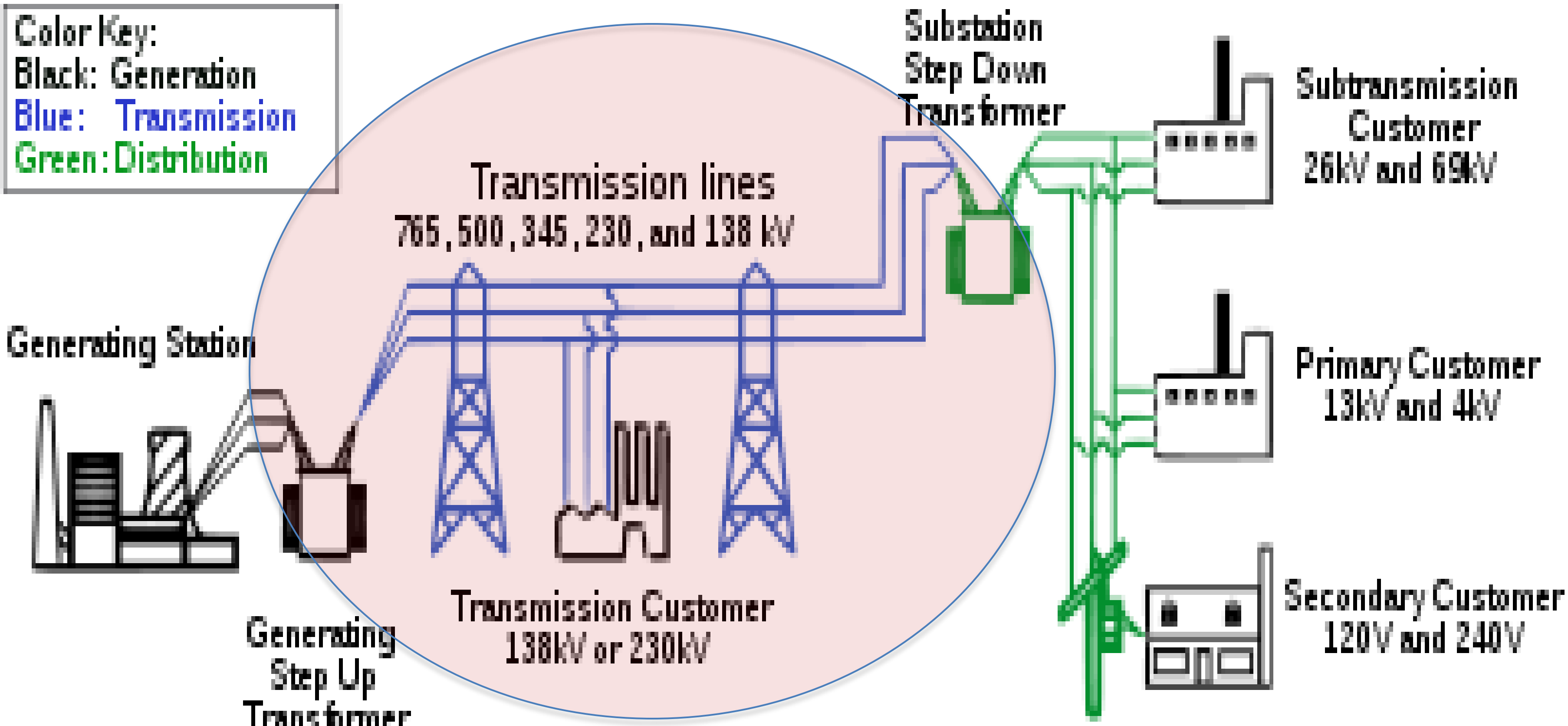
Modeling Electric Power Grid



Transmission grid: 3-phase balanced, high voltages, sparse meshed *small-world* topology, transmission lines, transformers and protective relays, etc.

"Electric" Topology of a Grid

- **Graph Laplacian**: $L = A^T A$

- **Admittance matrix:** $Y = A^T diag(y_1, \ldots, y_M) A$

  $y_l = 1/z_l, \ l = 1, \cdots, M$

- Branch-node Incidence Matrix A (M x N):

  branch *m*: node *i* – node *j*:

$$A_{m,i} = 1, A_{m,j} = -1$$

$$else, A_{m,k} = 0.$$

Location of Generators and Loads

- **DC Power Flow Model**:

$$P(t) = B'(t)\theta(t)$$
$$F(t) = \Lambda\left(y_l\right) A\theta(t)$$

- **Injected Power:**

$$P(t) = \left[P_G(t), -P_L(t), P_C\right]^T$$

- **Locations of G/L/C buses – bus type assignment**

$$\mathbb{T} = [\mathbb{T}_i]_{n\times 1}$$
$$\mathbb{T}_i = 1, 2, \text{ or } 3, \text{ if being a G/L/C bus}$$

Other Critical Electric Parameters

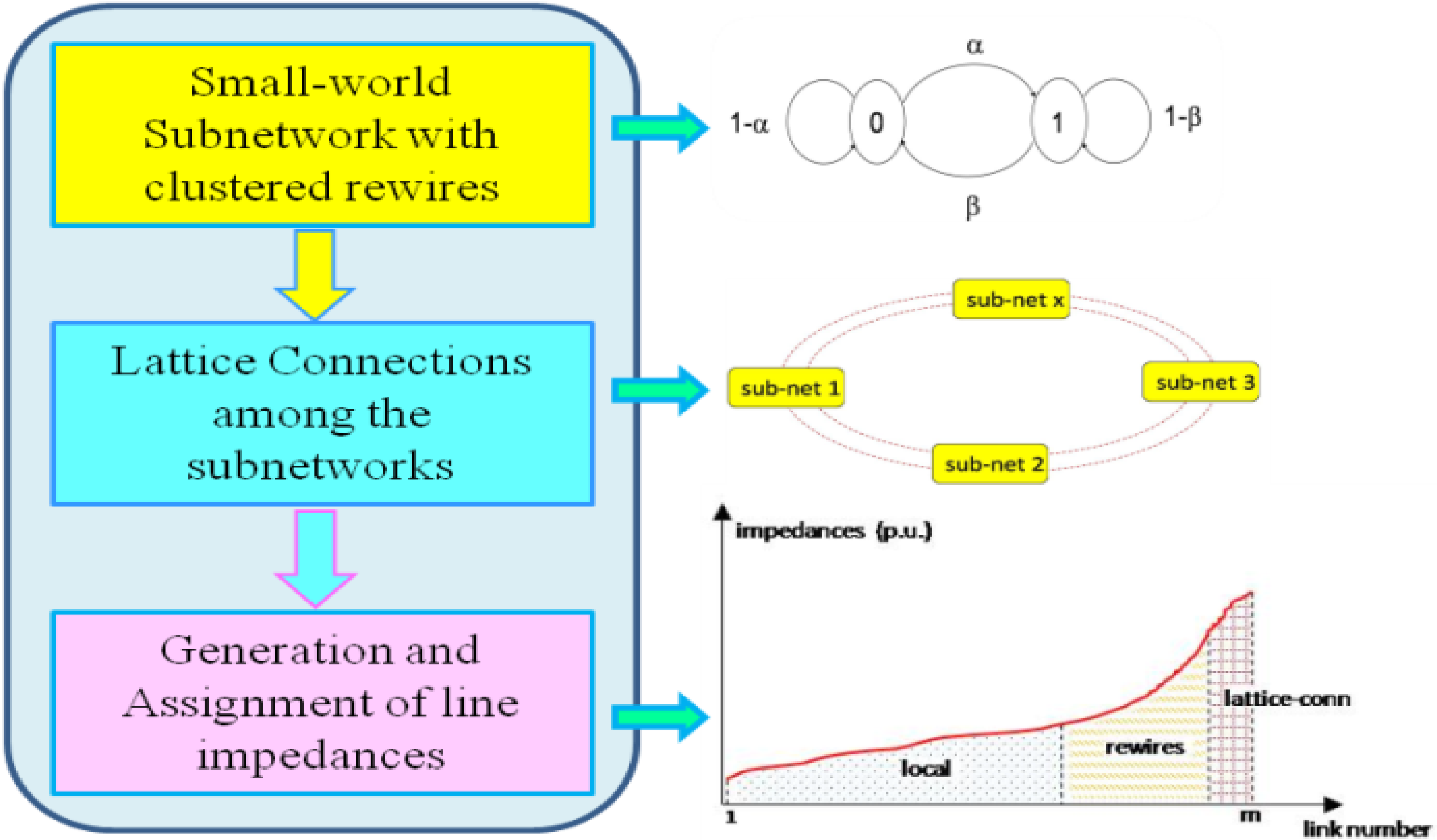
- **Generation Capacities**:
- **Transmission Constraints:**
- **Load Profiles**
- **etc**

$$P_G^{\min} \leq P_G \leq P_G^{\max},$$

$$P_L^{\min} \leq P_L \leq P_L^{\max},$$

$$F^{\min} \leq F \leq F^{\max}.$$

Generating the electric topology



Small-World power grid topology with impedances

Bus Type Assignment $\mathbb{T}$

- Three bus types in a grid:
  - Generation bus (20-40%)
  - Load bus (40-60%)
  - Connection bus (~20%).

- Real-world power grids have **Correlated** $\mathbb{T}$.
- **Randomized** $\widetilde{\mathbb{T}}$ causes the grid to behave differently and gives misleading results.

- *How to characterize a* **Correlated** $\mathbb{T}$ from the randomized ones ?

Defined Measure - *Bus Type Entropy*

$$W_1(\mathbb{T}) = -\Sigma_{k=1}^{3} \log(r_k) \times \mathfrak{n}_k - \Sigma_{k=1}^{6} \log(R_k) \times \mathfrak{m}_k$$

$$\mathfrak{n}_k = \Sigma_{i=1}^{n} \delta(\mathbb{T}_i - k), \;\; k = 1, 2, 3$$

$$r_k = \mathfrak{n}_k / n$$

Total number of G/L/C buses

Bus type ratios G/L/C

$$\mathfrak{m}_k = \Sigma_{j=1}^{m} \delta(\mathbb{L}_j - k), \;\; k = 1, 2, \cdots, 6$$

$$R_k = \mathfrak{m}_k / m$$

Total number of each type links i.e. {GG, GL, GC, LL, LC, CC}

Link type ratios

Two Additional Variations

$$W_1(\mathbb{T}) = -\Sigma_{k=1}^{3} \log(r_k) \times \mathfrak{n}_k - \Sigma_{k=1}^{6} \log(R_k) \times \mathfrak{m}_k$$

$$W_2(\mathbb{T}) = -\Sigma_{k=1}^{3} \log(r_k) - \Sigma_{k=1}^{6} \log(R_k)$$

$$W_3(\mathbb{T}) = -\Sigma_{k=1}^{3} \log(r_k) \times \frac{1}{\mathfrak{n}_k} - \Sigma_{k=1}^{6} \log(R_k) \times \frac{1}{\mathfrak{m}_k}$$

Empirical PDF of Randomized $\widetilde{\mathbb{T}}$

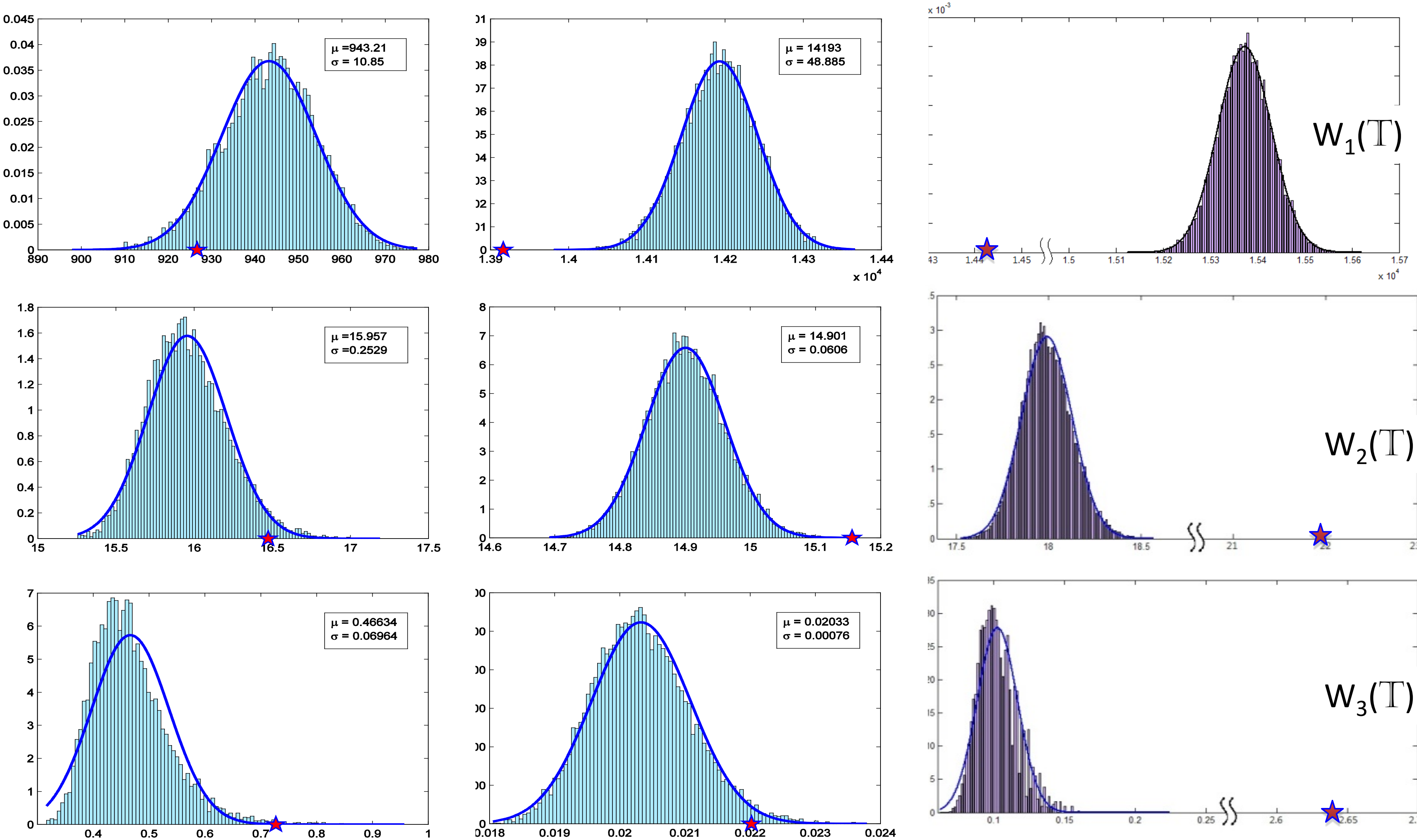
- Random permutation of original bus type assignment $\mathbb{T}_0$
- Evaluating of the bus type entropy
- Statistical analysis: normal fitting

$$f_W(x) = \frac{\Sigma_{k=1}^{k^{\max}} \delta_\Delta(W_k - x)}{k^{\max}}$$

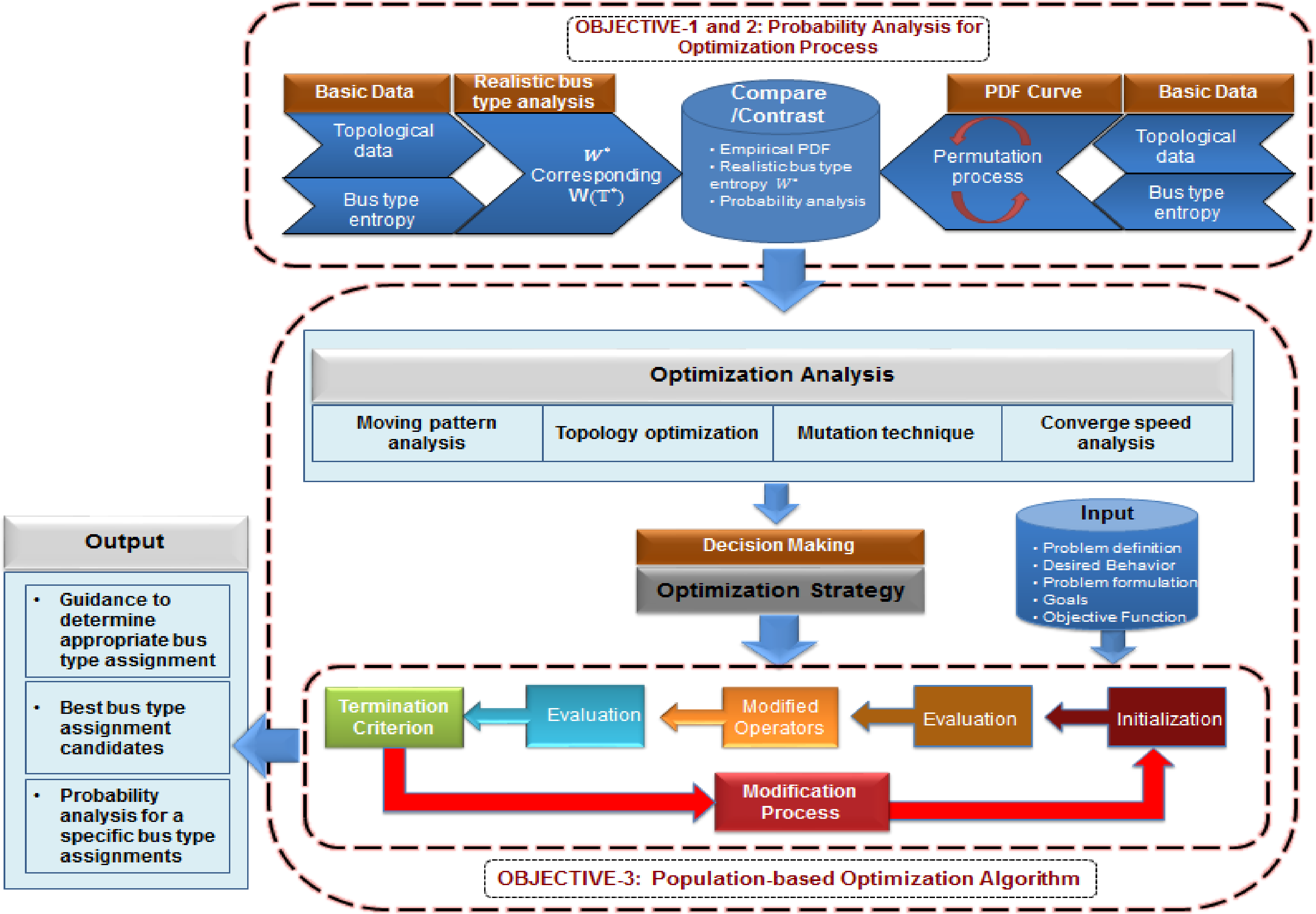
$$\delta_\Delta(x) = \begin{cases} \frac{1}{\Delta}, & -\frac{\Delta}{2} < x \leq -\frac{\Delta}{2} \\ 0, & \text{otherewise.} \end{cases}$$

Empirical and Fitting PDF of $W(\mathbb{T})$

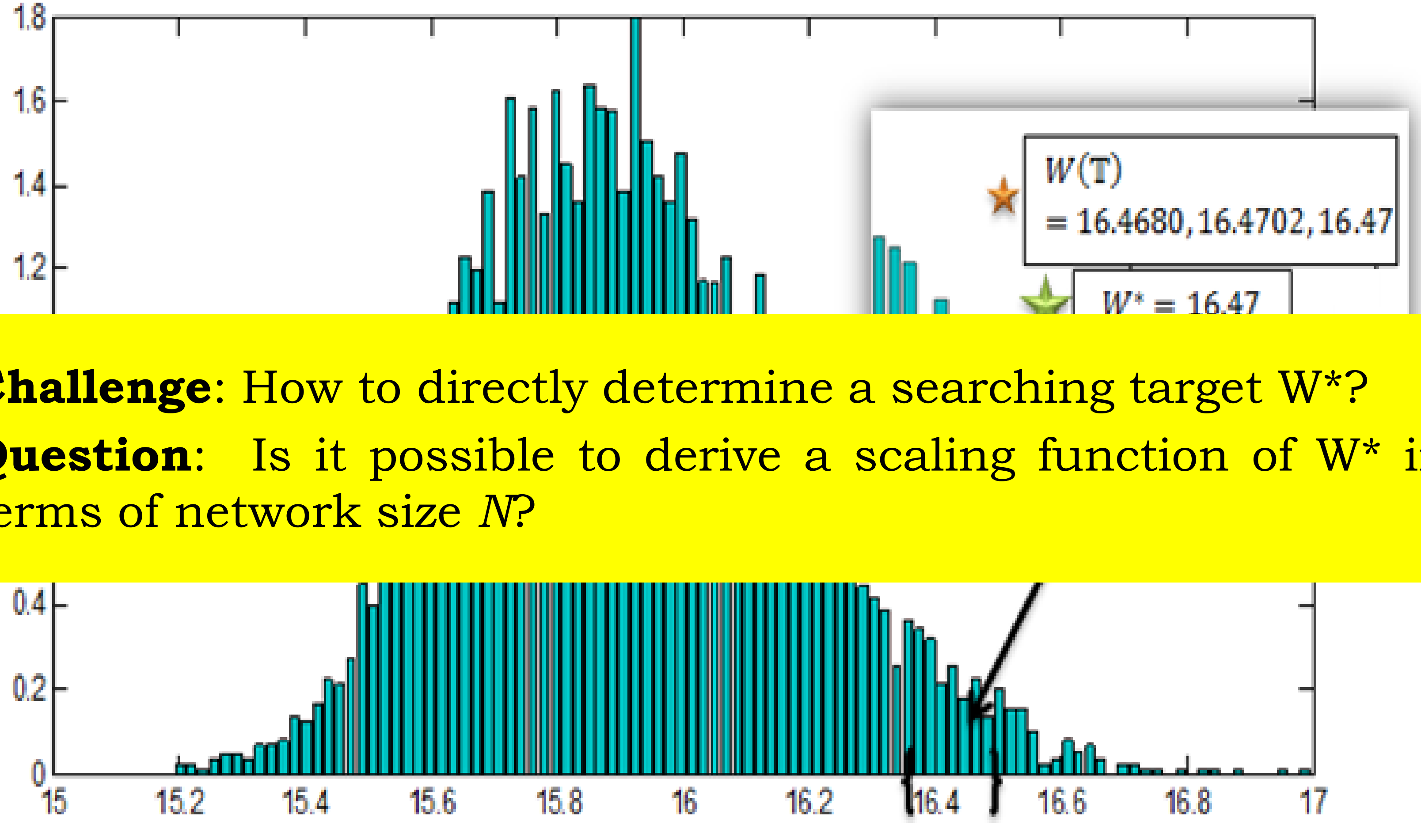
IEEE-300 | NYISO-2935 | ERCOT-5633



Multi-objective Optimization Algorithm



Numerical Results



Challenge: How to directly determine a searching target W*?

Question: Is it possible to derive a scaling function of W* in terms of network size *N*?

The best set of bus type assignments in for a 300 bus system - $W_2(\mathbb{T})$

Normal Fitting Parameters

TABEL I

The Parameters of Normal Distribution Fitting

	$W_1(\mathbb{T})$ $\mu/\sigma/W(\mathbb{T}^*)$	$W_2(\mathbb{T})$ $\mu/\sigma/W(\mathbb{T}^*)$	$W_3(\mathbb{T})$ $\mu/\sigma/W(\mathbb{T}^*)$
IEEE-300	943.21/10.58/927.5	15.95/0.252/16.47	0.466/0.069/0.726
NYISO	14193/48.8/13910	14.901/0.06/15.16	0.020/0.0007/0.022
ERCOT	15372/56.47/14428	17.99/0.13/22.32	0.102/0.0143/2.64

Normalized Distance of W($\mathbb{T}$*)

TABEL II

The Normalized Distance of Realistic Bus Type Entropy

	(N, M)	$W_1(\mathbb{T})$ d_{W_*}	$W_2(\mathbb{T})$ d_{W_*}	$W_3(\mathbb{T})$ d_{W_*}
IEEE-300	(300,409)	1.48	1.96	3.76
NYISO	(2935,6567)	5.78	28.72	2.42
ERCOT	(5633,7053)	16.71	33.30	177.48

$$d_{W_*} = |W(\mathbb{T}^*) - \mu| / \sigma$$

All the Test Cases Considered

- **IEEE Test cases**
  – 30, 57, 118, 300 buses
- **NYISO System**
  – 2935 buses
- **ERCOT System**
  – 5633 buses
- **WECC System**
  – 16994 buses

Revised Definitions

- **Relative Distance**: $d_W(\mathbb{T}^*, \widetilde{\mathbb{T}}) = \dfrac{W(\mathbb{T}^*) - \mu}{\sigma}$
- **Entropy Definition:**

$$W(\mathbb{T}) = -\frac{\Sigma_{i=1}^{n} \log(r_{\mathbb{T}_i})}{n} - \frac{\Sigma_{j=1}^{m} \log(R_{\mathbb{L}_j})}{m}$$

  – Equally can be written as:

$$W(\mathbb{T}) = -\Sigma_{k=1}^{3} r_k \log(r_k) - \Sigma_{k=1}^{6} R_k \log(R_k)$$

- **Advantages**:
  – Better Statistical Properties
  – Improved Numerical Stability

Empirical PDF

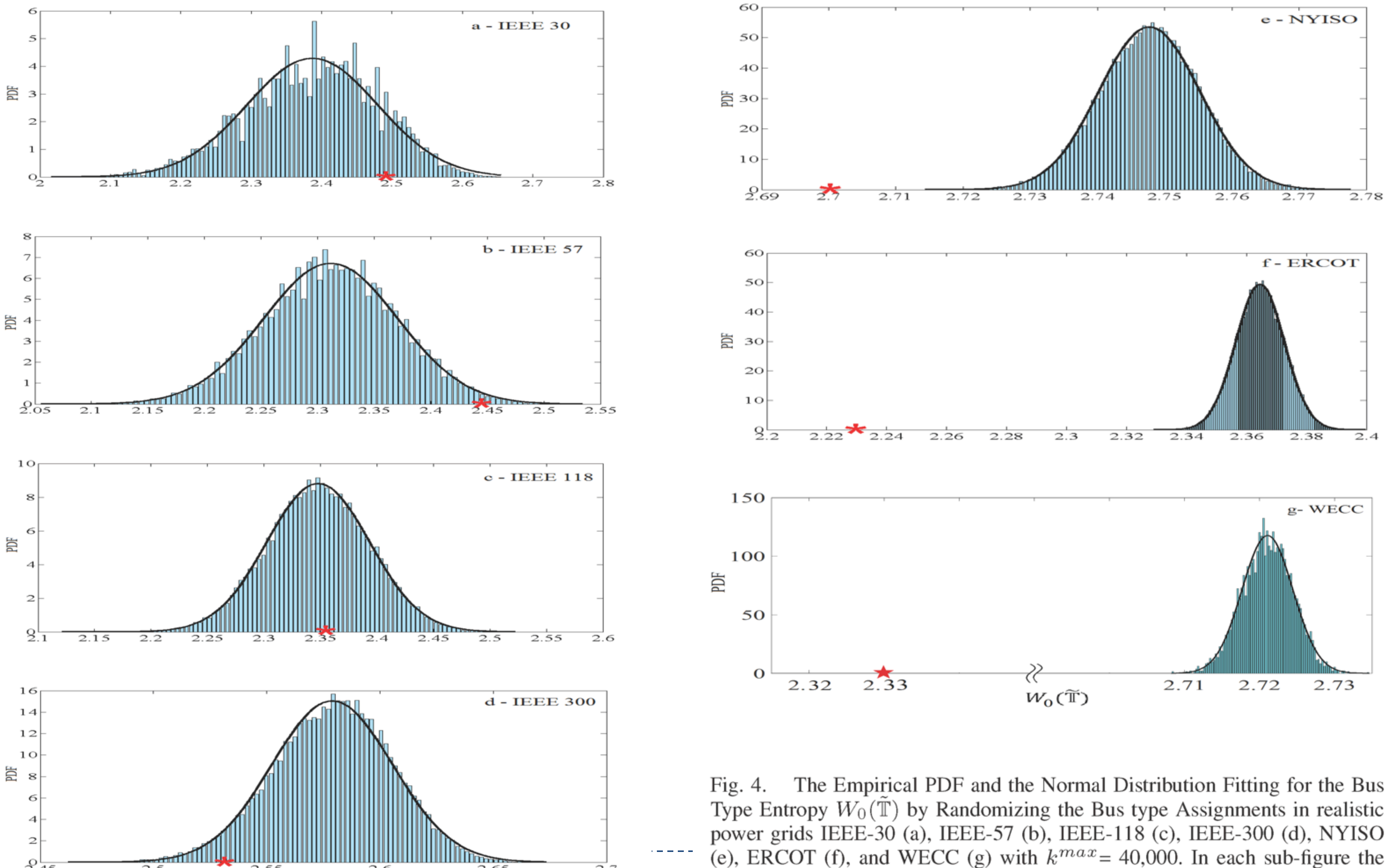


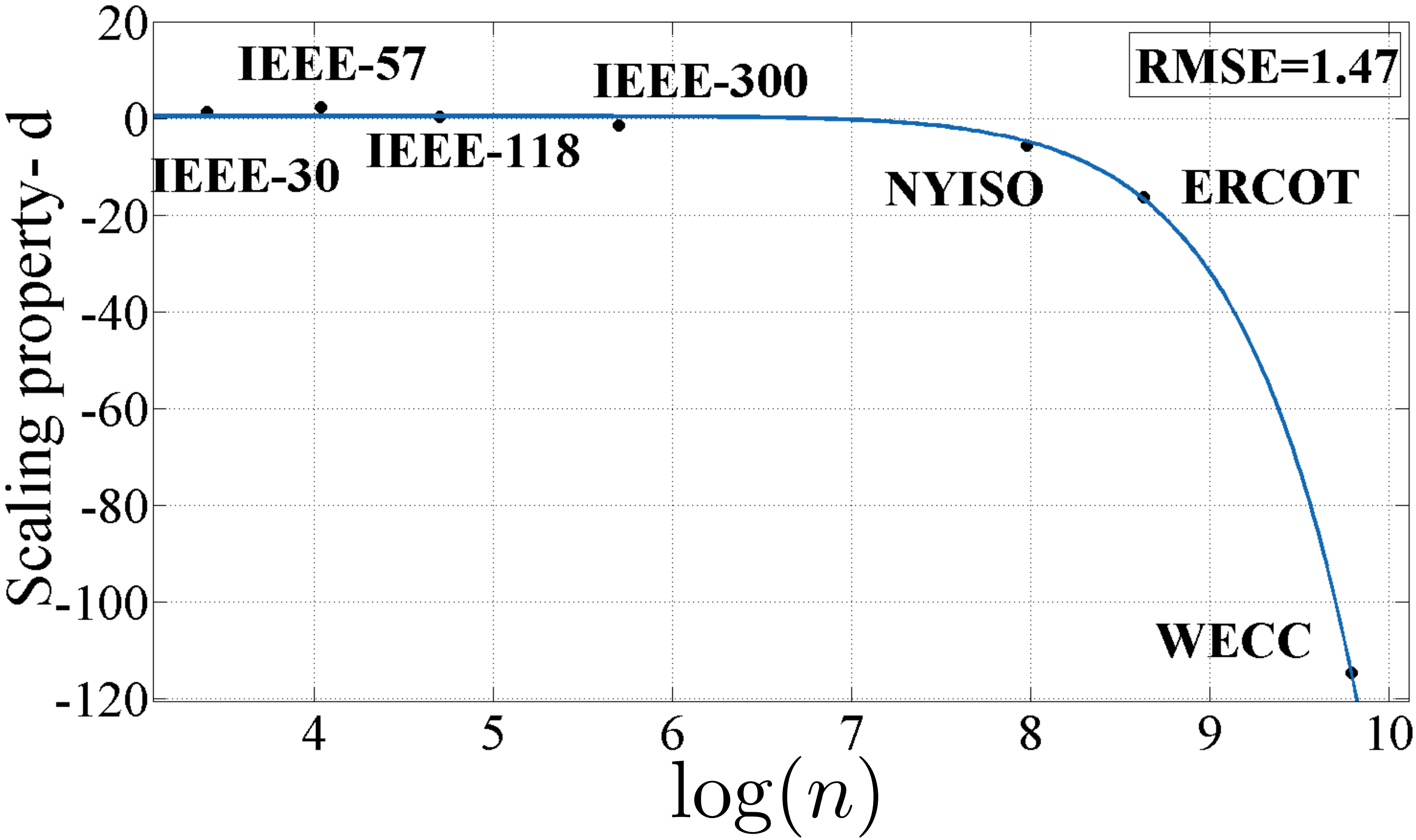
Fig. 4. The Empirical PDF and the Normal Distribution Fitting for the Bus Type Entropy $W_0(\tilde{\mathbb{T}})$ by Randomizing the Bus type Assignments in realistic power grids IEEE-30 (a), IEEE-57 (b), IEEE-118 (c), IEEE-300 (d), NYISO (e), ERCOT (f), and WECC (g) with k^{max}= 40,000. In each sub-figure the realistic bus type entropy $\mathbb{T}^*$and $W^* = W(\mathbb{T}^*)$ is marked by a red 'star'.

Normalized Distance vs. Network Size

TABLE I. THE NORMAL FITTING PARAMETERS OF THE EMPIRICAL PDF OF $W(\widetilde{\mathbb{T}})$ AND THE RELATIVE DISTANCE OF d_W

	(μ, σ)	$W(\mathbb{T}^*)$	$d_W(\mathbb{T}^*, \widetilde{\mathbb{T}})$
IEEE-30	(2.38, 9.0e-2)	2.49	1.22
IEEE-57	(2.31, 5.8e-2)	2.44	2.24
IEEE-118	(2.34, 4.5e-2)	2.35	0.22
IEEE-300	(2.57, 2.6e-2)	2.53	-1.53
NYISO-2935	(2.74, 7.3e-3)	2.70	-5.71
ERCOT-5633	(2.36, 8.1e-3)	2.23	-16.25
WECC-16994	(2.72, 3.4e-3)	2.33	-114.70

Scaling Function $d_W(n)$ vs. $\log(n)$



From $d_W(n)$ to $W^*(n)$

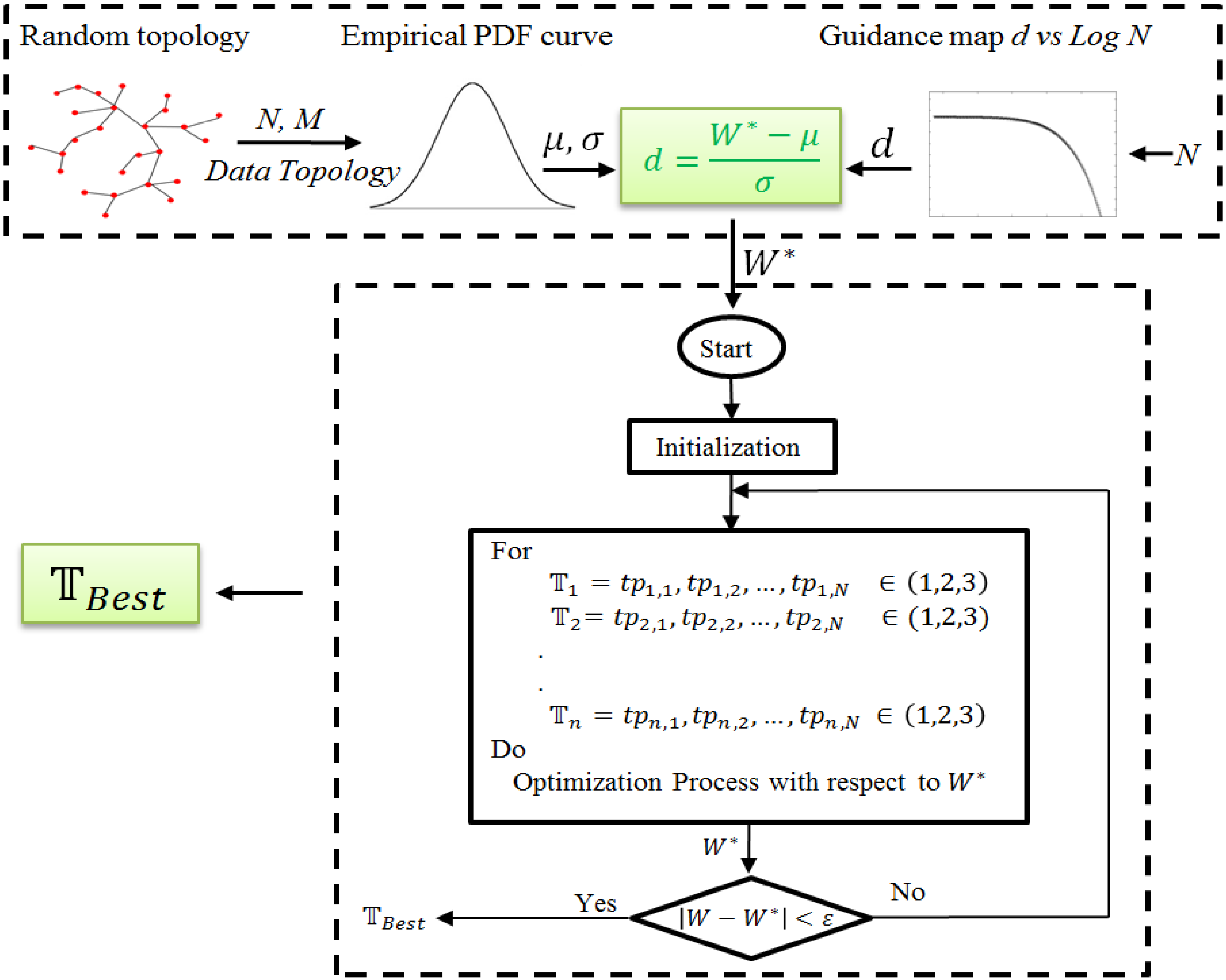
- **Scaling function of** $d_W(n)$

$$d_W(n) = \begin{cases} -1.39\log n + 6.79, & \log n \leq 8 \\ -1.25 \times 10^{-13}(\log n)^{15.1} + 0.43, & \log n < 8 \end{cases}$$

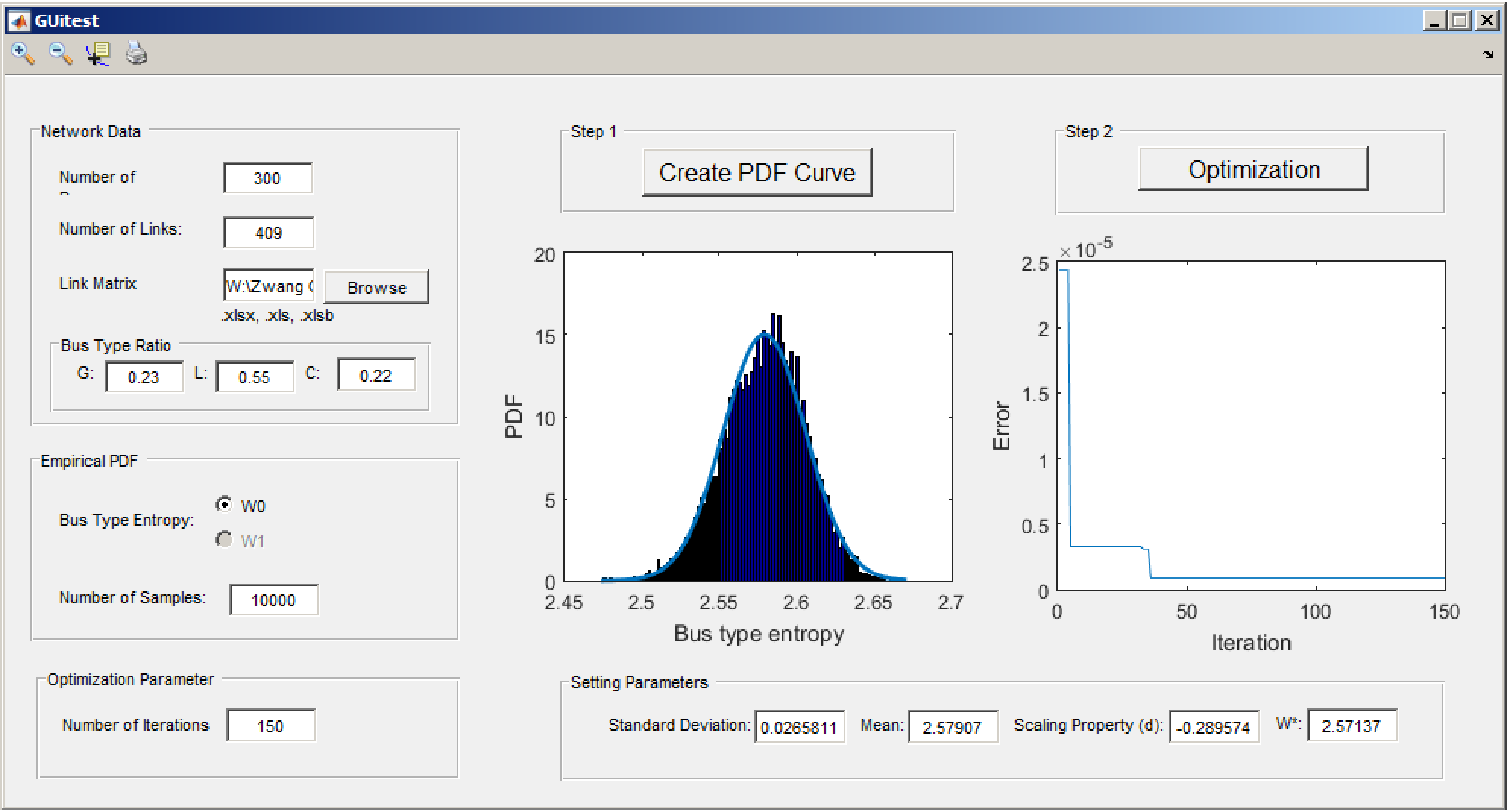
- **The Searching Target:**

$$W^*(n) = \mu + \sigma \cdot d_W(n)$$

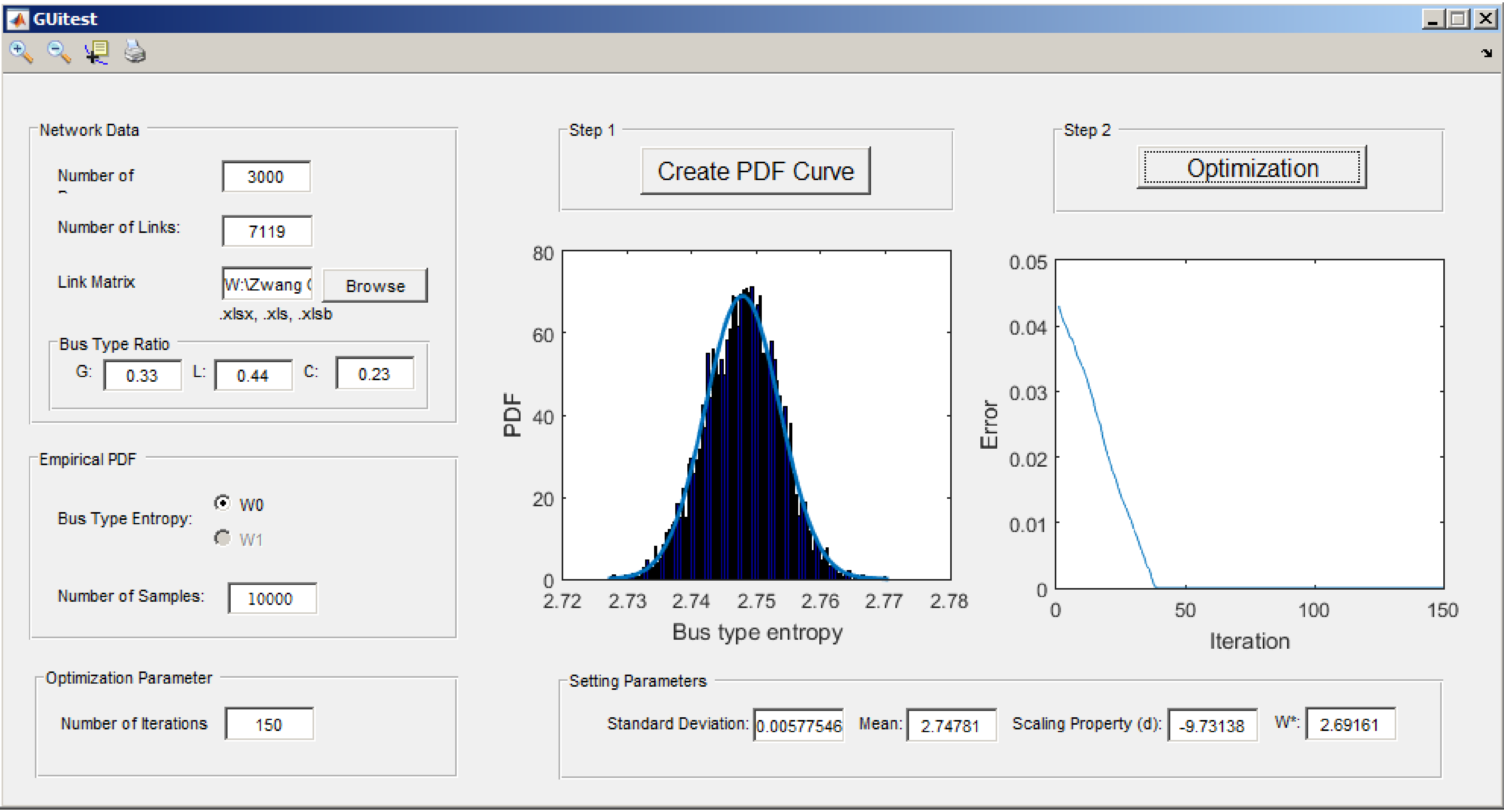
Revised Algorithm



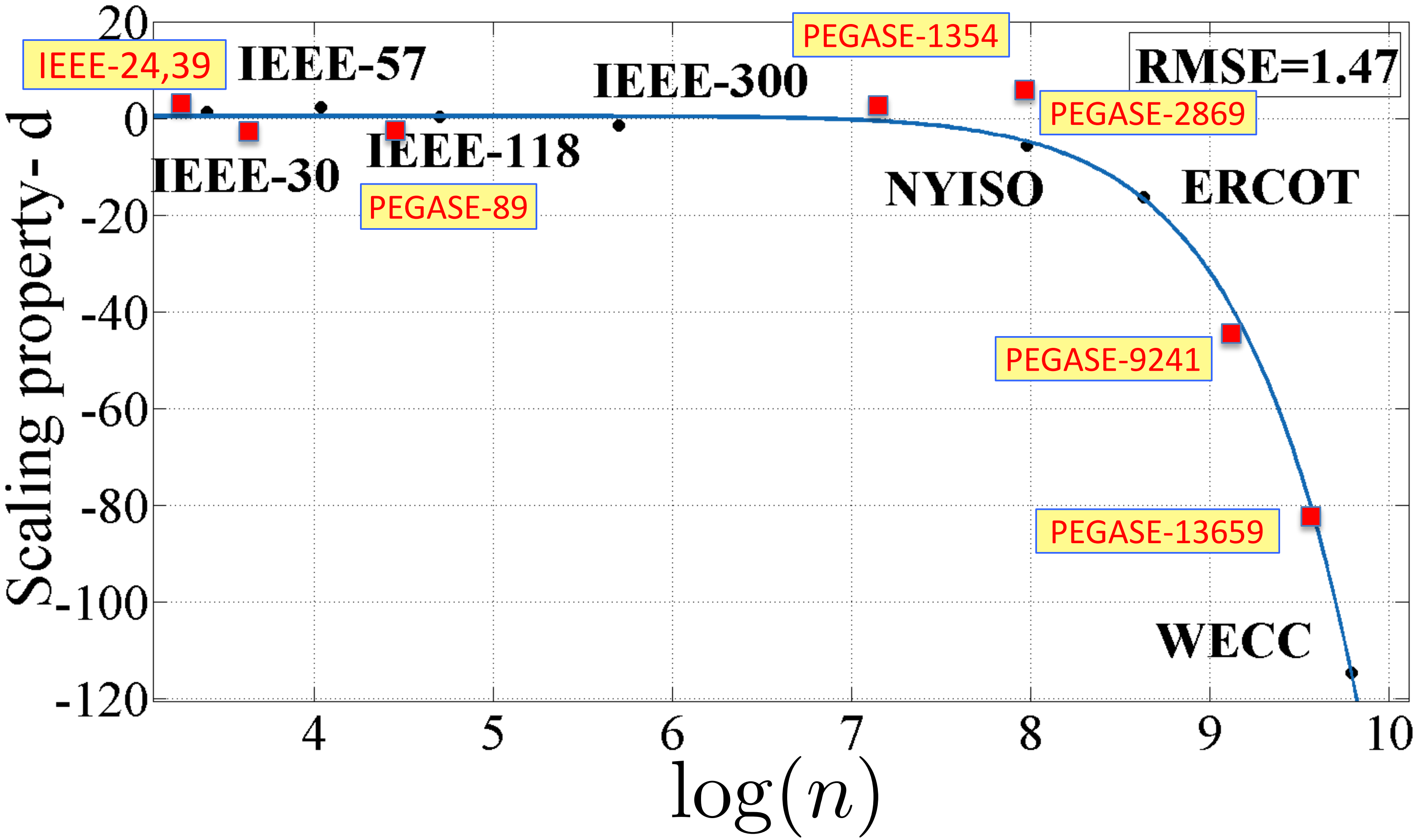
Simulation Platform with GUI



Simulation Platform with GUI



Scaling Function $d_W(n)$ vs. $\log(n)$



Conclusions & Future Works

- The RT-*nestedSmallWorld* model, to our best knowlege, is the most comprehensive and appropriate synthetic grid model in the literature to formulate a small-world connecting topology with line impedances of heavy-detailed distribution.
- The definition of a numerical measure called the *Bus Type Entropy*, is re-examined and re-defined to characterize the correlated bus type assignment in a grid, with the IEEE test cases, the NYISO, ERCOT, and WECC systems.
- The newly defined entropy has better statistical property and improved numerical stability.
- This measure enables our study of the scaling property of correlated bus type assignment with regard to network size, with the help of distribution parameters estimated from the non-segmented empirical PDF of a normal distribution of randomized bus type entropy.

Conclusions & Future Works

- With the derived scaling function of correlated bus type assignment versus network size, a more efficient search algorithm based on clonal selection procedure is developed to present more accurate bus type assignments of generation, load, and connection buses in our random-topology power grid model.
- The simulation platform is now implemented with graphic user interface (GUI).
- Next steps:
  - to verify the accuracy and effectiveness of the proposed bus type entropy;
  - to study other electrical parameters in the synthetic grid modeling such as generation capacities, load profiles, and transmission constraints, etc.

Questions? ☺

Thank You!