# A Note on the Fisher Ideal Index Decomposition for Structural Change in Energy Intensity

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*Index numbers have been used to decompose aggregate trends in energy intensity, i.e., the ratio of energy use to activity. By making a direct appeal to the theory underlying price index numbers used by the energy decomposition literature, this note proposes the chain weighted Fisher Ideal Index as a formula that solves the `residual problem.' The connection to index number theory also allows us to illustrate that the measures of activity used to define energy intensity need not be additive across the sectors that are involved in the decomposition. We give an empirical example using recent U.S. manufacturing data of the Fisher Ideal Index, compared to the Törnqvist Divisia index, a popular index in the energy literature.*

## INTRODUCTION

The changes in the composition of economic activities and its impact on aggregate measures of energy intensity have been the subject of empirical analysis since Myers and Nakamura (1978). This literature is reviewed by Ang and Zhang (2000). These studies have either implicitly, or explicitly, made the connection to the economic theory of index numbers. For example, many early studies used a fixed base year index that is analogous to the Laspeyres index, (for an example, see DOE, 1989). Boyd, McDonald et al. (1987) explicitly make an appeal to index numbers when they introduced the Divisia index approach and the Törnqvist approximation for this purpose. The Törnqvist and

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other forms of the Divisia have been widely used for this purpose in energy analysis since then.[1] Recently, there has been interest in developing official government statistics of energy intensity similar to those prepared for prices or productivity, using an index number decomposition (Lermit and Jollands, 2001; and Padfield, 2001).

Many studies discuss the problem of the residual term, or perfect decomposition. When an index number has a residual term, there is some portion of the change in energy intensity from the base period to the analysis period that remains unassigned to a particular index, i.e., it is "unexplained." The Laspeyres index and most applications of the Divisia index suffer from this problem. If the residual term is large enough, the empirical exercise may have little meaning. Ang and Zhang (2000) reports that the size of the residual in empirical studies varies dramatically, sometimes swamping the portion of the change that is explained. Ang and Choi (1997), Ang and Liu (2001) and Sun (1998a, b) offer two approaches to address this problem, the Log Mean Divisia method I and II, and the refined Laspeyres, respectively. By making a direct connection to index number theory, this paper offers another approach to the problem of the residual term, the Fisher Ideal index. This note focuses on the application of the Fisher Ideal index rather than the properties of the other proposed indices for energy analysis.

## 2. THEORETICAL BACKGROUND

Following Diewert (2001), the general form of the price index number problem arises from the following question.

*How do we express the change in a value aggregate*
$V_T/V_0 = S_i\, p_{i,T}\, q_{i,T} \,/\, S_i\, p_{i,0}\, q_{i,0}$ *in the form of two functions* $\boldsymbol{P}$ *and* $\boldsymbol{Q}$
*that satisfy* $V_T/V_0 = \boldsymbol{P}(p_0, p_T, q_0, q_T)\, \boldsymbol{Q}(p_0, p_T, q_0, q_T)$*?*

Where $p_{i,T}$ and $q_{i,T}$ are the price and quantity, respectively, of the $i^{th}$ commodity at times $(0,T)$.

To form a value aggregate in the form of the price index problem the energy intensity decomposition application of the index number approach relies on the identity

$$E_t = S_i\, y_{i,t}(e_{i,t}/y_{i,t}) = S_i\, y_{i,t}\, I_{i,t} \qquad (1)$$

where $E$ and $e_i$ denotes aggregate and sectoral energy use, $y_i$ denotes sectoral activity, and $I_i$ is sectoral energy intensity. This relationship can be expressed

1. As aggregator functions in production or consumption analysis they have been in wide use since the 1970s; see, for example, Diewert (1974) and Berndt, Morrison, and Watkins (1981).

in terms of aggregate energy intensity by dividing both sides by some aggregate measure of activity, denoted $Y$, and defining $S_{i,t}$ as the ratio of the $i^{th}$ sector to the aggregate activity measure at time $t$.

$$E_t / Y_t = S_i \, ( y_{i,t} / Y_t ) \, (e_{i,t} / y_{i,t}) = S_i \, S_{i,t} \, I_{i,t} \qquad (2)$$

In either form, it is easy to see the parallels between the price index problem and the energy (intensity) decomposition problem.

Focusing on the energy identity in (1) and paraphrasing Deiwert,

*How do we express the change in an energy aggregate* $E_T / E_0 = S_i \, y_{i,T} I_{i,T} \, / \, S_i \, y_{i,0} I_{i,0}$ *in the form of two functions* ***ACT*** *and* ***INT*** *that satisfy* $E_T / E_0 =$ ***ACT*** $(y_0, y_T, I_0, I_T)$ ***INT*** $(y_0, y_T, I_0, I_T)$*?*

Where $y_{i,T}$ and $I_{i,T}$ are the activity and energy intensity, respectively, of the $i^{th}$ sector at time $t$, and where the index functions, ***ACT*** and ***INT***, represent aggregate activity and intensity change, respectively.

Drawing on the parallels between the price and energy intensity index problem we can see what is required to define an index of activity and energy intensity change for a collection of energy using activities. The first requirement is a set of sub-sectors and measures of energy use that form a partition, whose sum measures the energy use in some aggregate sector. The second requirement is a set of activities, $y_{i,t}$, that correspond to the partition which provide "meaningful" measures of energy intensity for each of those sub-sectors. A common example in the literature would be the aggregate sector of manufacturing, with sub-sectors corresponding to the Standard Industrial Classification System (SIC), North American Industrial Classification System (NAICS), or other classification taxonomy and activity measured by the data obtained from government sources such as the value of shipments, gross output, or value added. However, energy use may occur in sectors where meaningful activity and intensity measures may not be easily derived from economic accounts like GDP, e.g., private transportation activity might be measured in passenger miles. Even when activity may be measured in the GDP accounts, e.g., various types of non-manufacturing retail, wholesale and service sectors, an alternative energy intensity measure denominated in terms of commercial floor space might be chosen instead. Conditional on the chosen set of energy and activity measures, one can take advantage of the parallel of the price index question and the energy index formulations based on (1) to provide an array of possible solutions.

The energy intensity version of the index number question is based on equation (2) and is obtained by dividing both sides by an aggregate measure of activity, $Y_t$. If the sectors cover all energy use in the economy, this aggregate activity is commonly measured using GDP. We wish to consider whether this changes the index number formulation, i.e.,

*How do we express the change in the energy intensity aggregate,* $(E_T/Y_T)/(E_0/Y_0) = S_i\, S_{i,T} I_{i,T} \,/\, S_i\, S_{i,0} I_{i,0}$*, in the form of two functions* ***STR*** *and* ***INT**** *that satisfy*

$$(E_T/Y_T)/(E_0/Y_0) = \mathbf{STR}\,(S_0, S_T, I_0, I_T)\,\mathbf{INT^*}\,(S_0, S_T, I_0, I_T)?$$

Multiplying the energy intensity index formulation on both sides by $(Y_T/Y_0)$ we can write,

$$(E_T/Y_T) \,/\, (E_0/Y_0) \times (Y_T/Y_0) = (Y_T/Y_0)\,\mathbf{STR}(S_0, S_T, I_0, I_T)\,\mathbf{INT^*}\,(S_0, S_T, I_0, I_T) \tag{3a}$$

and from the energy index formulation,

$$E_T/E_0 = \mathbf{ACT}\,(y_0, y_T, I_0, I_T) \times \mathbf{INT}\,(y_0, y_T, I_0, I_T) \tag{3b}$$

If we consider the activities and intensities in (1) to be analogous to quantities and prices, respectively, the parallel to the price index problem is scaling the quantities by a constant. Index number theory proposes a number of desirable properties that the indices should have. One property is that the index should be *invariant* to scaling of the quantities (Diewert 2001; p26), e.g., changing the units of measurement. If we wish our intensity index to have a similar property this means that dividing our activities, $y_i$, by an aggregate should also leave the intensity index unchanged, so that $\mathbf{INT^*}\,(s_0, s_T, I_0, I_T) = \mathbf{INT^*}\,(y_0, y_T, I_0, I_T)$ and also $\mathbf{INT}\,(y_0, y_T, I_0, I_T) = \mathbf{INT}\,(S_0, S_T, I_0, I_T)$. Such a property is quite desirable for our intensity index, since it implies that the aggregate intensity index does not depend on the magnitude of the activities, only the mix of those activities. It is quite natural then to expect

$$\mathbf{INT^*}\,(y_0, y_T, I_0, I_T) = \mathbf{INT}\,(y_0, y_T, I_0, I_T) \tag{4}$$

and therefore,

$$(Y_T/Y_0)\,\mathbf{STR}\,(S_0, S_T, I_0, I_T) = \mathbf{ACT}\,(y_0, y_T, I_0, I_T) \tag{5}$$

This connection between the activity index, ***ACT***, and the "structure" index, ***STR,*** provides a direct interpretation. The activity index may further be decomposed into the effect of structure, i.e., the changing mix of activities, and the overall growth in activity. We investigate below whether the invariance property holds for the commonly used index number formulae presented.

The implication of (4) is that while the intensity version of the index number is commonly expressed in terms of "shares" of economic activity, i.e., $S_{i,t} = ( y_{i,t} / Y_t ) = ( y_{i,t} / S_i\, y_{i,t} )$, this need not be the case. The resulting energy intensity index number is invariant to whether the value $s_{i,t}$ can be interpreted as a *share* or simply a *shift* between the sector level activity measure and the aggregate level activity measure. It does not matter that the underlying measures of activity in the industrial example can be summed to equal an aggregate measure of activity for that sector, while measure like floor space and passenger miles cannot. All that matters is that intensity, $(e_{i,t} / y_{i,t} ) = I_{i,t}$, which is analogous to prices in the index number theory, is well defined, i.e., is a "good" measure of the sector level energy intensity. In fact, in applications of the price index number problem, neither quantities nor prices can be summed directly to equal an aggregate. One cannot directly add the quantities of automobiles consumed to those of jars of jelly. That aggregation problem is precisely what the price and quantity index does. Similarly, the intensity index, ***INT***, and the corresponding activity index, ***ACT***, or structure index, ***STR***, provides the aggregation across sectors for intensity (analogous to prices) and either total activity or structure (analogous to quantities).

## 3. INDEX NUMBER FORMULAE

The index number formulae below are given in terms of the energy intensity index formulation. The Laspeyres approach is quite common in energy intensity applications. For economic applications, this approach uses a base period fixed weight for the prices or quantities. In terms of energy intensity we have,

**Laspeyres**

$$L_{Str} = S_i\, S_{i,T}\, I_{i,0} / S_i\, S_{i,0}\, I_{i,0} \qquad (6)$$

$$L_{Int} = S_i\, S_{i,0}\, I_{i,T} / S_i\, S_{i,0}\, I_{i,0} \qquad (7)$$

where $L_{Str}$ is the 'structure' index and $L_{Int}$ is the 'intensity' index. By reversing the roles of the base period ($t = 0$) and the end period ($t = T$) we can obtain the Paasche index.

**Paasche**

$$P_{Str} = S_i\, S_{i,T}\, I_{i,T} / S_i\, S_{i,0}\, I_{i,T} \qquad (8)$$

$$P_{Int} = S_i\, S_{i,T}\, I_{i,T} / S_i\, S_{i,T}\, I_{i,0} \qquad (9)$$

The basic formulae for Törnqvist approximation to the Divisia are:

**Törnqvist Divisia**

$$D_{Str} = \exp\left[\sum_i \frac{(w_{i,T} + w_{i,0})/2}{\ln(S_{i,T}/S_{i,0})}\right] \tag{10}$$

$$D_{Int} = \exp\left[\sum_i \frac{(w_{i,T} + w_{i,0})/2}{\ln(I_{i,T}/I_{i,0})}\right] \tag{11}$$

Where $S_{i,t}$ is the $i^{th}$ activity share $w_{iT}$ is the $i^{th}$ energy share in period $T$. The Törnqvist Divisia formulae given above have also been called arithmetic mean Divisia (AMD), due to the use of $(w_{i,T} + w_{i,0})/2$ as the weighting function. Ang and Liu (2001) introduce the Log-mean Divisia Index Method I (LMDI) and show that it has perfect decomposition and consistency in aggregation. This modification to the Divisia index is replacement of the arithmetic average weighting function with one based on the log mean. The log mean $L$ of two numbers, $x$ and $y$ is

$$L(x, y) = (y - x)/\ln((y/x) \text{ for } x \neq y, \quad \text{and} \quad L(x, x) = x \tag{12}$$

The LMDI formulae are now:

**Log-mean Divisia**

$$D^*_{Str} = \exp\left[\sum_i \frac{L(E_{i,0}, E_{i,T})/L(E_0, E_T)}{\ln(S_{i,T}/S_{i,0})}\right] \tag{13}$$

$$D^*_{Int} = \exp\left[\sum_i \frac{L(E_{i,0}, E_{i,T})/L(E_0, E_T)}{\ln(I_{i,T}/I_{i,0})}\right] \tag{14}$$

This approach can be easily extended to an additive decomposition and multiple factors. Robustness to zero values can be handled by replacing zero values in the data with a small positive (non-archemidian) number with convergence of the index obtained as this number approaches zero.

The Fisher Ideal index is the geometric average of the Laspeyres and Paasche indices,

**Fisher Ideal**

$$F_{Str} = (L_{Str}\, P_{Str})^{1/2} \tag{15}$$

$$F_{Int} = (L_{Int}\, P_{Int})^{1/2} \tag{16}$$

Fisher identifies a number of desirable properties that an economic price or quantity index should have. The *invariance* property was discussed above. It is easy to verify that all of the above intensity indices are invariant to scaling $S_{i,T}$ by $Y_T$. The other important property we focus on is *factor reversal*. This property states if the functional form of the price index $\boldsymbol{P}(p_0, p_T, q_0, q_T)$ is acceptable, then it should also be a good form for the quantity index, with the roles of the price and quantity vectors simply reversed. This quantity index, i.e., $\boldsymbol{Q}(p_0, p_T, q_0, q_T) = \boldsymbol{P}(q_0, q_T, p_0, p_T)$, must satisfy $V_T/V_0 = \boldsymbol{P}(p_0, p_T, q_0, q_T)\, \boldsymbol{Q}(p_0, p_T, q_0, q_T)$. This condition is equivalent to perfect decomposition. Fisher (1921) shows that the Fisher Ideal Index is the only index that satisfies *factor reversal* and three other weak axioms of index number theory, *positivity, time reversal, and quantity reversal (or quantity weights symmetry)*. Thus, the Fisher Ideal index provides perfect decomposition.[2] It also follows that any other index that provides perfect decomposition is either equivalent to the Fisher Ideal index or fails to satisfy these other axioms.

## 4. EMPIRICAL EXAMPLE

To illustrate the application of the Fisher Ideal index we compare it to the Divisia decomposition for the manufacturing data from 1983 to 1998 used by Boyd and Laitner (2001). They employ a chained Törnqvist approximation of the Divisia index, which has also been called the rolling year Arithmetic Mean Divisia (AMD).[3] We use data from the Energy Information Administration (1999) the Bureau of Labor Statistics (Andreassen and Chentrens, 1999), and Argonne National Laboratory (Ross et al. 1993) to examine recent trends in energy use, focusing on the relationship between non-transportation energy use and economic activity.

2. We wish to thank an anonymous referee for the proof specifically presented for the energy intensity formulation (see Appendix A).

3. Greening, Davis et al. (1997) refer to chained indices as rolling year indices. When the Törnqvist approximation of the Divisia index was introduced by Boyd et al. (1987) for energy decomposition they use a chained index, but do not specifically identify it as such.

**Table 1. Comparison of the Chain Weighted Fisher Ideal Index and Törnqvist Approximation of the Divisia index for U.S. Manufacturing**

Electricity

| | | Fisher Ideal | | Törnqvist Divisia | |
|---|---|---|---|---|---|
| | Aggregate | Intensity | Structure | Intensity | Structure |
| 1983 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 1984 | 1.00444 | 1.02754 | 0.97752 | 1.02738 | 0.97769 |
| 1985 | 0.95534 | 1.05358 | 0.90675 | 1.05213 | 0.90803 |
| 1986 | 0.94904 | 1.01044 | 0.94856 | 1.00899 | 0.94997 |
| 1987 | 0.94275 | 0.94722 | 0.99528 | 0.94622 | 0.99630 |
| 1988 | 0.93210 | 0.89933 | 1.03644 | 0.89845 | 1.03746 |
| 1989 | 0.94171 | 0.92528 | 1.01776 | 0.92435 | 1.01878 |
| 1990 | 0.97421 | 0.96951 | 1.00485 | 0.96851 | 1.00588 |
| 1991 | 1.00212 | 1.01989 | 0.98258 | 1.01884 | 0.98359 |
| 1992 | 0.95824 | 0.98180 | 0.97601 | 0.98078 | 0.97702 |
| 1993 | 0.96543 | 0.98194 | 0.98319 | 0.98092 | 0.98420 |
| 1994 | 0.94337 | 0.96723 | 0.97533 | 0.96622 | 0.97632 |
| 1995 | 0.88588 | 0.96472 | 0.94881 | 0.96367 | 0.94935 |
| 1996 | 0.88588 | 0.95189 | 0.92774 | 0.95066 | 0.92795 |
| 1997 | 0.82839 | 0.91345 | 0.90688 | 0.91233 | 0.90711 |
| 1998 | 0.76488 | 0.87998 | 0.86920 | 0.87882 | 0.86949 |

Non-Electric Energy

| | | Fisher Ideal | | Törnqvist Divisia | |
|---|---|---|---|---|---|
| | Aggregate | Intensity | Structure | Intensity | Structure |
| 1983 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 1984 | 0.93935 | 0.95237 | 0.98632 | 0.95238 | 0.98631 |
| 1985 | 0.85645 | 0.89976 | 0.95187 | 0.89971 | 0.95189 |
| 1986 | 0.87212 | 0.84552 | 1.03146 | 0.84550 | 1.03143 |
| 1987 | 0.86981 | 0.81469 | 1.06766 | 0.81466 | 1.06760 |
| 1988 | 0.85143 | 0.75361 | 1.12980 | 0.75359 | 1.12972 |
| 1989 | 0.87084 | 0.80653 | 1.07973 | 0.80649 | 1.07970 |
| 1990 | 0.90126 | 0.87213 | 1.03340 | 0.87206 | 1.03341 |
| 1991 | 0.89421 | 0.87762 | 1.01890 | 0.87761 | 1.01883 |
| 1992 | 0.88852 | 0.86006 | 1.03309 | 0.86010 | 1.03302 |
| 1993 | 0.94428 | 0.89649 | 1.05330 | 0.89654 | 1.05323 |
| 1994 | 0.92868 | 0.89168 | 1.04150 | 0.89171 | 1.04143 |
| 1995 | 0.88890 | 0.88912 | 0.99975 | 0.88915 | 0.99969 |
| 1996 | 0.84615 | 0.87434 | 0.96776 | 0.87437 | 0.96770 |
| 1997 | 0.78325 | 0.83214 | 0.94126 | 0.83217 | 0.94120 |
| 1998 | 0.71308 | 0.79673 | 0.89500 | 0.79677 | 0.89494 |

We separately examine the trends in electric and fossil fuel[4] use relative to the gross domestic product (GDP) in manufacturing. Specifically, we examine trends using the *energy intensity* formulation, i.e., the ratio of electricity consumption in kilowatt-hour (kWh) or fossil fuel consumption in mMBTU, to GDP from manufacturing. The sectors include 19 industrial sectors based on the LIEF model categories (Ross et al. 1993). BLS data on chain-weighted constant dollar value added were used as activity. The LIEF model energy data, supplemented by data from the Annual Survey of Manufacturers were used to provide the industrial sector energy data for this analysis.[5]

The first issue is the comparison between the Törnqvist and the Fisher Ideal index. While the failure of the Törnqvist to provide perfect decomposition is an important theoretical issue, it appears of minor importance in this application. This is illustrated in Table 1. In most years, the difference between the two indices occurs in the third significant digit, or lower.[6] This empirical observation is also supported by Diewert (2001), who shows that the Törnqvist approximation of the Divisia index closely approximates the Fisher Ideal index. This should be some comfort to analysts, since the Törnqvist version of the Divisia index has been very popular for energy decomposition since its introduction.

The empirical significance of the decomposition for U.S. manufacturing is more easily illustrated in Figures 1 and 2 for electricity and non-electric energy, respectively. Both electric and non-electric energy intensity are relatively flat during the late eighties and early nineties. When the composition of industry is accounted for, we observe that the reduction in electric intensity in the nineties is about half structural and half real intensity. For non-electric energy, the story is a bit more complicated. The recovery of heavy industry in the early eighties tended to increase the aggregate energy intensity. Real intensity actually fell much faster, until 1988, when the trends reversed. During the nineties sectoral shift accounted for most of the decline in aggregate energy intensity, even offsetting a rise in real intensity between 1988 and 1990. In the late nineties, real energy intensity does begin to fall again, but it is not clear if this is a new emerging trend or not.

4. The term non-electric energy would be more accurate, since we include by-product and biomass based fuels in this category. The term fossil fuel is used for ease in exposition.

5. The complete dataset is available from the authors on request.

6. Boyd and Laitner (2001) mention in the appendix that there is little empirical difference between the Log Mean Divisia method I (LMDI), which also provides a perfect decomposition, and their computations for the Törnqvist.

**Figure 1. Manufacturing Electricity Intensity and the Fisher Ideal Indices for Structural Shift and Real Intensity**

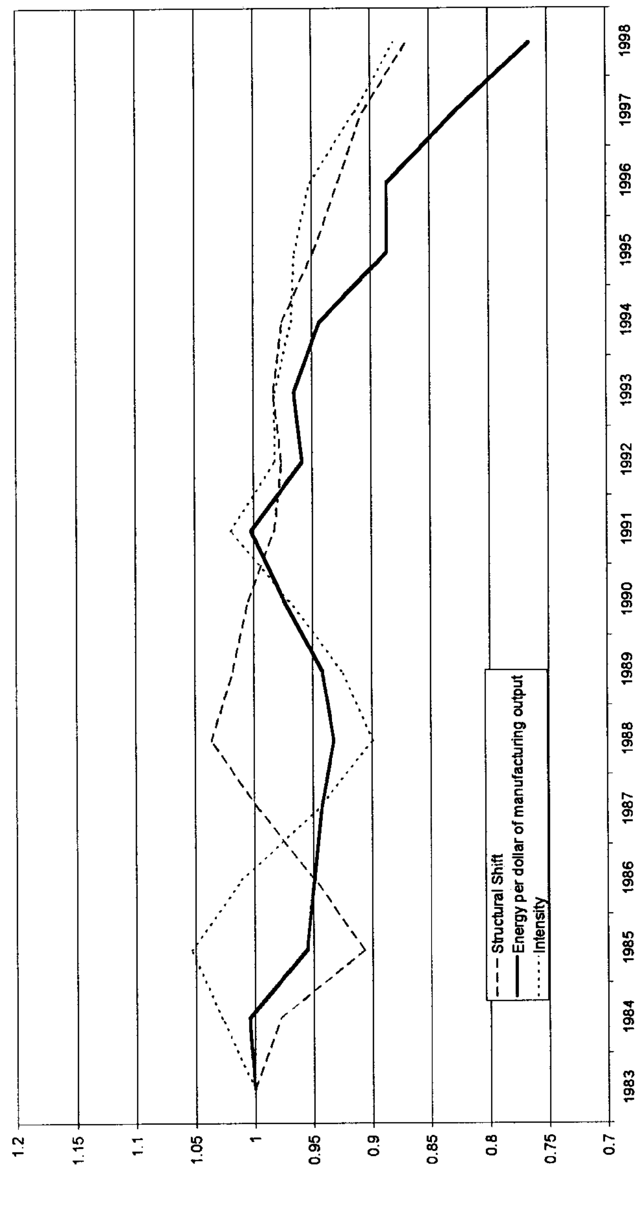
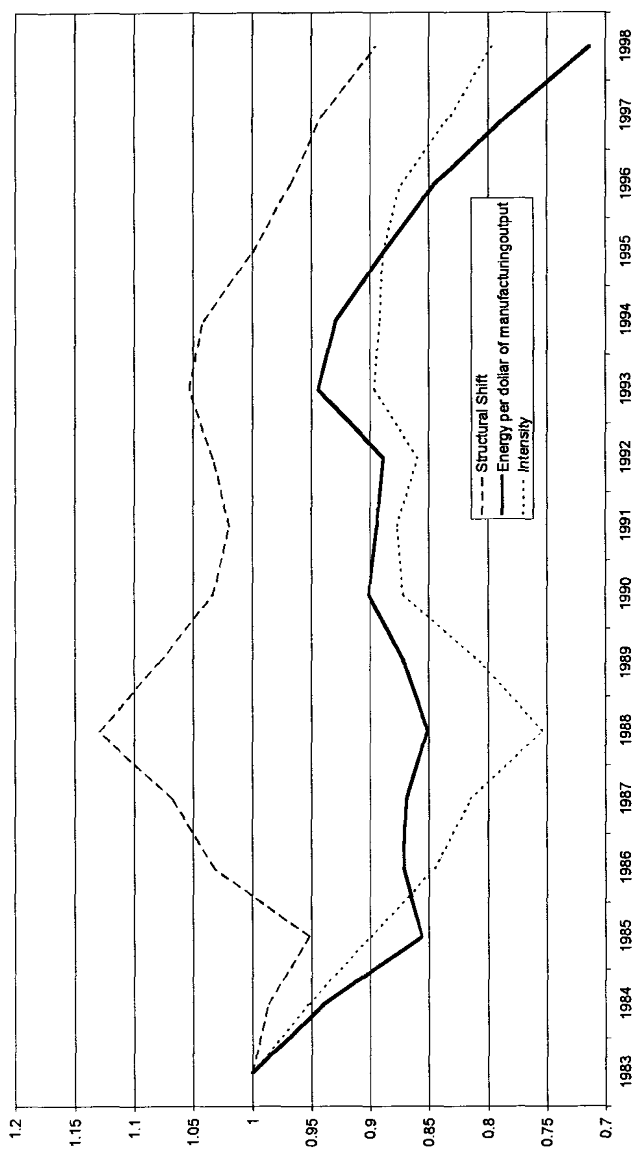


**Figure 2. Manufacturing Non-Electric Energy Intensity and the Fisher Ideal Indices for Structural Shift and Real Intensity**



## 4.1 Manufacturing Electricity/GDP Trends: 1983-1998

The drop in electricity intensity that occurred in the late nineties occurred during a period of very rapid economic expansion. Figure 1 shows the recent trends in electricity efficiency once we account for sectoral shifts. The volatility in electricity/GDP ratio in the late eighties was driven by sectoral shift; specifically, production swings in primary aluminum, steel, and refining.[7] Sectoral shift accounted for about half of the overall –0.3% annual change in energy intensity during the period. Sectoral shift was more stable in the nineties, accounting for nearly all of the -0.4% annual intensity change during that period. The average contribution of intensity improvements remained nearly the same over the entire time period, except in the last three years. In 1995-1998, shift contributed about –0.3% to annual decline, the same as it did from 1990-1999. However, the average rate of real energy intensity change accelerated to –1.1%, compared with an overall rate of less than –0.2%.

## 4.2 Manufacturing Fossil Fuel/GDP Trends: 1983-1998

The decline in the ratio of aggregate manufacturing non-electric energy use to GDP is much larger than that of electricity to GDP, averaging –1.8% annually (see Figure 2). Sectoral shift contributed to increasing energy intensity from 1983-1988, effectively slowing the decline of aggregate energy intensity by offsetting some large increases in non-electric energy intensity. After 1988, sectoral shift accounted for nearly all (-1.4%) of the annual decline in aggregate intensity (-1.7%). However, in 1997 and 1998, aggregate intensity declined dramatically at –6.0% annually. From 1997 to 1998, sectoral shift caused a –2.7% annual rate of energy intensity, with an additional –3.3% remaining. In the previous 10 years, real intensity had averaged only -0.2% annual change.

## 4.3 Observations on Energy Intensity Changes from 1996 to 1998

Compared with trends in prior years, energy trends in the more recent years looked quite different. The recent years showed a marked acceleration of energy intensity decline. If we look back to the point where energy prices took major downward turn (1983 for electricity and 1986 for non-electric energy), an interesting picture emerges.

7. These sectors exhibited very volatile patterns in the eighties. For example, the annual growth rate in the aluminum industry was –55% in 1985 and 44% in 1987. Although not as dramatic as aluminum growth rates, annual growth rates in steel and refining ranged from –21% to 28% in the late eighties and very early nineties.

During the15-year period of 1983-1998, the rate of aggregate electricity intensity change was -0.3%, about half of which was sectoral shift and half was real intensity. During 1997 and 1998, electricity intensity changed by an annual rate of –1.8%. Sectoral shift doubled, from –0.13% to –0.26%. After accounting for the sectoral shift, we estimate the decline in real intensity as –1.6%.

For non-electric energy use in the manufacturing sector, the rate of change in aggregate energy intensity was –1.3% from 1986-1998. Almost four fifths, -1.0%, was sectoral shift; the remainder of the change was decline in real intensity, -0.2%. Between 1996 and 1998, the impact of sectoral shift increased to –2.7%, almost a factor of three. Real intensity declined even more dramatically, to -3.3%.

## 5. CONCLUSION

This note reinvents an old wheel by appealing to price index number theory and introducing the Fisher Ideal index as a tool in the area of energy intensity indicators. The chain weighted Fisher Ideal index has recently become the index number of choice in economic data series, due to a number of highly desirable properties. One of these properties is *factor reversal,* which implies *perfect decomposition*. As a consequence the Fisher Ideal index eliminates the residual problem that has occurred in many previous energy decomposition studies, as well as in price index applications.

The energy literature has applied the index number approach to decomposition of both *energy use* and *energy intensity*. We observe that the index number theory requires that the measures of energy must form an additive partition of total energy, but that the sectoral activities that are used to construct the measure of intensity (i.e., the denominator) need not be additive. This concept is the core of the price index problem, and so provides guidance to energy decomposition as well.

We compare the chain weighted Fisher Ideal index to the Törnqvist approximation to the Divisia using data from Boyd and Laitner (2001). The Fisher Ideal index provides perfect decomposition, but when compared to the Divisia application[8] this property is of minor empirical significance for this dataset. This should be some comfort to analysts, since the Törnqvist version of the Divisia index has been very popular for energy decomposition analysis since its introduction by Boyd et al. (1987). However, the empirical importance of perfect decomposition depends on the specific application, so when complete time series data are available, the Fisher Ideal index represents a "new" old index that can be applied in energy studies.

Empirically, the decomposition presented here reveals some interesting stylized facts about energy intensity for U.S. Manufacturing. For electricity there

8. This may be more to do with the chained approach used than the particular Divisia formulae that is used. Chain weighted, or rolling year indices, are more data intensive, so may not be used in instances when data are not available routinely.

was little net change in the eighties, but both structure and intensity factors lead to a decline in electricity intensity in the nineties. This is a change from the trends analyzed by Boyd et al. (1987), suggesting a possible reversal of a historical shift toward electrification. For non-electric energy, the growth in heavy industry obscures a large drop in intensity through the mid-eighties. However, during the late eighties and early nineties, a period of falling energy prices, the index of real intensity has risen, albeit declining slightly at the end of the period analyzed.

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## APPENDIX A

### Proof That the Fisher Ideal Provides Perfect Decomposition

Begin with the equation for energy intensity:

$$E_t / Y_t = \sum_i (y_{i,t} / Y_t)(e_{i,t} / y_{i,t})$$

The index for energy intensity is:

$$(E_T / Y_T) / (E_0 / Y_0) = (\sum_i S_{iT} I_{iT}) / (\sum_i S_{i0} I_{i0})$$

For the Fisher Ideal Index:

$$F_{Str} = (L_{Str} P_{Str})^{1/2}$$

$$F_{Int} = (L_{Int} P_{Int})^{1/2}$$

and thus:

$$F_{Tot} = F_{Str} F_{Int} = \{(L_{Str} P_{Str})(L_{Int} P_{Int})\}^{1/2}$$

$$= \left\{ \left( \frac{\sum_i S_{iT} I_{i0}}{\sum_i S_{i0} I_{i0}} x \frac{\sum_i S_{iT} I_{iT}}{\sum_i S_{i0} I_{iT}} \right) \left( \frac{\sum_i S_{i0} I_{iT}}{\sum_i S_{i0} I_{i0}} x \frac{\sum_i S_{iT} I_{iT}}{\sum_i S_{iT} I_{i0}} \right) \right\}^{1/2}$$

$$= (\sum_i S_{iT} I_{iT}) / (\sum_i S_{i0} I_{i0})$$