U.S. DEPARTMENT OF ENERGY
Electricity Delivery & Energy Reliability

Advanced Grid Modeling 2014 Peer Review

Chance-constrained OPF – Incorporating High-Performance Computing into Power Grid Operations

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June 18, 2014

Operational challenges in renewable incorporation

CIGRE -International Conference on Large High Voltage Electric Systems '09:

- *Large, random fluctuations in wind power must be balanced by other power sources, possibly located far away*
- *This causes large power flows through the transmission system*
- *Control is difficult – e.g. flow reversal observed*
- *Expand transmission capacity? Difficult, expensive, takes time*
- *Problems **already observed** when penetration is high*
- Our work: to develop a robust control scheme that is foundationally strong, computationally practicable and easy to incorporate into existing power engineering practice

Presentation Outline

1. ***Project purpose**: develop robust, modern mathematical methodologies for use in grid operations, principally OPF and Unit Commitment*
2. ***Significance and Impact**: safe, economic operation of the grid under high renewable penetration and high transmission levels*
3. ***Technical approach**: use of chance-constrained and robust optimization; fast optimization algorithms*
4. ***Technical accomplishments** (so far): a fast, scalable, robust chance-constrained optimization approach that scales well to real-world power transmission systems.*

OPF:

$$\min \; c(p) \quad \text{(a quadratic)}$$

s.t.

$$B\theta = p - d \tag{1}$$

$$|y_{ij}(\theta_i - \theta_j)| \;\leq\; u_{ij} \quad \text{for each line } ij \tag{2}$$

$$P_g^{min} \;\leq\; p_g \;\leq\; P_g^{max} \quad \text{for each generator bus } g \tag{3}$$

Notation:

$p \;=\;$ vector of generations $\in \mathbb{R}^n$, $\quad d \;=\;$ vector of loads $\in \mathbb{R}^n$

$B \;\in \mathbb{R}^{n \times n}$, $\quad$ (bus susceptance matrix)

$$\forall i, j : \quad B_{ij} = \begin{cases} -y_{ij}, & ij \;\in\; \mathcal{E} \text{ (set of lines)} \\ \sum_{k;\{k,j\} \in \mathcal{E}} y_{kj}, & i = j \\ 0, & \text{otherwise} \end{cases}$$

OPF + Real-Time Control

$$\min \; c(p) \quad \text{(a quadratic)}$$

s.t.

$$B\theta = p - d$$
$$|y_{ij}(\theta_i - \theta_j)| \leq u_{ij} \quad \text{for each line } ij$$
$$P_g^{min} \leq p_g \leq P_g^{max} \quad \text{for each bus } g$$

How does OPF handle short-term fluctuations in **demand** (d)?

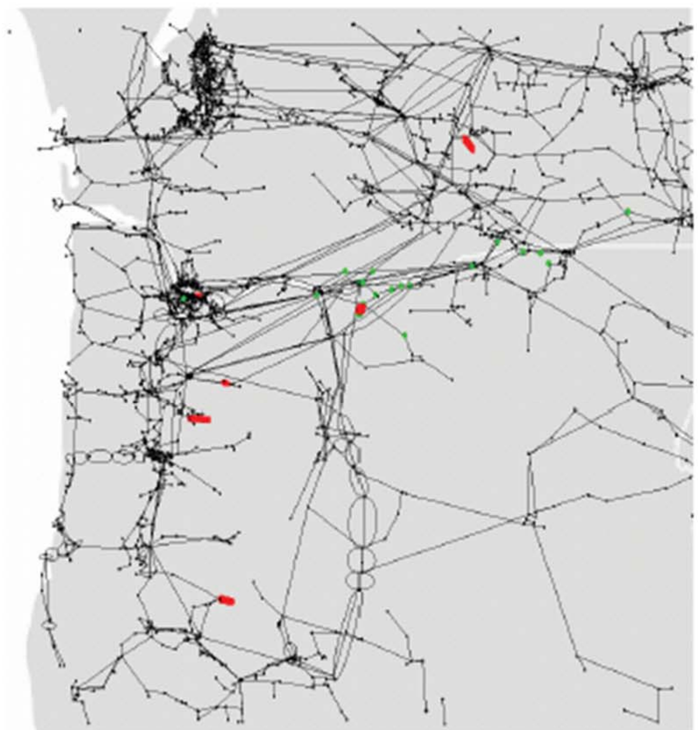
Frequency control:

- Automatic control: primary, secondary
- Generator output varies up or down **proportionally** to **aggregate change**
  Each participating generator has its own preset constant

Experiment—OPF + Real-Time Control

Bonneville Power Administration data, Northwest US

- data on wind fluctuations at planned farms
- with standard OPF, 7 lines exceed limit $\geq 8\%$ of the time



Line limits and line tripping

If power flow on a line exceeds its limit, the line becomes compromised and may 'trip'. But process is complex and time-averaged:

- Thermal limit is most common
- Thermal limit includes capabilities of terminal equipment
- Wind strength and direction contributes to line temperature. 'Exact' process governed by heat equation (IEEE 738).
- In 2003 Northeast U.S. and Canada blackout event, many critical lines tripped due to thermal reasons, but **well short** of their limits.

Take away:

Extremely difficult to precisely model line tripping as a function of line overloads.

Practicable proxy for line protection

- **Summary of above**: it is bad for a line to exceed its limit for too long; exact process complex and data-challenging

- **Want:** "fraction time a line exceeds its limit to be small“

- **Proxy**: Prob(violation on line i) < ε_i for each line i

Goals for Control Under Uncertainty

- **Familiar control**: if possible, similar to current power engineering practice
- **Aware of line and generator limits,** through chance constraints, i.e. probabilistic reliability
- **But not too conservative**
- **Computationally practicable:** should run fast on a current workstation even on large examples

Model for Real-Time Control *Between OPFs*

The control specifies, for each generator ***i***, two parameters

- $\overline{p_i}$ = mean output at ***i***
- α_i = response parameter, nonnegative

Real-time output of generator ***i*** :

$$p_i = \overline{p}_i - \alpha_i \sum_j \Delta\omega_j$$

Here $\Delta\omega_j$ = deviation from mean output of renewable j . We impose

$$\sum_i \alpha_i = 1$$

to emulate the action of primary and secondary frequency control

Parallels existing engineering practice, **BUT** we optimize over the control parameters in risk-aware fashion (chance constraints)

Computing line flows, under DC approximation

B = *bus susceptance matrix,* B^+ = *pseudo-inverse of* B

wind power at bus i: $\mu_i + \mathbf{w}_i$ — Wind generation fluctuations

DC approximation

- $B\boldsymbol{\theta} = \overline{p} - d + \mu + \mathbf{w} - (\sum_{i \in G} \mathbf{w}_i)\alpha$
- $\boldsymbol{\theta} = B^+(\bar{p} - d + \mu) + B^+(I - \alpha e^T)\mathbf{w}$
- flow is a linear combination of bus power injections:

$$\mathbf{f_{ij}} = y_{ij}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j)$$

Boldface = *random variables*

Computing fluctuating line flows, under DC approximation

$$\mathbf{f_{ij}} = y_{ij}\left((B_i^+ - B_j^+)^T(\bar{p} - d + \mu) + (A_i - A_j)^T\mathbf{w}\right),$$

$$A = B^+(I - \alpha e^T)$$

Fluctuating power flows due to wind and system response

Given distribution of wind can calculate moments of line flows:

- $E\mathbf{f_{ij}} = y_{ij}(B_i^+ - B_j^+)^T(\bar{p} - d + \mu)$
- $var(\mathbf{f_{ij}}) = y_{ij}^2 \sum_k (A_{ik} - A_{jk})^2 \sigma_k^2$ (assuming independence)
- and higher moments if necessary

From chance constraints to deterministic model

- chance constraint: $P(\mathbf{f_{ij}} > f_{ij}^{max}) < \epsilon_{ij}$ **and** $P(\mathbf{f_{ij}} < -f_{ij}^{max}) < \epsilon_{ij}$
- from moments of $\mathbf{f_{ij}}$, can get conservative approximations using e.g. Chebyshev's inequality
- for Gaussian wind, can do better, since $\mathbf{f_{ij}}$ is Gaussian :

$$|E\mathbf{f_{ij}}| + var(\mathbf{f_{ij}})\phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{max}$$

Chance Constrained Optimal Power Flow

Choose control parameters so as to minimize expected cost, with overload probability kept small

$$\min_{\overline{p},\alpha} \mathbb{E}[c(\overline{p})]$$ Min Cost

$$\text{s.t.} \sum_{i \in G} \alpha_i = 1,\ \alpha \geq 0$$ Freq. regulation model

$$B\delta = \alpha, \delta_n = 0$$

$$\sum_{i \in G} \overline{p}_i + \sum_{i \in W} \mu_i = \sum_{i \in D} d_i$$ Avg. power balance

$$\overline{f}_{ij} = y_{ij}(\overline{\theta}_i - \overline{\theta}_j),$$ Line flows

$$B\overline{\theta} = \overline{p} + \mu - d,\ \overline{\theta}_n = 0$$ DC power flow

$$s_{ij}^2 \geq y_{ij}^2 \sum_{k \in W} \sigma_k^2 (B_{ik}^+ - B_{jk}^+ - \delta_i + \delta_j)^2$$ Auxiliary constraint

$$|\overline{f}_{ij}| + s_{ij}\phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{max}$$ Chance constraint

Polish 2003-2004 "winter peak"

- 2746 buses, 3514 branches, 8 wind sources
- 5 – 20 % penetration, $\sigma = .3\mu$ at each wind source
- Formulation has 36625 variables
- 38507 constraints, 6242 conic constraints
- 128538 non-zeros, 87 dense columns
- *Piece of cake?*

Polish 2003-2004 “winter peak”

CPLEX:

- Total time on 16 threads = 3393 seconds
- “optimization status 6”
- Solution is very infeasible

Gurobi:

- Time = 31.1 seconds
- “Numerical trouble encountered”

Basic cutting-plane algorithm

conic constraint:

$$\sqrt{x_1^2 + x_2^2 + ... + x_k^2} = \|x\|_2 \leq y$$

candidate solution:

$$(x^*, y^*)$$

cutting-plane (linear constraint):

$$\|x^*\|_2 + \frac{{x^*}^T}{\|x^*\|_2}(x - x^*) = \frac{{x^*}^T x}{\|x^*\|_2} \leq y$$

Reduces conic program to a sequence of *linearly* constrained QPs

Basic cutting-plane algorithm

Polish 2003-2004 case

CPLEX: infeasible after 3300 seconds

Gurobi: "numerical trouble"

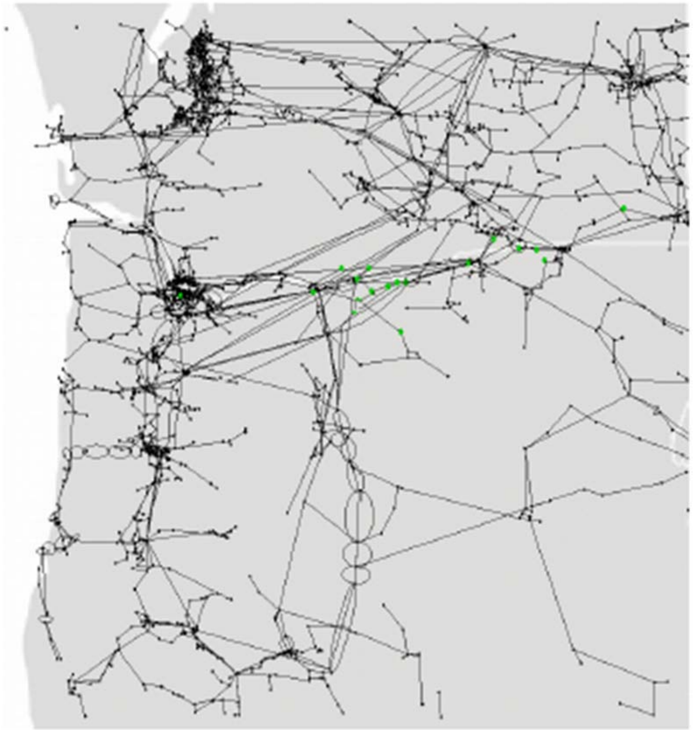
Cutting-plane algorithm: 33 seconds

Iteration	Max rel. error	Objective
1	1.2e-1	7.0933e6
4	1.3e-3	7.0934e6
7	1.9e-3	7.0934e6
10	1.0e-4	7.0964e6
12	8.9e-7	7.0965e6

Back to motivating example

BPA case:

- Standard OPF: cost 235603, 7 lines unsafe ≥ 8% of the time
- CC-OPF: cost 237297, every line safe ≥ 98 % of the time
- Runtime = 9.5 seconds



Robustness? Data errors? Model error?

$$s_{ij}^2 \geq y_{ij}^2 \sum_{k \in W} \sigma_{\mathbf{k}}^2 (B_{ik}^+ - B_{jk}^+ - \delta_i + \delta_j)^2$$

$$|\overline{f}_{ij}| + s_{ij}\phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{max}$$

(the $\overline{f}_{ij}$ implicitly incorporate the $\boldsymbol{\mu}_{\mathbf{i}}$)

What if the $\boldsymbol{\mu}_{\mathbf{i}}$ or the $\boldsymbol{\sigma}_{\mathbf{k}}$ are incorrect? ... What happens to

$$Prob(\mathbf{f}_{\mathbf{ij}} > f_{ij}^{max})?$$

Robustness? Data errors? Model error?

Let the *correct* parameters be $\tilde{\mu}_i$, $\tilde{\sigma}_i$ for each farm i.

Theorem: Suppose there are parameters $M > 0$, $V > 0$ such that

$$|\bar{\mu}_i - \mu_i| < M\mu_i \text{ and } |\ \bar{\sigma}_i^2 - \sigma_i| < V\sigma_i$$

for all i. Then:

$$Prob(\mathbf{f_{ij}} > f_{ij}^{max}) \ < \ \epsilon_{ij} + O(V) + O(M)$$

Here, the $O()$ "hides" some constants dependent on e.g. reactances

In other words, model deteriorates in a controlled manner.

How about small data errors?

Robust optimization

Polyhedral data error model:

$$|\tilde{\sigma}_i - \sigma_i| \leq \gamma_i \ \forall i, \quad \sum_i \frac{|\tilde{\sigma}_i - \sigma_i|}{\gamma_i} \leq \Gamma.$$

Ellipsoidal data error model:

$$(\tilde{\sigma} - \sigma)^T A(\tilde{\sigma} - \sigma) \ \leq \ b$$

Here $A \succeq 0$ and $b > 0$ are parameters.

Robust handling of chance constraints

Nominal case: $|E\,\mathbf{f_{ij}}| + var(\mathbf{f_{ij}})\phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{max}$

$\rightarrow$ a conic constraint

Robust case: $\max_{\mathcal{E}} \{|E\,\mathbf{f_{ij}}| + var(\mathbf{f_{ij}})\phi^{-1}(1 - \epsilon_{ij})\} \leq f_{ij}^{max}$

($\mathcal{E}$ **:** data error model)

How do we solve the robust-constrained case?

Traditional robust-optimization (duality) approach yields a nonconvex problem

Theorem. The robust problem is a convex optimization problem and can be solved in polynomial time in the polyhedral and ellipsoidal data cases.

An "ambiguous chance-constrained problem"

Conclusion

Chance Constrained Optimal Power Flow is a control formulation/algorithm that enables:

Computationally practicable probabilistic reliability: No sampling required—runs large examples on current workstations

Fully network aware: Considers all individual lines and generators

Integrates with current practice

Tunable conservatism

Future Work—FY15

- **Time-extended formulation:** Chance constraints on individual generator ramping between OPF periods
- **Fluctuating voltage magnitudes:** Increasing levels of sophistication of approximations to voltage fluctuations
  - Full linearization
  - Multi-linear convexification
  - Quadratic convexification
  - In collaboration with U. of Michigan (Ian Hiskens)

Acknowledgements/Contacts

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