CERTS
Consortium for Electric Reliability Technology Solutions

Random Topology Power Grid Modeling and the Simulation Platform

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Motivation

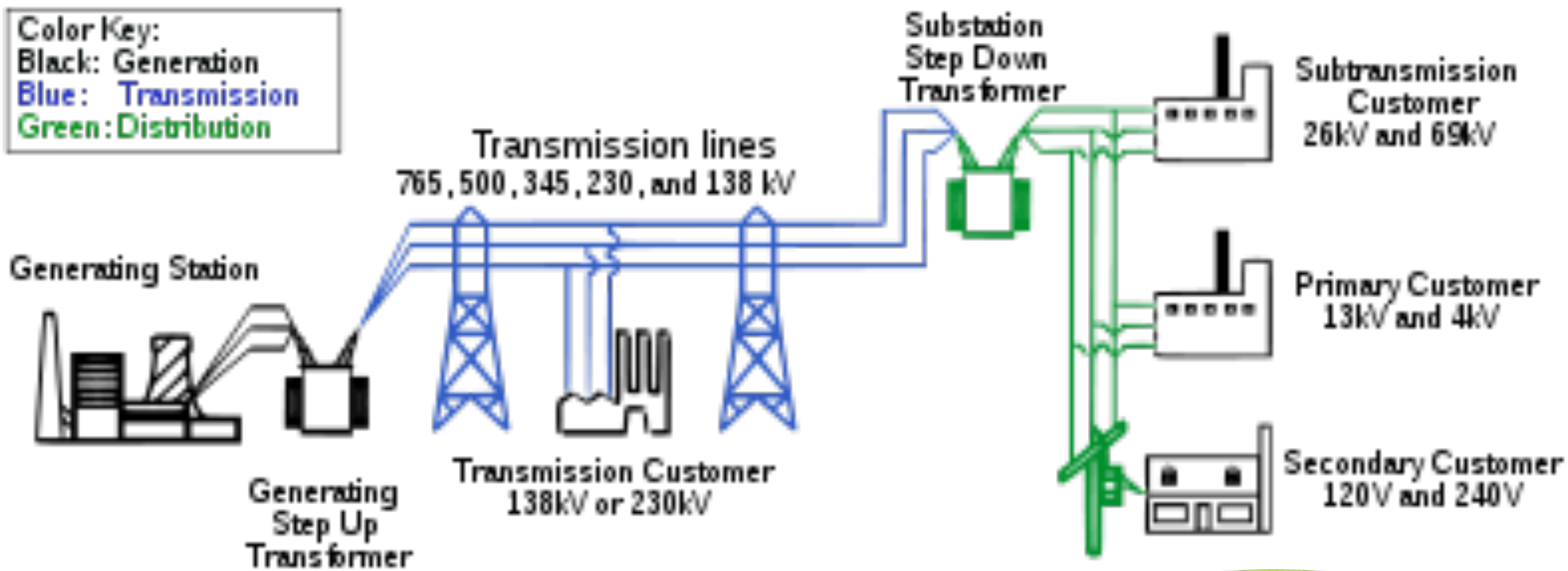
- Appropriate randomly generated grid network topologies necessary to test new concepts and methods.
  - If the random networks are truly representative and if the concepts or methods test well in this environment they would test well on any instance of such a network.

- Current situation
  - difficult to obtain realistic grid data
  - limited reference test cases
  - existing models with shortcomings

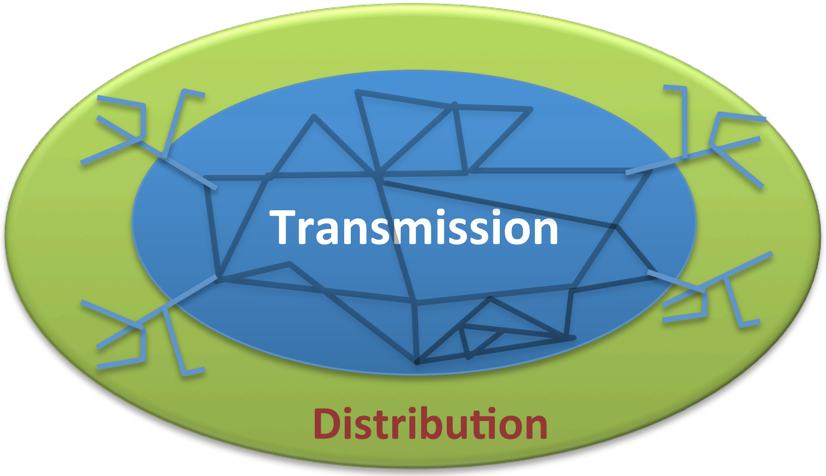
Critical Applications for the Grid

- Renewable generation interconnection
- PMU placements to facilitate fast state estimation and real-time state awareness
- Transmission expansion planning
- Grid vulnerability and security analysis
- Transient stability controls
- Electricity market strategy experiments
- Smart grid communication infrastructure

Electric Power Grid Network



- 3 sections in transmission
  - High, Medium and Low voltage sections



Power Grid vs. Graph Network

- Line-Node Incidence Matrix A (M x N):

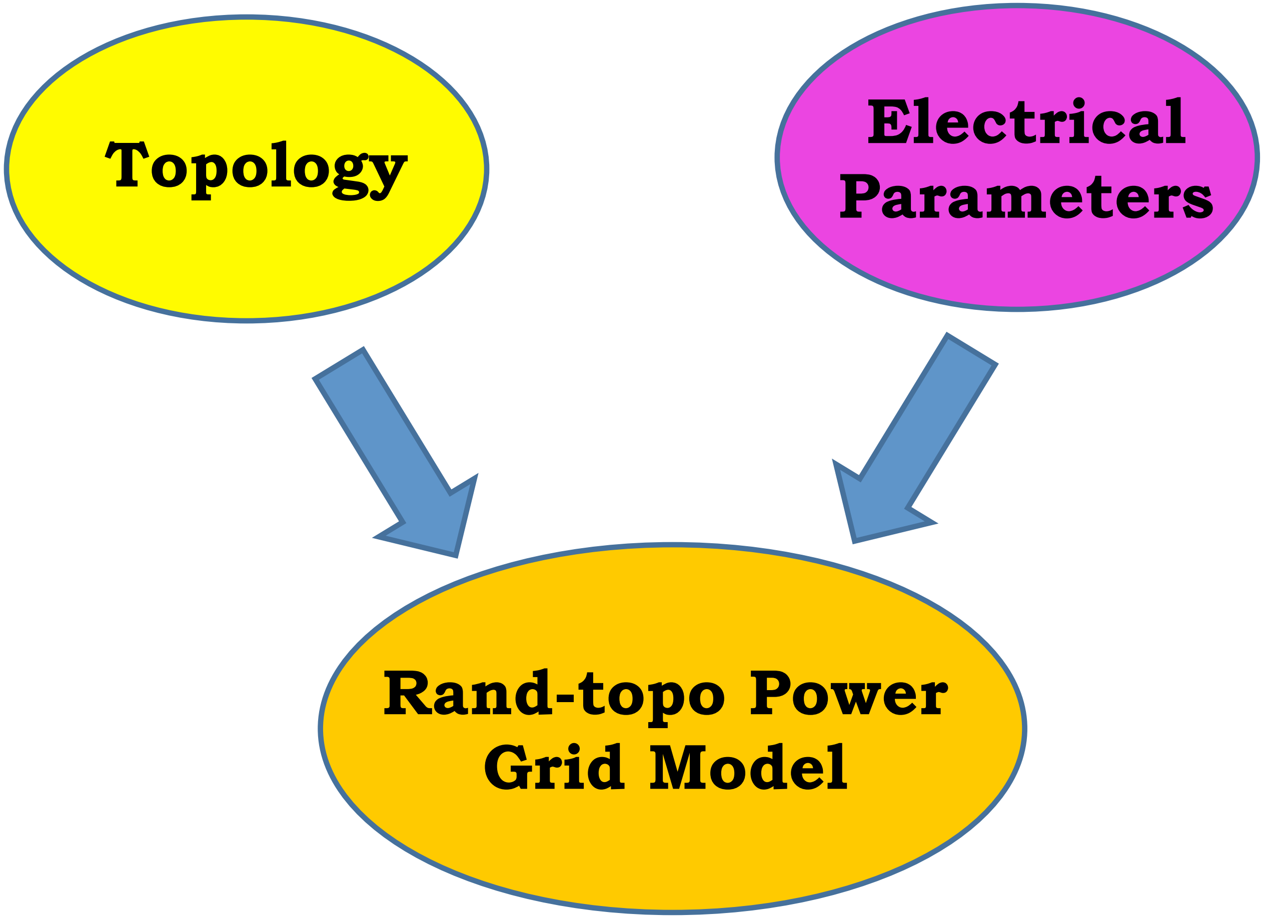
  Line *m*: node *i* – node *j* → $A_{m,i} = 1, A_{m,j} = -1$
  $else, A_{m,k} = 0.$

- **Admittance matrix**

$$Y = A^T diag(y_1, \ldots, y_M) A$$

- **Graph Laplacian**: $L = A^T A$
- Observation: ***Y*** **is a complex-weighted Laplacian!**
- Complex weights given by the admittances of the lines $y_l = 1/z_l = 1/(r_l + jx_l)$

Statistical Modeling of Power Grid



Power Grid – Network Topology

Topology →

- ✓ Small-world Properties
- ✓ Node Degree Distribution
- ✓ Connectivity Scaling
- ✓ Correlated Rewiring
- ✓ Graph Spectral Density
- etc

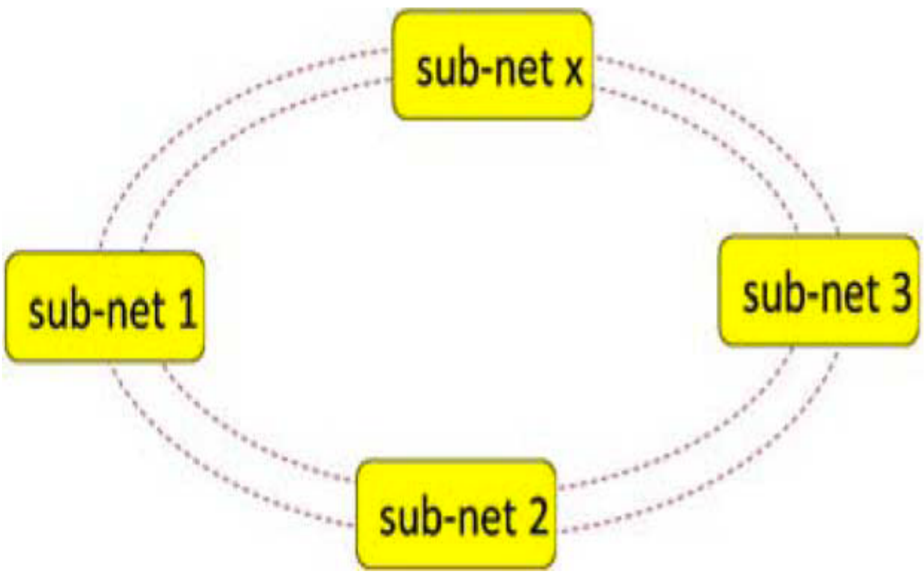
Power Grid – Electrical Parameters

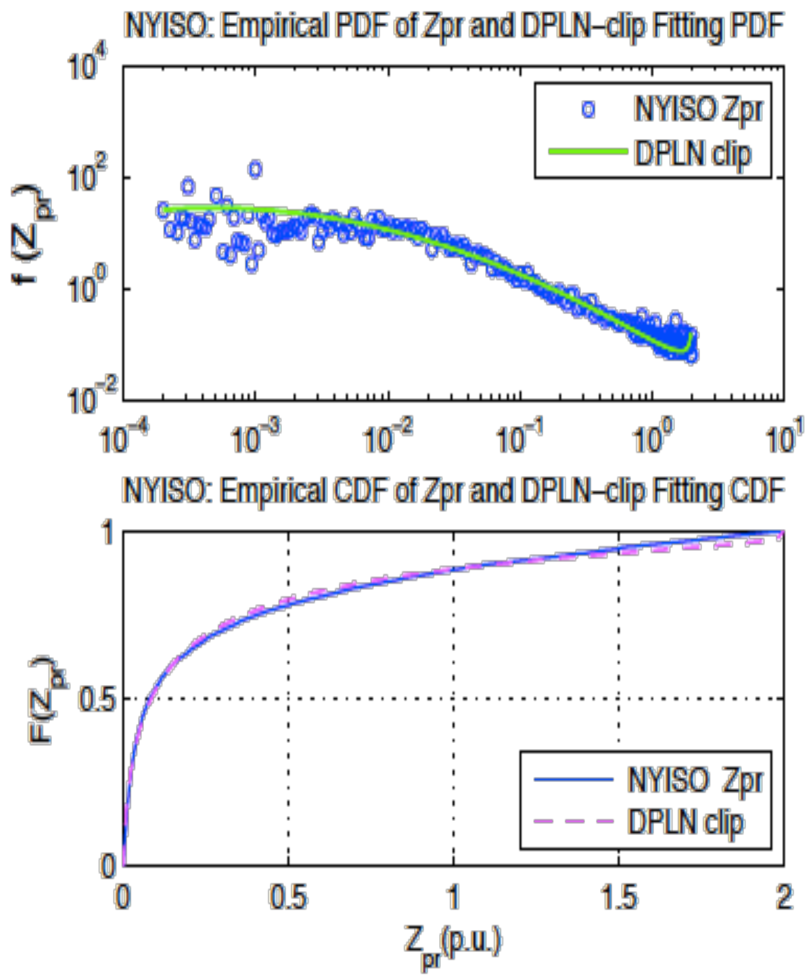
Electrical Parameters ⇒

- ✓ Line impedances – heavy-tailed distribution
- ✓ Generation and load settings
- Bus type assignments
- Dynamic evolution
- etc

Plausible Electrical Topology

- The proposed model that matches observed properties is what we call ***RT-nested-Small-world***.
- IEEE → SW subnet 30; NYISO & WECC → SW sub-net 300





Bus Type Assignment T

- Three bus types in a grid:
  - Generation bus
  - Load bus
  - Connection bus
- *RT-nestedSmallWorld:* **Random or Correlated** T **?**

TABLE I
RATIO OF BUS TYPES IN REAL-WORLD POWER NETWORKS

	(n, m)	$r_{G/L/C}(\%)$
IEEE-30	(30,41)	20/60/20
IEEE-57	(57,78)	12/62/26
IEEE-118	(118,179)	46/46/08
IEEE-300	(300, 409)	23/55/22
NYISO	(2935,6567)	33/44/23

Bus Type vs. Node degree

- Correlation between bus types and node degree

TABLE II
CORRELATION BETWEEN NODE DEGREE AND BUS TYPES IN REAL-WORLD POWER NETWORKS

	$\langle k \rangle$	$\langle k \rangle_G$	$\langle k \rangle_L$	$\langle k \rangle_C$	$\rho(t, k_t)$
IEEE-30	2.73	2.00	2.61	3.83	0.4147
IEEE-57	2.74	3.86	2.54	2.67	-0.2343
IEEE-118	3.03	3.56	2.44	3.40	-0.2087
IEEE-300	2.73	1.96	2.88	3.15	0.2621
NYISO	4.47	4.57	5.01	3.33	-0.1030
WECC	2.67	-	-	-	-

Bus Type vs. Clustering Coefficient

TABLE III
BUS TYPES AND CLUSTERING COEFFICIENTS OF REAL-WORLD POWER NETWORKS AND RANDOM GRAPH NETWORKS

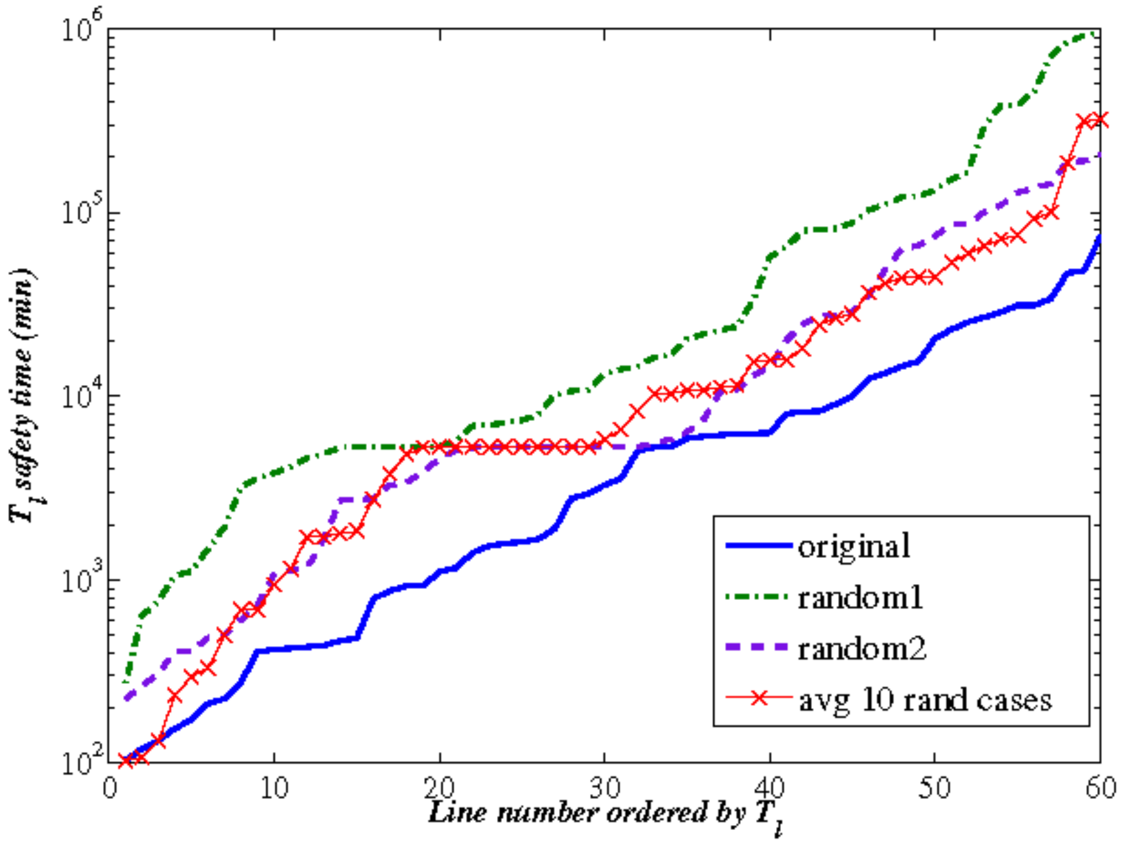
	$C(R)$	C_{all}	C_G	C_L	C_C	$\rho(t, C_t)$
IEEE-30	0.0943	0.2348	0.1944	0.2537	0.2183	0.0210
IEEE-57	0.0489	0.1222	0.1524	0.1352	0.0778	-0.1064
IEEE-118	0.0260	0.1651	0.1607	0.1969	0.0167	-0.0538
IEEE-300	0.0091	0.0856	0.1227	0.0895	0.0364	-0.1428
NYISO-2935	0.0015	0.2134	0.2693	0.2489	0.0688	-0.2382
WECC-4941	0.0005	0.0801	-	-	-	-

Bus Type vs. Degree Distribution

TABLE IV
ESTIMATE COEFFICIENTS OF THE TRUNCATED GEOMETRIC AND THE IRREGULAR DISCRETE FOR THE NODE DEGREES IN THE NYISO SYSTEM

node groups	$\max(\underline{k})$	p	k_{max}	k_t	$\{p_1, p_2, \cdots, p_{k_t}\}$
All	37	0.2269	34	3	0.4875, 0.2700, 0.2425
Gen	37	0.1863	36	1	1.000
Load	29	0.2423	26	3	0.0455, 0.4675, 0.4870
Conn	21	0.4006	18	3	0.0393, 0.4442, 0.5165

Bus Type Assignment vs. Grid Vulnerability



IEEE-300 bus system, given the same topology, G/L/C ratios, and generation and load statistical settings, the test cases with random bus type assignments tend to have larger expected safety time than that of the realistic grid settings.

Bus Type Entropy

$$W_1(\mathbb{T}) = -\Sigma_{k=1}^{3} \log(r_k) \times \mathfrak{n}_k - \Sigma_{k=1}^{6} \log(R_k) \times \mathfrak{m}_k$$

$$\mathfrak{n}_k = \Sigma_{i=1}^{n} \delta(\mathbb{T}_i - k), \;\; k = 1, 2, 3$$

$$r_k = \mathfrak{n}_k / n$$

Total number of G/L/C buses

Bus type ratios G/L/C

$$\mathfrak{m}_k = \Sigma_{j=1}^{m} \delta(\mathbb{L}_j - k), \;\; k = 1, 2, \cdots, 6$$

$$R_k = \mathfrak{m}_k / m$$

Total number of each type links i.e. {GG, GL, GC, LL, LC, CC}

Link type ratios

Two Variations

$$W_1(\mathbb{T}) = -\Sigma_{k=1}^{3} \log(r_k) \times \mathfrak{n}_k - \Sigma_{k=1}^{6} \log(R_k) \times \mathfrak{m}_k$$

$$W_2(\mathbb{T}) = -\Sigma_{k=1}^{3} \log(r_k) - \Sigma_{k=1}^{6} \log(R_k)$$

$$W_3(\mathbb{T}) = -\Sigma_{k=1}^{3} \log(r_k) \times \frac{1}{\mathfrak{n}_k} - \Sigma_{k=1}^{6} \log(R_k) \times \frac{1}{\mathfrak{m}_k}$$

Empirical PDF of Randomized T

- Random permutation of original bus type assignment T_0
- Evaluating of the bus type entropy
- Statistical analysis: normal fitting

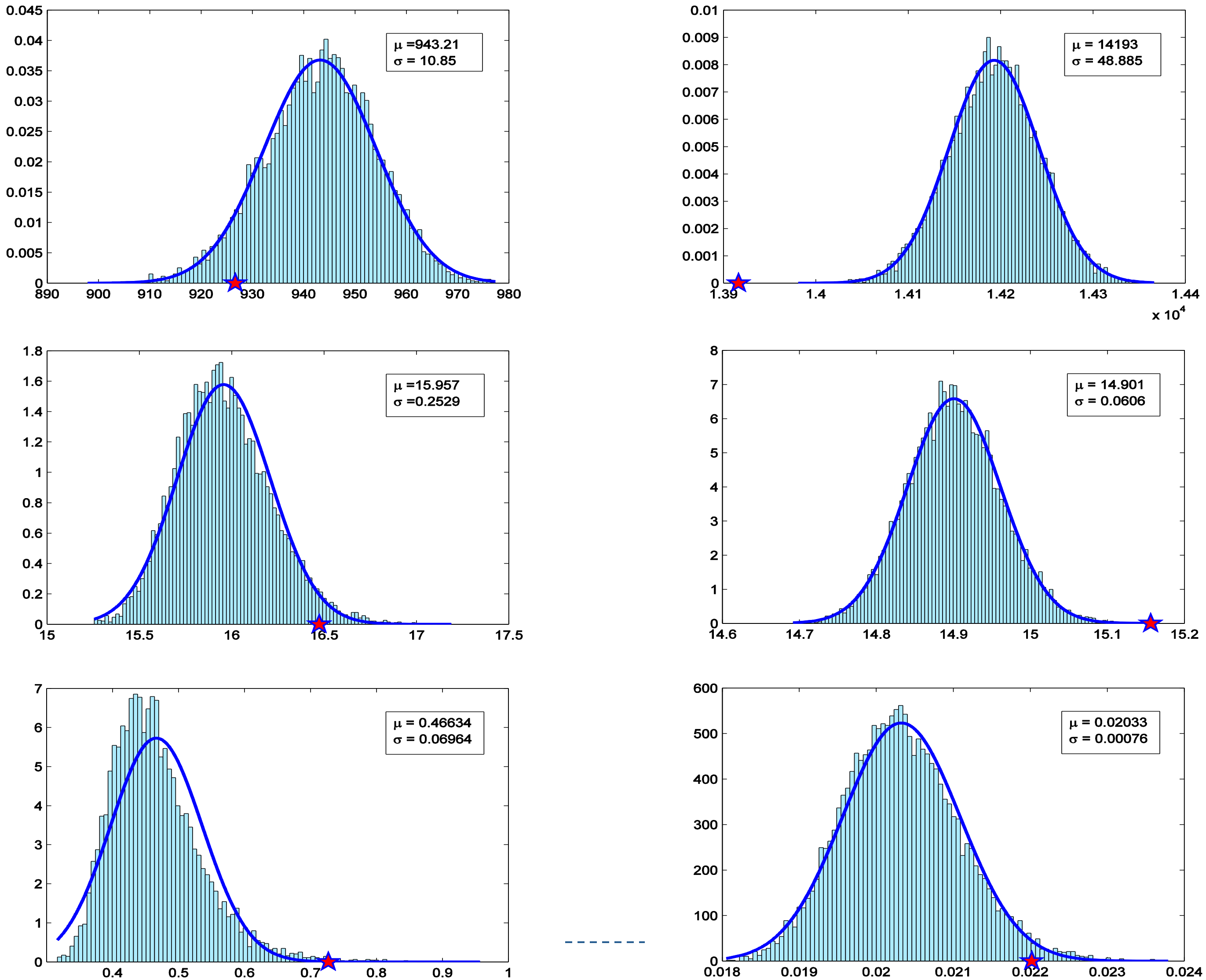
$$f_W(x) = \frac{\Sigma_{k=1}^{k^{\max}} \delta_\Delta(W_k - x)}{k^{\max}}$$

$$\delta_\Delta(x) = \begin{cases} \frac{1}{\Delta}, & -\frac{\Delta}{2} < x \leq -\frac{\Delta}{2} \\ 0, & \text{otherewise.} \end{cases}$$

Empirical and Fitting PDF of W(T)

IEEE-300

NYISO-2935



$W_1(T)$

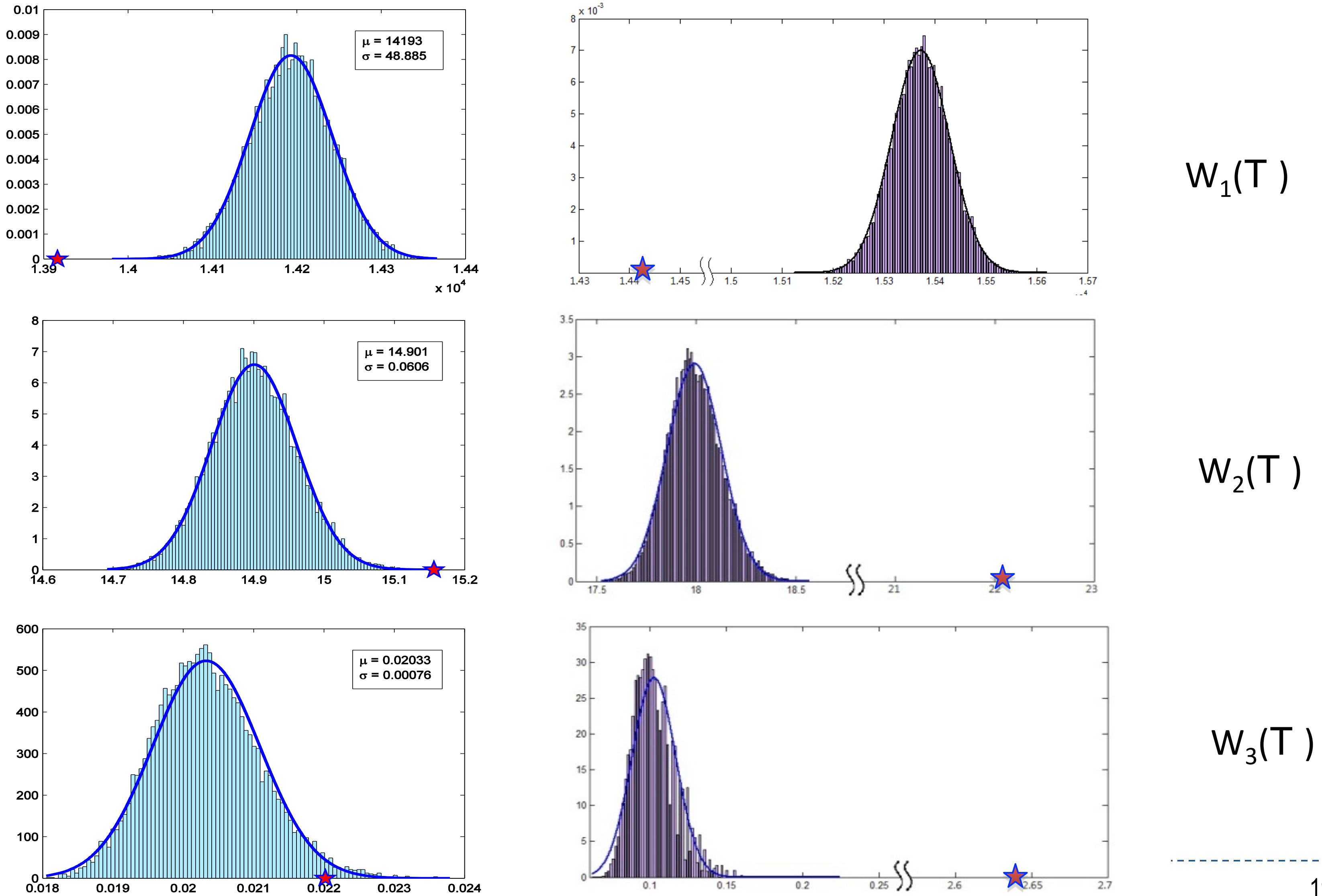
$W_2(T)$

$W_3(T)$

Empirical and Fitting PDF of W(T)

NYISO-2935

MPC -5633



Normal Fitting Parameters

TABEL I
The Parameters of Normal Distribution Fitting

	$W_1(\mathbb{T})$ $\mu/\sigma/W(\mathbb{T}^*)$	$W_2(\mathbb{T})$ $\mu/\sigma/W(\mathbb{T}^*)$	$W_3(\mathbb{T})$ $\mu/\sigma/W(\mathbb{T}^*)$
IEEE-300	943.21/10.58/927.5	15.95/0.252/16.47	0.466/0.069/0.726
NYISO	14193/48.8/13910	14.901/0.06/15.16	0.020/0.0007/0.022
MPC	15372/56.47/14428	17.99/0.13/22.32	0.102/0.0143/2.64

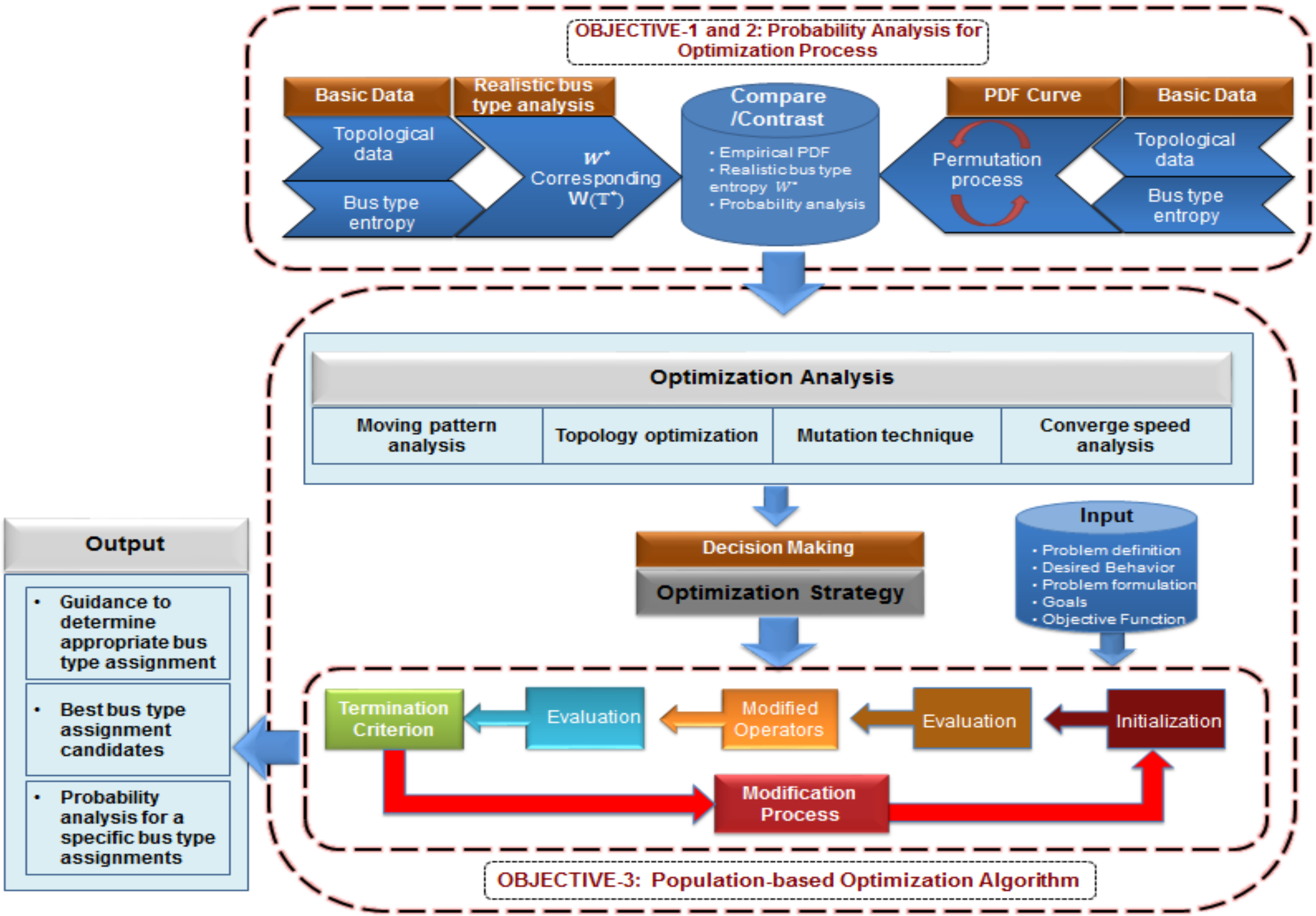
Normalized Distance of W(T*)

TABEL II
The Normalized Distance of Realistic Bus Type Entropy

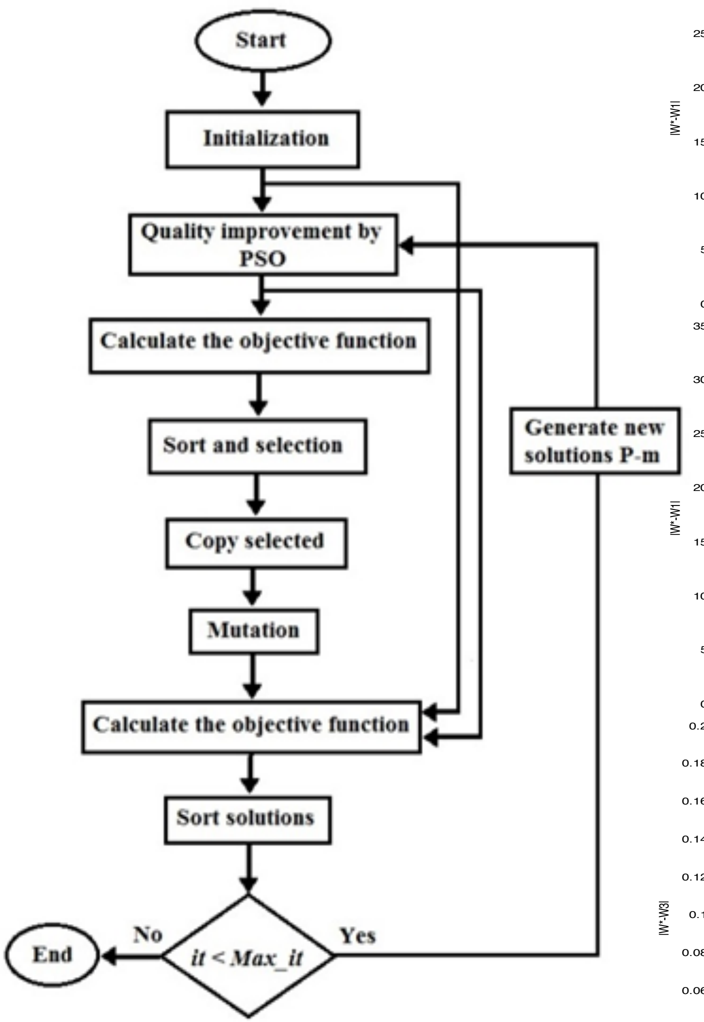
	(N, M)	$W_1(\mathbb{T})$ d_{W_*}	$W_2(\mathbb{T})$ d_{W_*}	$W_3(\mathbb{T})$ d_{W_*}
IEEE-300	(300,409)	1.48	1.96	3.76
NYISO	(2935,6567)	5.78	28.72	2.42
MPC	(5633,7053)	16.71	33.30	177.48

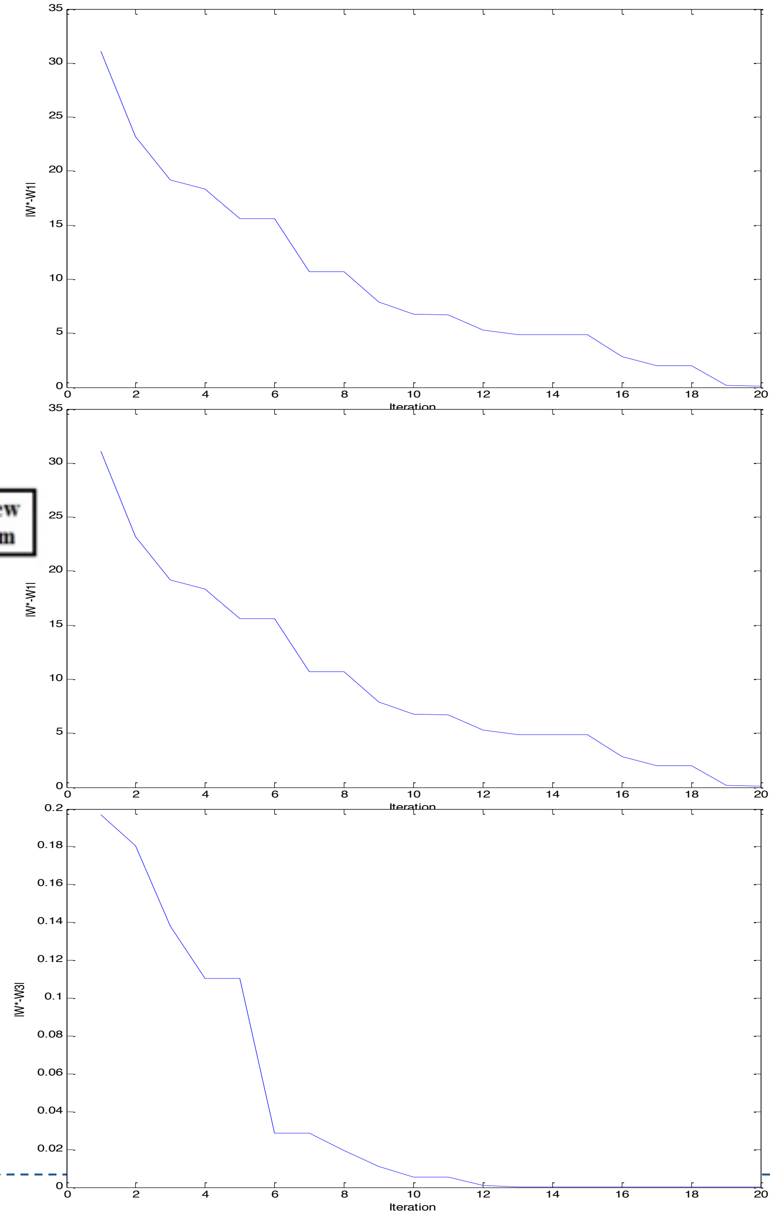
$$d_{W_*} = |W(\mathbb{T}^*) - \mu| / \sigma$$

Multi-objective Optimization Algorithm



Clonal Selection Algorithm



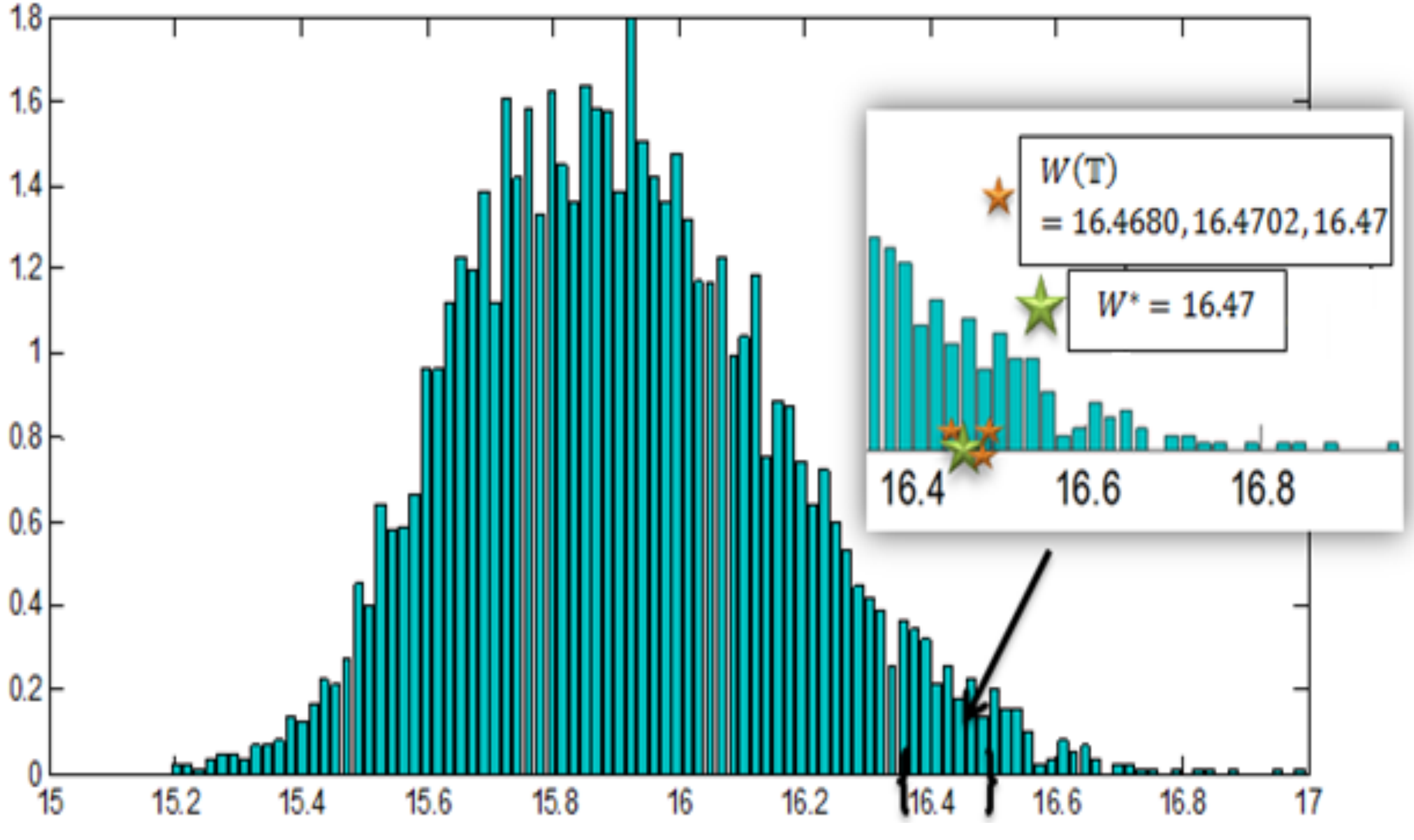


$W_1(T)$

$W_2(T)$

$W_3(T)$

Numerical Results



The best set of bus type assignments in for a 300 bus system - $W_2(T)$

Conclusions & Future Works

- The bus type (G/L/C) assignment of a realistic power system is not random but correlated.
- A novel measure W(T), called the *Bus Type Entropy*, is defined to characterize the correlated bus type assignment in a grid.
- Statistical analysis on the three realistic and synthetic grids verify the effectiveness of W(T):
  - W(T*) of a realistic power grid always stands out from those of random bus type assignments.
  - Consistent trend of the W(T*) is observed in all the test cases.
  - which is even more obvious for a large grid.

Conclusions & Future Works

- A multi-objective optimization algorithm is formulated to assign the bus types (G/L/C) that have the entropy values close to that of a realistic grid.
- The scaling property of W(T) the proposed entropy measure will be further studied versus the grid size and other electrical or topological metrics.
- Numerical simulation will be done to verify the effectiveness of W(T) in electrical aspects:
  - System vulnerability to cascading failures,
  - Other options?

Questions? ☺

Thank You!