U.S. DEPARTMENT OF ENERGY
Electricity Delivery & Energy Reliability

Advanced Grid Modeling 2014 Peer Review

Power Grid Optimization under Uncertainty:
Formulations, Algorithms, and High-Performance Computing

Jianhui Wang
Computational Engineer – Energy Systems
Decision and Information Sciences Division
Argonne National Laboratory

Victor M. Zavala
Assistant Computational Mathematician
Mathematics and Computer Science Division
Argonne National Laboratory

Outline of the Talk

Project I: **Power System Control and Optimization**

Project II: **Optimization Methodologies for Large-Scale Power Systems**

Project I: Power System Control and Optimization

Project purpose: The purpose of this project is to enhance the reliability and economics of the power grid by advanced optimization methods.

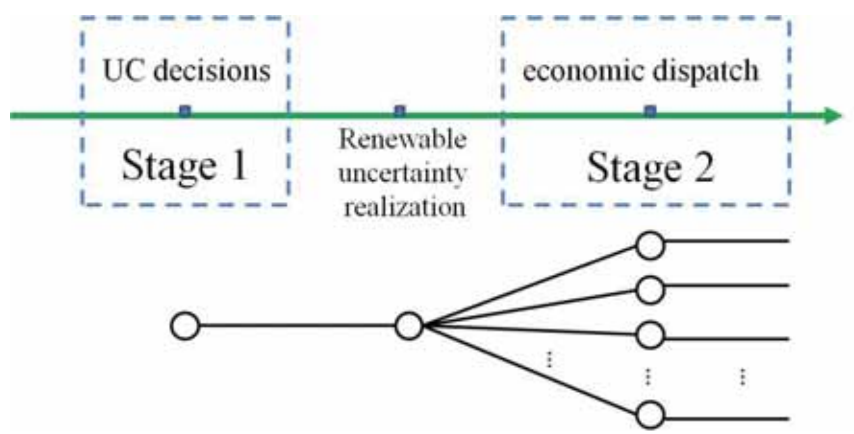
Technical Challenges:

- Uncertainty and variability of renewable generation
- Integrated transmission and distribution analysis
- Dynamic line rating
- Complexity of power system operation and optimization (non-linear, large networks, multi-timescale, stochastic, etc.)

Classical Stochastic Optimization Approach

- Two-stage stochastic UC: large-sized (renewable power) scenarios.
  - Stochastic UC formulations. (Philpott 06, Sen 06)
  - Active management of uncertainties.

  (Bouffard and Galiana 08, Ruiz et al. 09, Wang et al. 08, Oren et al. 10)
  - Chance constrained optimization (Wang et al. 12, 13a, 13b)
- Potential challenges:
  - How to precisely estimate the (joint) probability distributions?
  - How to solve the large-sized extensive formulation?



Robust Unit Commitment

- Robust optimization concept
- An uncertainty set that can capture the wind power “ramp” events
- A robust optimal solution that minimizes the total cost under the worst wind power fluctuation, while ensuring high utilization of wind power

(NOM)

$$\begin{aligned} \min \quad & c(y) + f(x) & \\ \text{s.t.} \quad & Ay \le b & (1) \\ & Gx + Hq \le d & (2) \\ & Wy + Tx \le h & (3) \\ & y \in \{0, 1\} & \end{aligned}$$

Objective function:

- c(y) represents the start-up/shut-down and fixed costs.
- f(x) represents the fuel and operational costs.

Constraints:

- Min-up/-down time, start-up/shut-down operations, and system spinning reserves.
- Power flow balance, transmission capacity limits.
- Power generation upper/lower bounds and ramp-rate limits.

Two-Stage Robust UC Model

- To minimize the total cost under the worst-case scenarios.
- Describe renewable output by an uncertainty set D.
- Problem decomposed into two stages:
  - 1st stage: UC decisions.
  - 2nd stage: economic dispatch under the worst-case scenario.

$$
\begin{aligned}
& \min_{y} && R(y) = c(y) + \max_{q \in \mathcal{D}} \min_{x \in \mathcal{X}(y,q)} f(x) \\
\text{(RUC)} \quad & \text{s.t.} && Ay \le b, \\
& && y \in \{0,1\}.
\end{aligned}
$$

$$\text{Notation} : \mathcal{X}(y,q) = \Big\{ x : \text{constraints (2) and (3)} \Big\}$$

Uncertainty Set Definition

- Each renewable output q_{bt} running in interval $[Q_{bt}^n - Q_{bt}^v, Q_{bt}^n + Q_{bt}^u]$.
  - E.g., the interval can be generated based on .05- and .95-percentiles of q_{bt} .
  - $z_{bt}^{\pm}$ describe the magnitude of increase/decrease from the forecasted value.
- Integer $\Gamma_b \in [0, T]$ restricts the number of deviations:
  - $\Gamma_b = 0 \rightarrow$ all $q_{bt} = Q_{bt}^n$; $\Gamma_b = 2T \rightarrow$ all q_{bt} free.
  - Γ_b : "budget of uncertainty."

Uncertainty set

$$\mathcal{D} := \Big\{ q \in \mathbb{R}^{|B| \times T} : \sum_{t=1}^{T} (z_{bt}^+ + z_{bt}^-) \leq \Gamma_b,$$
$$q_{bt} = Q_{bt}^n + z_{bt}^+ Q_{bt}^u - z_{bt}^- Q_{bt}^v, \ \forall t, \forall b \in B \Big\}$$

Reformulation of the 2nd-Stage Problem

- η, λ dual variables, N dual feasible region.
- The 2nd-stage problem $\Leftrightarrow$ a bilinear program.

$$
\begin{aligned}
R(y) =& c(y) + \max_{q \in \mathcal{D}} \min_{x \in \mathcal{X}(y,q)} f(x) \\
=& c(y) + \max_{q \in \mathcal{D}} \max_{(\eta,\lambda) \in \mathcal{N}} \left\{ \eta^T (d - Hq) + \lambda^T (h - Wy) \right\} \\
=& c(y) + \max_{(q,\eta,\lambda) \in \mathcal{D} \times \mathcal{N}} \left\{ -\eta^T Hq - \lambda^T Wy + \eta^T d + \lambda^T h \right\} \\
=& c(y) + \theta(y),
\end{aligned}
$$

$$
\text{where} \quad \theta(y) = \max_{(q,\eta,\lambda) \in \mathcal{D} \times \mathcal{N}} \left\{ -\eta^T Hq - \lambda^T Wy + \eta^T d + \lambda^T h \right\}.
$$

Benders' Decomposition Algorithm

- Benders' decomposition framework
  - Solve the bilinear sub-problem.
  - Add any violated constraints (4) as Benders' cuts.

(RUC)

$$
\begin{aligned}
\min \quad & c(y) + \theta(y) \\
\text{s.t.} \quad & Ay \leq b \\
& \theta(y) \geq -(\lambda^T W)y + (\eta^T d - \eta^T Hq + \lambda^T h), \qquad (4) \\
& \forall (q, \eta, \lambda) \in \mathcal{D} \times \mathcal{N}.
\end{aligned}
$$

Simulation Results : IEEE 118-bus System

- Ratio = $\frac{\text{Hydro capacity}}{\text{Average renewable output}}$
- Γ = Budget of uncertainty
- Opt. Gap: optimality gaps
- WV Gap: worst-case performance difference between robust UC and deterministic solutions

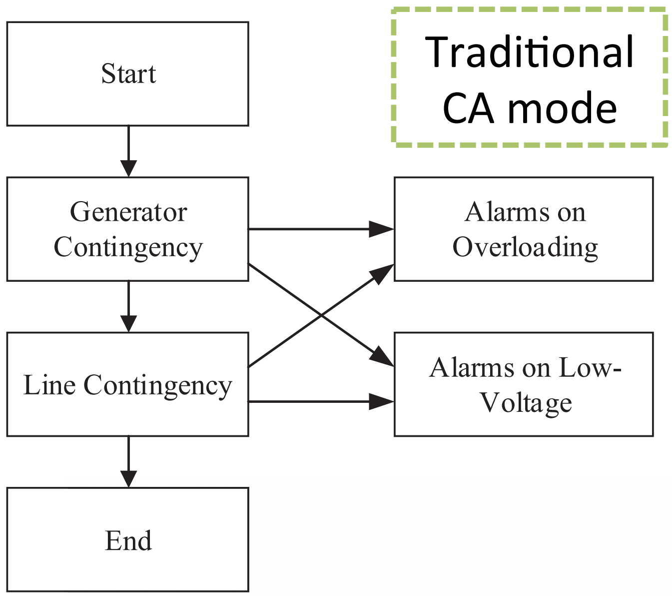
Ratio		Γ				
		2	4	6	8	10
0	Opt. Gap(%)	0.06	0.09	0.09	0.10	0.08
	WV Gap(%)	8.10	14.06	18.02	20.32	20.32
	Time (s)	194	353	578	1024	1876
0.1	Opt. Gap(%)	0.08	0.09	0.10	0.10	0.09
	WV Gap(%)	7.52	12.77	15.57	16.41	16.40
	Time (s)	101	309	546	997	542
0.2	Opt. Gap(%)	0.08	0.07	0.08	0.08	0.08
	WV Gap(%)	7.97	11.78	14.70	16.23	16.32
	Time (s)	145	331	908	1852	2053
0.3	Opt. Gap(%)	0.09	0.08	0.09	0.09	0.10
	WV Gap(%)	7.05	12.11	14.52	16.36	16.46
	Time (s)	178	361	736	1116	3101
0.4	Opt. Gap(%)	0.08	0.09	0.08	0.09	0.10
	WV Gap(%)	6.07	9.87	12.00	12.98	13.12
	Time (s)	218	463	1113	1446	1141
0.5	Opt. Gap(%)	0.08	0.09	0.09	0.09	0.07
	WV Gap(%)	5.65	9.29	11.33	12.30	12.49
	Time (s)	178	686	1114	995	3594

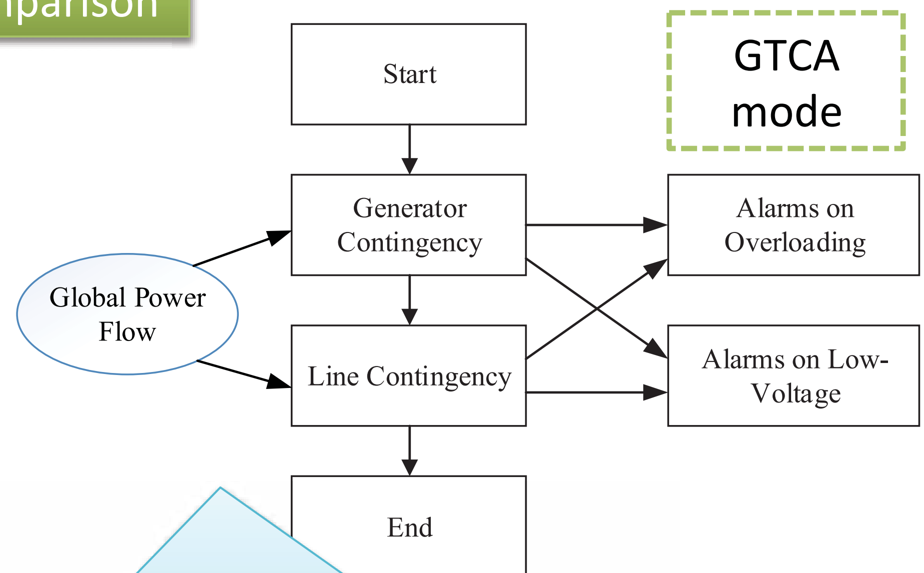
Integrated Transmission and Distribution Analysis

Motivation

- Increasing DG penetration and Arizona-Southern California outage on Sep. 8, 2011, etc.
- Traditional contingency analysis (CA) may give wrong alarms when a transmission power system (TPS) is connected with an electrically looped distribution system (DPS).
- A new global power flow based CA method (GTCA) is proposed to solve this problem.

Comparison





A global power flow in which both the TPS and DPS power flow are solved is used to reflect the transmission and distribution interactions

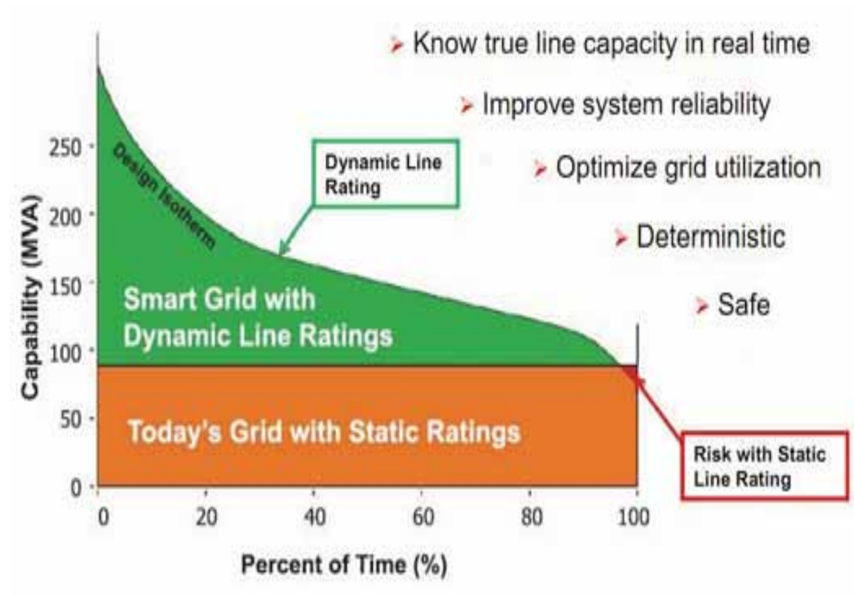
Key Findings

A number of numerical examples including IEEE 5, 14, 30, 118 bus systems have been tested to show the difference of the two CA methods

- Numerical simulations validate the accuracy and convergence of the proposed method, in particular when the distribution grid has loops or distributed generators.
- For a 6-bus TPS connected with a 3-feeder DPS with 2 loops , **2 wrong alarms are given by the traditional CA**.
- For a 118-bus TPS connected with a 6-feeder DPS with 3 loops, **5 wrong alarms are given by the traditional CA**.
- If the DPS is radial, the difference of the two CA methods is not big, but can lead to the overlook of the potential overloading occasionally as well.
- The running time of GTCA is longer than traditional CA, but that can be further shortened by several techniques like screening.

Dynamic Line Rating (DLR)

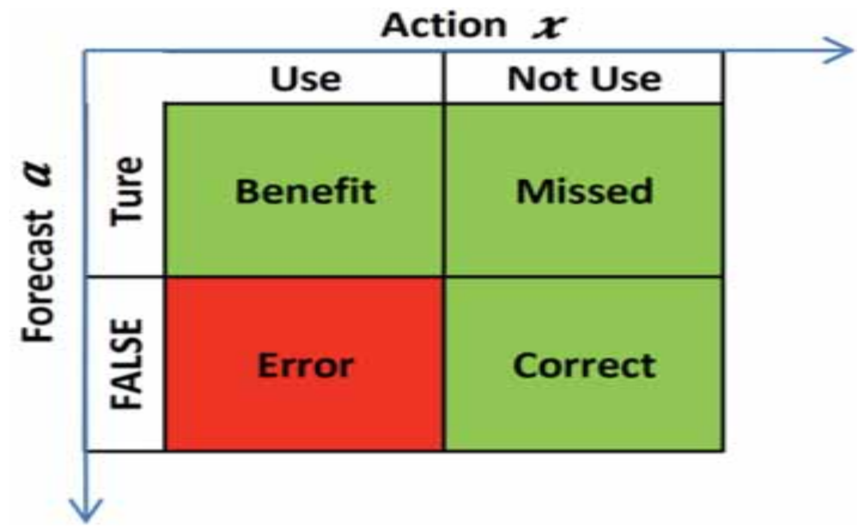
- Transmission lines connect generation and demand
- Line rating: the maximal allow-able flow on a transmission line
- Static line rating is conservative
  - Capacity in green zone is wasted
- Dynamic line rating may provide additional transmission capacity at no cost
  - Monitor real-time ambient environment (e.g., temperature, wind speed, line tension)
  - Forecast real-time transmission capacity
  - Alleviate transmission congestion



Source: http://www.neuralenergy.info/2009/06/circuit-ratings.html

A Binary Rating Forecast

- A binary rating forecast (Nexans - The Valley Group)
  - Forecast whether there will be certain amount of extra capacity on a line
  - Example: there is a 95% chance of 15% or more extra capacity above the static rating on line ℓ
- Forecasts can be wrong !
  - $x = 1$ represents that forecasted rating is used
  - When $(\tilde{a} = 0, x = 1)$, overloading occurs



- System-level overloading risk
  - Forecasting errors on different lines can be correlated
  - Overloading on multiple lines may lead to cascading blackouts
  - A risk measure at the system level (**The probability that more than *k* lines are overloaded)**

Modeling the Overloading Risk

- Model the random events
  - Forecast error: Bernoulli random number, $\tilde{a}_\ell = 1$ represents error occurs
  - If $(1 - \tilde{a}_\ell)x_\ell = 1$ then overloading occurs on line ℓ
- A reliability requirement

$$P_s := \mathbb{P}(\text{more than } k \text{ lines are overloaded}) = \mathbb{P}(\sum_{\ell \in L}(1 - \tilde{a}_\ell)x_\ell \geq k + 1) \leq \epsilon,$$

- Exact evaluation of the risk P_s is not practical
  - Incomplete distribution information in practice, i.e., only joint distributions up to level *m* are known, e.g., marginal and pair-wise joint distributions (*m* = 2)
- Risk requirement becomes ambiguous with incomplete information
  - Incomplete distribution information defines a family P of distributions
  - Which distribution function to use to evaluate the risk?
- A distributionally robust perspective (worst-case logic)

$$\sup_{\xi \in \mathcal{P}} \{\mathbb{P}_\xi(\sum_{\ell \in L}(1 - \tilde{a}_\ell)x_\ell \geq k + 1)\} \leq \epsilon$$

  - No ambiguity but still difficult to characterize

A Deterministic Approximation

- An inner approximation
  - Let $U(x) \geq F(x) := \sup_{\xi \in \mathcal{P}} \{\mathbb{P}_\xi(\sum_{\ell \in L}(1 - \tilde{a}_{\ell,t})x_{\ell,t} \geq k+1)\}$
  - Let $X := \{x \in \{0,1\}^{|L|} : F(x) \leq \epsilon\}$ and $\overline{X} := \{x \in \{0,1\}^{|L|} : U(x) \leq \epsilon\}$. Then

$$\overline{X} \subseteq X.$$

- Calculation of $U(x)$ [Prekopa 1990]
  - Let $s_0(x) = 1$, $s_i(x) = \sum_{C \subseteq L:|C|=i} p_C \prod_{j \in C} x_j$, and $\binom{j}{0} = 1$; $v_i \geq 0$

$$U(x) = \max\{\textstyle\sum_{j=k+1}^{|L|} v_j : \sum_{j=i}^{|L|} \binom{j}{i} v_j = s_i(x) \;\; i = 0...m\}$$

- Linearization
  - Let $\pi \in \mathbb{R}^{m+1}$ be the dual multipliers for the maximization problem

$$\overline{X} = \{x \in \{0,1\}^{|L|} : \exists \pi \in \mathbb{R}^{m+1} : \pi^\top S(x) \leq \epsilon, \pi^\top T \geq e_k^\top\}.$$

  - $\pi^\top S(x)$ can be linearized by McCormick linearization
  - Now $\bar{X}$ can be formulated as a mixed-integer linear program

Congestion Management with DLR

- Congestion management
  - Only consider economic dispatch
  - Dynamic line rating forecasts for the line capacities
- Only use dynamic ratings of those lines that
  - reduce congestion most effectively
  - do not exceed system overloading risk requirement
- Mathematical formulation

$$
\begin{aligned}
\min & \sum_{g\in G}\sum_{t\in T} c_g(p_{g,t}) + \sum_{n\in N}\sum_{t\in T} h_{n,t}q_{n,t} \\
\text{s.t. } & G(p_{g,t}, p_{\ell,t}, q_{n,t}) \geq 0 \\
& -SLR_\ell \cdot (1+\alpha_{\ell,t}x_{\ell,t}) \leq p_{\ell,t} \leq SLR_\ell \cdot (1+\alpha_{\ell,t}x_{\ell,t}) \quad \forall \ell \in L \; \forall t \in T \\
& \left[\sup_{\xi\in\mathcal{P}} \left(\mathbb{P}_\xi(\sum_{\ell\in L}(1-\tilde{a}_{\ell,t})x_{\ell,t} \geq k+1)\right) \leq \epsilon \Rightarrow \pi^\top S(x) \leq \epsilon, \pi^\top T \geq e_k^\top\right] \quad \forall t \in T
\end{aligned}
$$

MILP Formulation

$$\min \sum_{g\in G}\sum_{t\in T} c_g(p_{g,t}) + \sum_{n\in N}\sum_{t\in T} h_{n,t}q_{n,t}$$ ← Objective

$$\text{s.t. } G(p_{g,t}, p_{\ell,t}, q_{n,t}) \geq 0$$ ← Dispatch Const.

$$-SLR_\ell \cdot (1+\alpha_{\ell,t}x_{\ell,t}) \leq p_{\ell,t} \leq SLR_\ell \cdot (1+\alpha_{\ell,t}x_{\ell,t}) \quad \forall \ell \in L\ \forall t \in T$$ ← DLR

$$\pi_{0t} + \sum_{h=i}^{m} \binom{i}{h} \pi_{ht} \leq e_k^i \quad i = 1, ...n, t \in T$$

$$\pi_{0t} + \sum_{h=1}^{m} \sum_{C_t \in \mathcal{I}_t^h} p_{C_t}^h y_{C_t}^h \geq \epsilon, t \in T$$

Risk Req.

$$y_{C_t}^h \leq M_h^+ x_j \quad \forall j \in C_t, \forall C_t \in \mathcal{I}_t^h, h = 1, ..., m, t \in T$$

$$y_{C_t}^h \geq M_h^- x_j \quad \forall j \in C_t, \forall C_t \in \mathcal{I}^h, h = 1, ..., m, t \in T$$

$$y_{C_t}^h \leq \pi_{ht} - M_h^-(h - \sum_{j\in C_t} x_j) \quad \forall C_t \in \mathcal{I}_t^h, h = 1, ..., m, t \in T$$

$$y_{C_t}^h \geq \pi_{ht} - M_h^+(h - \sum_{j\in C_t} x_j) \quad \forall C_t \in \mathcal{I}_t^h, h = 1, ..., m, t \in T$$

Linearization

$$\pi_{0t} \leq 0;\ \pi_{ht} \text{ free } h = 1, ..., n;\ y_{C_t}^h \text{ free } \forall C_t \in \mathcal{I}_t^h, h = 1, ..., m$$

$$x_{\ell,t} \in \{0,1\}; p_{g,t}, q_{n,t} \geq 0.$$

Var. Bounds

$$\text{where } \mathcal{I}_t^h := \{S \subseteq \{1, ..., n\} : |S| = h\}$$

Case Study – Load Shedding Reduction

- Experiment settings
  - m = 2, i.e., only marginal and pair-wise joint distributions are available
  - k = 3, evaluating the overloading risk on 3 or more lines
  - Two sets of forecasting data:
    1) lower ratings (15% over static rating) with higher confidence levels
    2) higher ratings (30% over static rating) with lower confidence levels
  - Significant load shedding reduction achieved on the IEEE 73 (RTS 96)-bus system ϵ

Threshold		L.S. Reduction	Avg. # of Lines	k-Overloading Risk
0.15	0.01	**67%**	5.25	0.007
	0.05	**69%**	6.25	0.016
0.3	0.01	**68%**	3.25	0.009
	0.05	**80%**	5.00	0.030

K-Overloading risk: an upper bound on the actual overloading risk of more than k lines

Collaboration and Outreach

- **Publications** - more than 20 IEEE Transactions and journal papers and a number of conference presentations. Work also widely cited (more than 100 citations in the past two years)
- **Invited seminars and talks** – more than 10 including the ones at FERC, IBM, Northwestern University, University of Sydney, Iowa State, Sandia National Labs, University at Buffalo
- **Professional community involvement**– Editor for 9 academic journals (e.g., IEEE Transactions on Power Systems, IEEE Transactions on Smart Grid), guest editor for 10 special issues for various journals, Chair of the IEEE Power and Energy Society Power System Operational Methods subcommittee and panel session chair for the past 5 years
- **Collaboration** - working closely with the other Argonne divisions (e.g., MCS), industry (e.g., PJM, Alstom Grid) and other research institutions (e.g., University of Florida, Auburn University, University of Tennessee, Tsinghua University)

Representative Publications

- B. Chen, J. Wang, L. Wang, Y. He, Z. Wang, **Robust Optimization for Transmission Expansion Planning: Minimax Cost vs. Minimax Regret**, IEEE Transactions on Power Systems, In press.
- L. Fan, J. Wang, R. Jiang, Y. Guan, **Min-Max Regret Bidding Strategy for Thermal Generator Considering Price Uncertainty**, IEEE Transactions on Power Systems, In press.
- C. Liu, J. Wang, Y. Fu, V. Koritarov, **Multi-area Optimal Power Flow with Changeable Transmission Topology**, IET Generation, Transmission & Distribution, In Press, 2014.
- Y. Guan, J. Wang, **Uncertainty Sets for Robust Unit Commitment**, IEEE Transactions on Power Systems, In Press.
- R. Jiang, J. Wang, M. Zhang, Y. Guan**, Two-Stage Minimax Regret Unit Commitment Considering Wind Power Uncertainty**, IEEE Transactions on Power Systems, Vol. 28, No. 3, pp. 2271-2282, 2013.
- C. Zhao, J. Wang, J.P. Watson, Y. Guan, **Multi-Stage Robust Unit Commitment Considering Wind and Demand Response Uncertainties**, IEEE Transactions on Power Systems, Vol. 28, No. 3, pp. 2708-2717, 2013.
- J. Wang, J. Wang, C. Liu, J. Ruiz, **Stochastic Unit Commitment with Sub-hourly Dispatch Constraints**, Applied Energy, Vol. 105, pp. 418-422, 2013.
- J. Ostrowski, J. Wang, C. Liu, **Transmission Switching with Connectivity-Ensuring Constraints**, IEEE Transactions on Power Systems, In Press, 2014.
- C. Zhang, J. Wang, **Optimal Transmission Switching Considering Probabilistic Reliability**, IEEE Transactions on Power Systems, In Press, 2014.

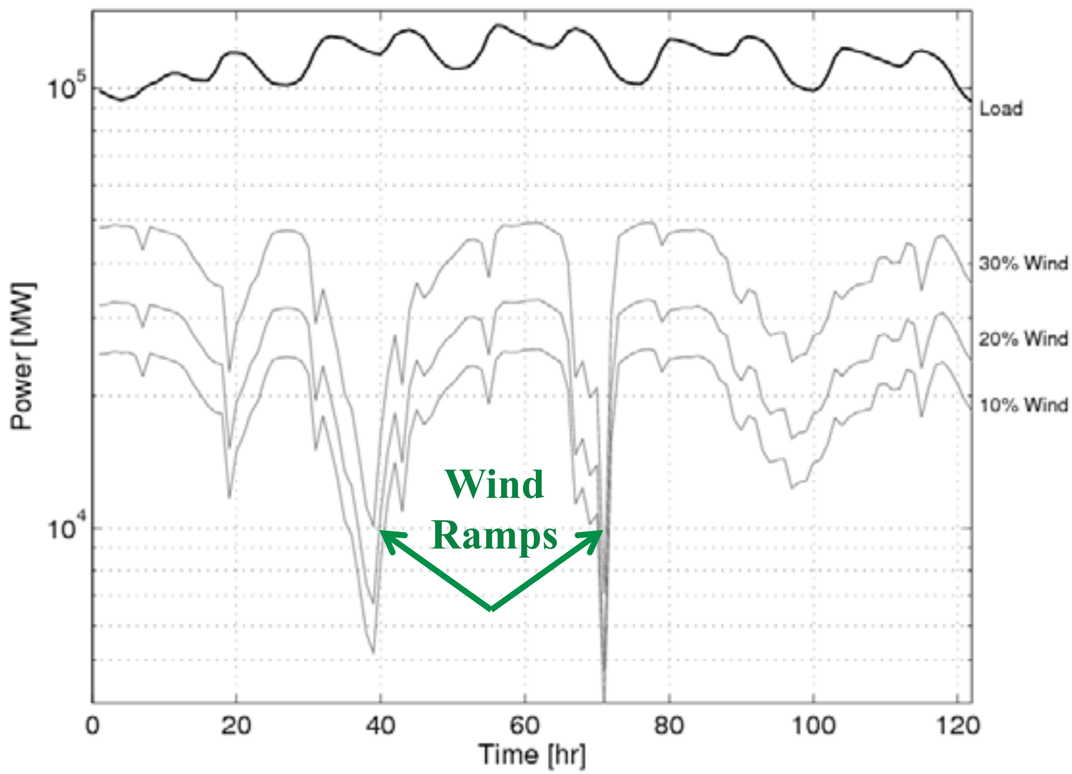
Outline of the Talk

Project I: **Power System Control and Optimization**

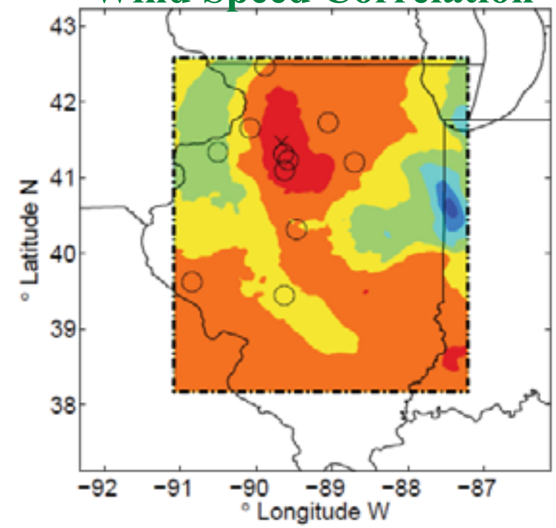
Project II: **Optimization Methodologies for Large-Scale Power Systems**

Renewable Power Adoption

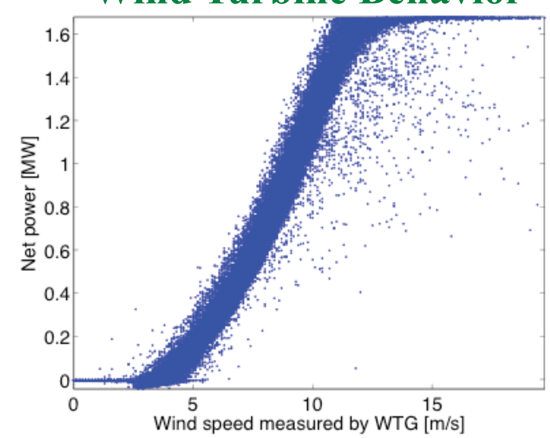
- **Adoption Levels of 20% in Upcoming 20 Years**
- **Spatio-Temporal Patterns & Scales Significantly Different Than Those of Loads**



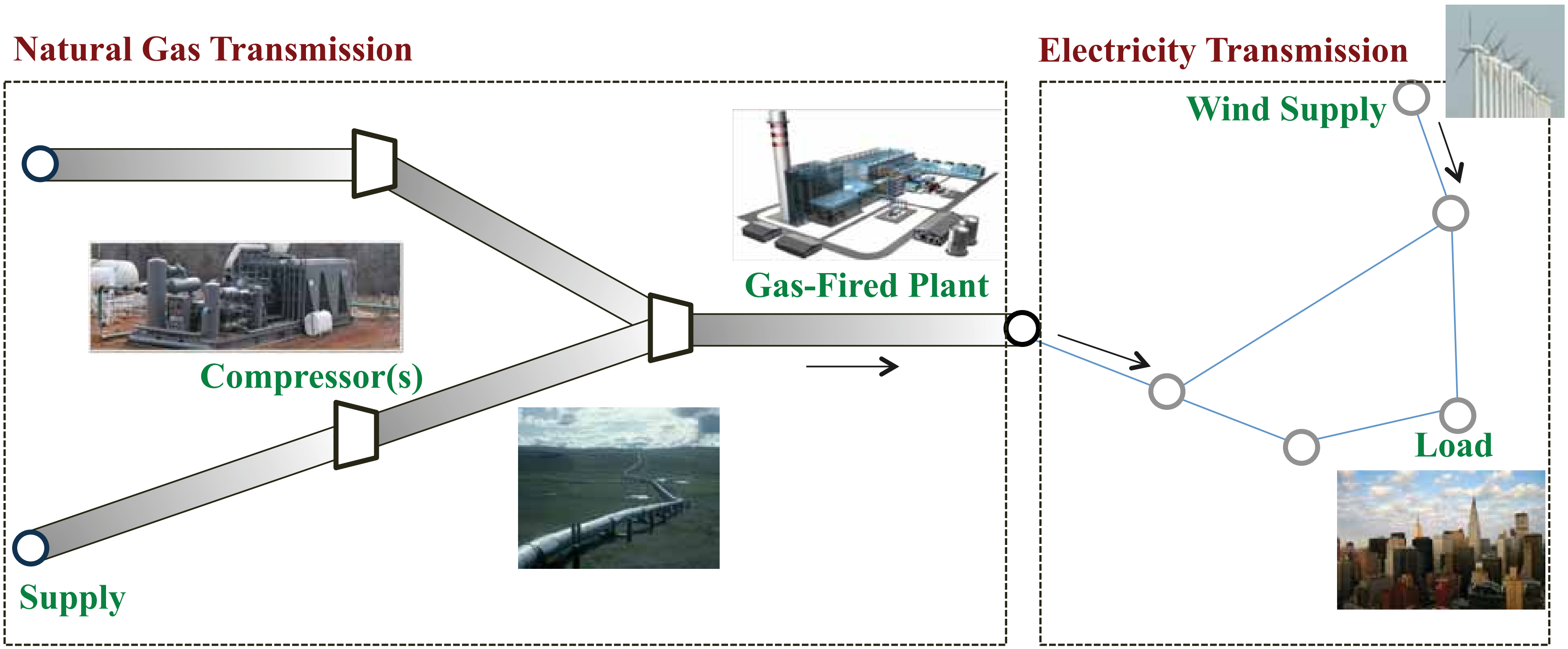
Wind Speed Correlation



Wind Turbine Behavior



Interdependent Infrastructures



- Gas Pipelines Provide Storage Capacity to Mitigate Fluctuations (Ramps & Capacity)
- But, Gas Travels at <u>30-50 mph</u> (Storage Has to be Built up <u>Well in Advance</u>)
- Aggressive Gas Withdrawals Cascade Upstream the Pipeline (Compromises <u>Stability</u>)
- Shortages of Natural Gas for Power Plants Occuring More Frequently

Emerging Behavior

Forbes Magazine Article on Polar Vortex in Midwest (Feburary, 2014)
http://tinyurl.com/mrvuqfl

– **California**'s electric grid operator asked **power generators to reduce their gas usage**. Southern California Gas asked its customers to power down where possible.

– In **Texas**, which **produces more natural gas than any other state**, the Electric Reliability Council of Texas (ERCOT) called a **state of emergency** on Thursday out of concern there wouldn't be **enough gas for power plants**. Earlier in the week ERCOT had asked to **fire up two big coal-fired plants** that are normally but on standby during the winter.

– In **New England**, tight supplies during the dreaded Polar Vortex caused **the price of natural gas to soar 20-fold** to more than $100 per thousand cubic feet. Gas was in such short supply in late January that New England's grid operators told power generators **to fire up not just their coal burners, but even their peaking plants that run on oil and even jet fuel**.

Questions & Technical Challenges

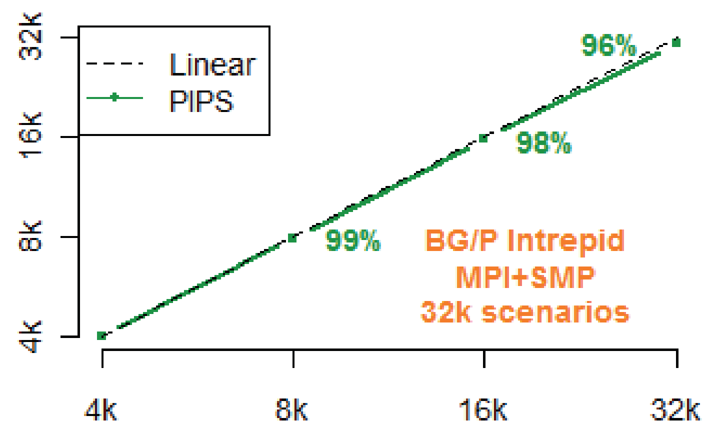
- **Economic Questions:** **Do Markets Spread Risk & Incentives in a Fair Manner?**
- **Reliability Questions:** **How to Characterize Uncertainties? How to Mitigate Them?**
- **Complexity:** **Nonconvex, Mixed-Integer, Huge Networks, Huge Probability Spaces**
- **Emerging Features:** **Multi-Scale, Multi-Physics, Stochastic, Optimal Control, Networks**

Parallel Interior-Point Solver (PIPS)

Interior Point Algorithm
Convex Multi-Stage Stochastic Programs

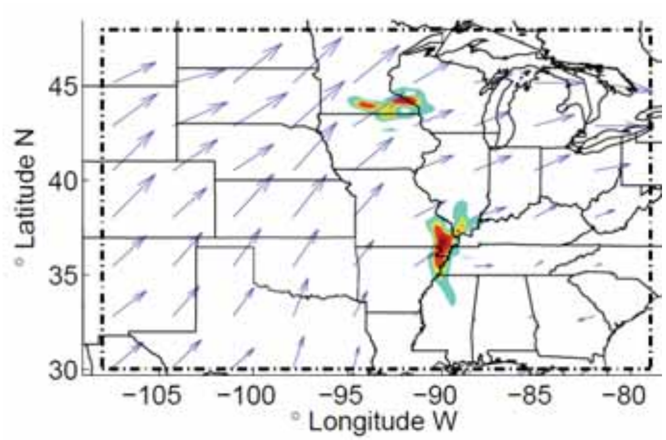
Argonne's Blue Gene/P
32,000 Nodes

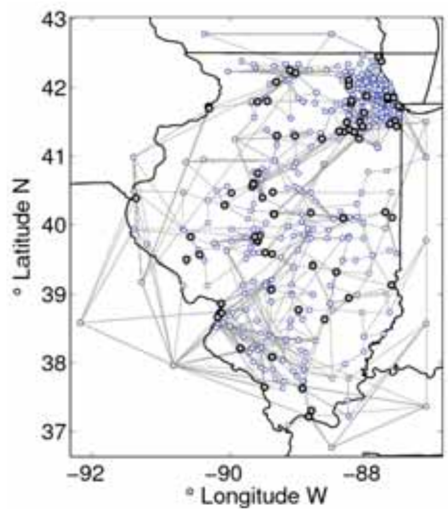
Stochastic Economic Dispatch for Illinois with
Up To 2 Billion Variables & 10,000 Scenarios Solved in Under an Hour

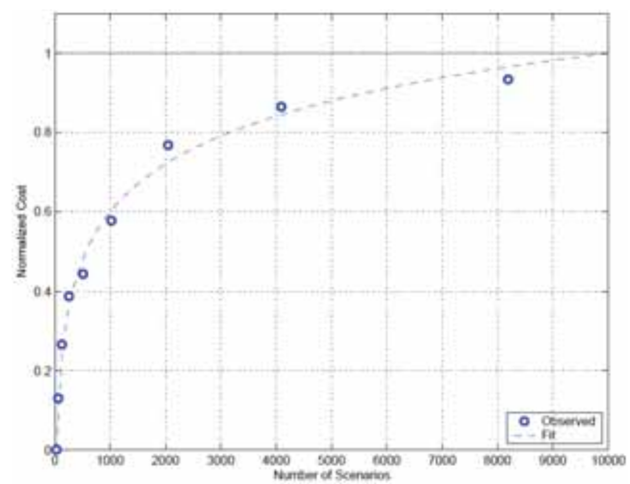


Pushed Advances in Numerical LinearAlgebra Library
PARDISO (O. Schenk)

Strong Scaling on Several Platforms:
BlueGene/P/Q (Argonne), Titan (Oak Ridge)





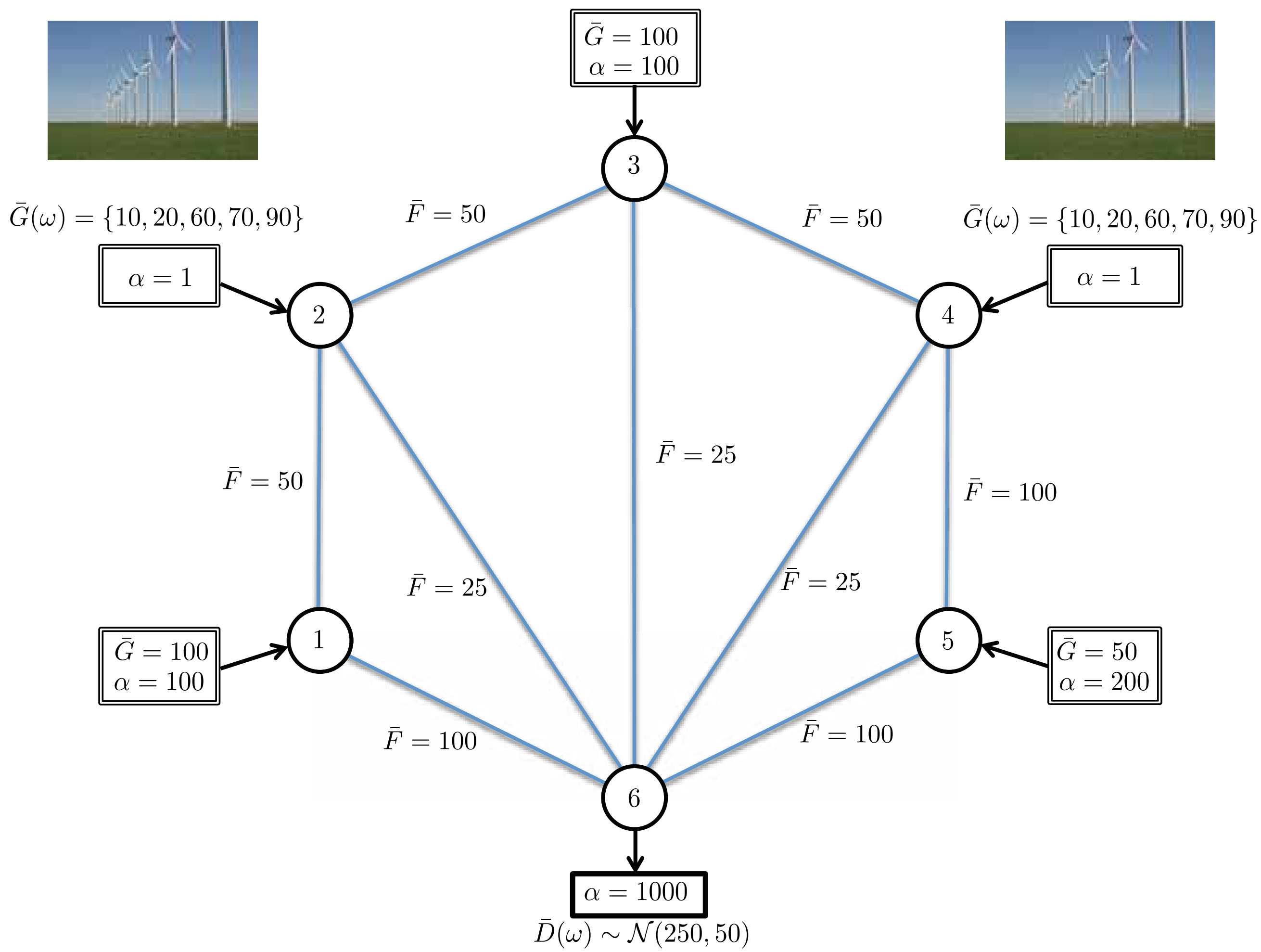


Stochastic Day-Ahead Market Clearing

$$
\begin{aligned}
\min_{d_j, D_j(\cdot), g_i, G_i(\cdot), f_\ell, F_\ell(\cdot)} \varphi^{sto} := \mathbb{E}&\left[\sum_{i \in \mathcal{G}} \alpha_i^g G_i(\omega) + \Delta\alpha_i^g |G_i(\omega) - g_i|\right] \\
-\,\mathbb{E}&\left[\sum_{j \in \mathcal{D}} \alpha_j^d D_j(\omega) - \Delta\alpha_j^d |D_j(\omega) - d_j|\right] \\
+\mathbb{E}&\left[\sum_{\ell \in \mathcal{L}} \Delta\alpha_\ell^f |F_\ell(\omega) - f_\ell|\right] \\
\text{s.t.} \sum_{\ell \in \mathcal{L}_n^{rec}} f_\ell - \sum_{\ell \in \mathcal{L}_n^{snd}} f_\ell + \sum_{i \in \mathcal{G}_n} g_i - \sum_{i \in \mathcal{D}_n} d_i &= 0, \;\; (\pi_n) \qquad n \in \mathcal{N} \\
\sum_{\ell \in \mathcal{L}_n^{rec}} (F_\ell(\omega) - f_\ell) - \sum_{\ell \in \mathcal{L}_n^{snd}} (F_\ell(\omega) - f_\ell) + \sum_{i \in \mathcal{G}_n} (G_i(\omega) - g_i) & \\
- \sum_{j \in \mathcal{D}_n} (D_j(\omega) - d_j) &= 0, \;\; (p(\omega)\Pi_n(\omega)) \qquad \omega \in \Omega, n \in \mathcal{N} \\
-\bar{F}_\ell(\omega) \le F_\ell(\omega) \le \bar{F}_\ell(\omega), & \qquad \omega \in \Omega, \ell \in \mathcal{L} \\
0 \le G_i(\omega) \le \bar{G}_i(\omega), & \qquad \omega \in \Omega, i \in \mathcal{G} \\
0 \le D_i(\omega) \le \bar{D}_j(\omega), & \qquad \omega \in \Omega, j \in \mathcal{D}
\end{aligned}
$$

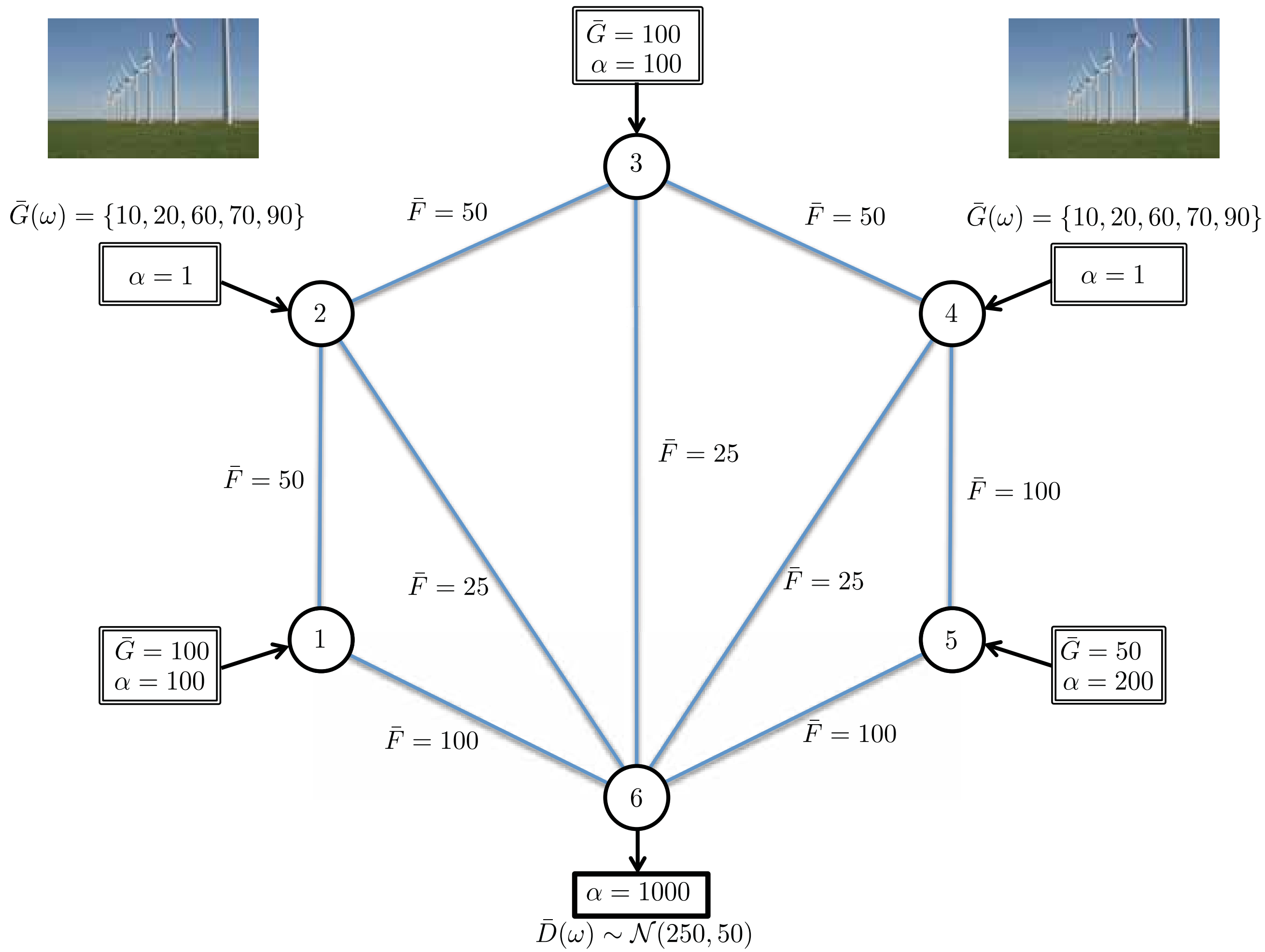
- Deterministic Clearing Induces Price Distortions That Bias Incentives & Revenue Adequacy
- New Stochastic Clearing Formulation Guarantees Fairness & Revenue Adequacy

Illustrative Study : Wind Uncertainty



	Social Surplus φ	Per Node Premia $\mathcal{M}_n^{\pi}$	Day-Ahead Prices π_n
Deterministic	-217529	{9,9,9,-205,-208,-273}	{100,100,100,100,100,100}
Stochastic	-217628	{0.001,0.001,0.001,0,0,0}	{96,96,96,307,310,374}
Stochastic-WS	-218266	-	-

Illustrative Study : Transmission Contingencies

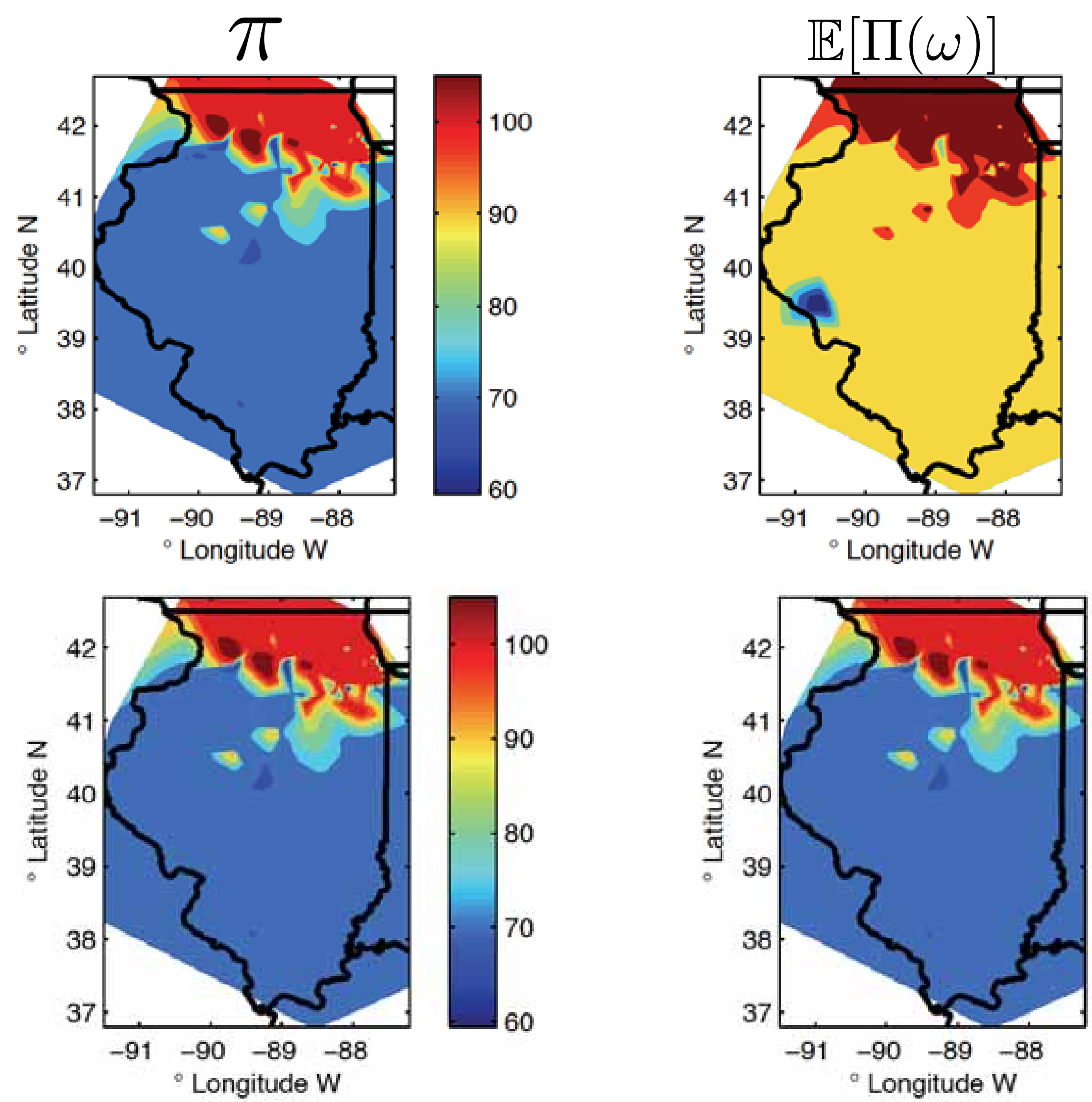


	Payments $\mathbb{E}[P_i^g(\omega)]$	$\mathbb{E}[C_i^g(\omega)]$	ISO Revenue $\mathcal{M}^{ISO}$
Deterministic	{5803,4723,5100,-919,9627}	{5231,50,3876,47,3387}	40570
Stochastic	{5107,3955,3683,7371,9623}	{5107,51,3683,46,3383}	-118103
Stochastic-WS	{4951,3888,3422,7170,9479}	{4951,47,3422,45,3079}	-118283

Computational Study : Illinois

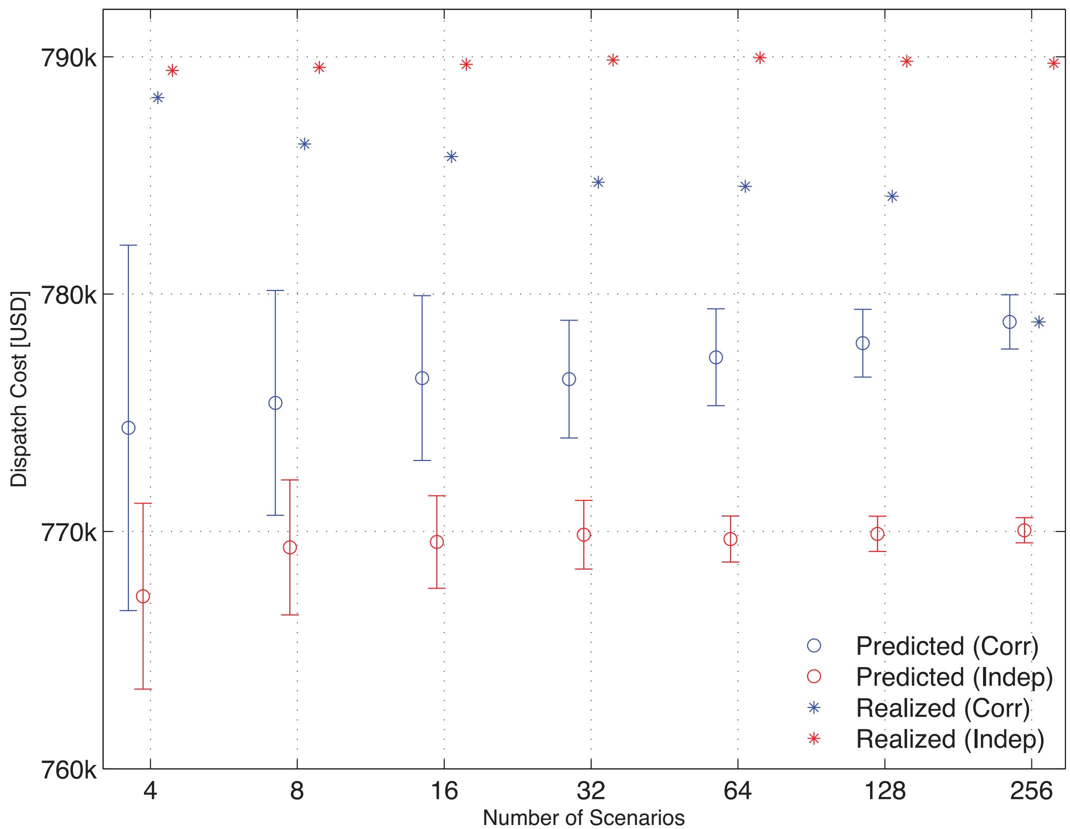
Deterministic

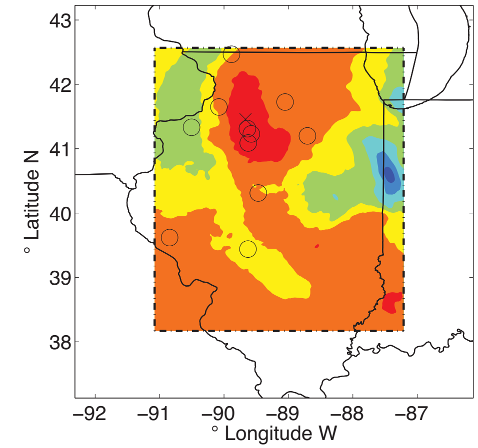
Stochastic



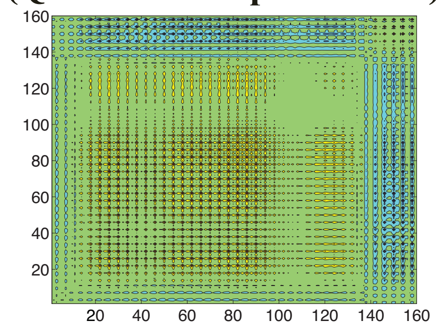
Computational Study : Uncertainty Characterization Illinois

- How To Characterize Uncertainty? Central vs. Distributed?
- What if ISO Neglects Long-Range Correlations?

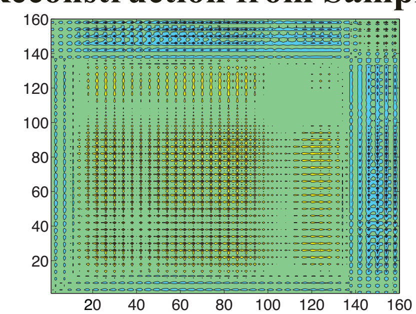


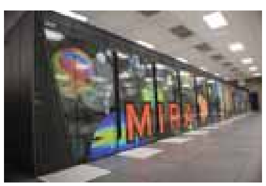


True Covariance
(Quasi-Geostrophic Model)



Rao-Blackwell-Ledoit-Wolf
(Reconstruction from Samples)

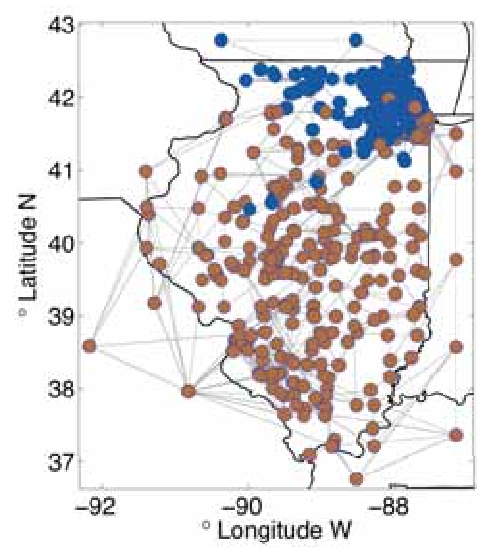




- Argonne's Mira BlueGene/Q : 16,384 Nodes
- Covariance Reconstruction From Numerical Weather Prediction Ensembles using RBLW Estimators

Nonconvex PIPS

- **Nonconvex Interior-Point Algorithm** : Advances in Theory (Office of Science Funding)
- **Exploits Heterogeneous Structures :** Stochastic, PDEs, Networks, Multi-Stage…
- **Enables Distributed Model Construction :** Privacy Concerns
- **Applications:** Gas-Electric, Security-Constrained & Multi-Period ACOPF, Predictive Control for AGC…



Stochastic, Network

$$\min \sum_{\omega\in\Omega} f_\omega(x, x_\omega)$$

$$\text{s.t. } g_\omega(x, x_\omega) \geq 0, \omega \in \Omega$$

$$\begin{bmatrix} K_0 & B_1^T & B_2^T & \dots & B_\Omega^T \\ B_1 & K_1 & & & \\ B_2 & & K_2 & & \\ \vdots & & & \ddots & \\ B_\Omega & & & & K_\Omega \end{bmatrix} \qquad K_\omega = \begin{bmatrix} H_\omega & J_\omega^T \\ J_\omega & \end{bmatrix}$$

Reduced Space (PDE)

$$\min f(u, x)$$

$$\text{s.t. } g(u, x) \geq 0$$

$$J_x \text{ invertible}$$

$$\begin{bmatrix} H_{xx} & H_{xu} & J_x^T \\ H_{ux} & H_{uu} & J_u^T \\ J_x & J_u & \end{bmatrix} \longrightarrow \left[\begin{array}{cc|c} J_x & & J_u \\ H_{xx} & J_x^T & H_{xu} \\ \hline H_{ux} & J_u^T & H_{uu} \end{array}\right]$$

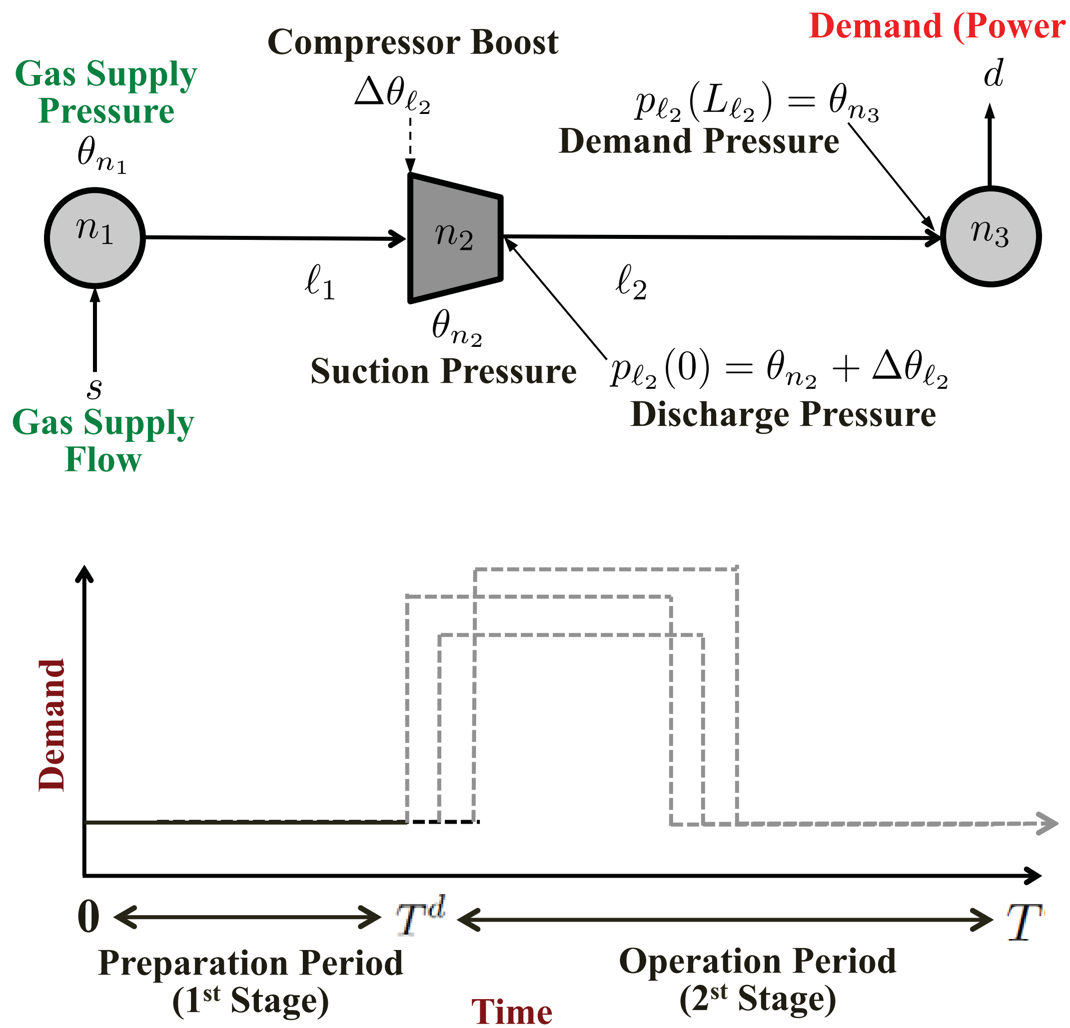
Stochastic PDE Network

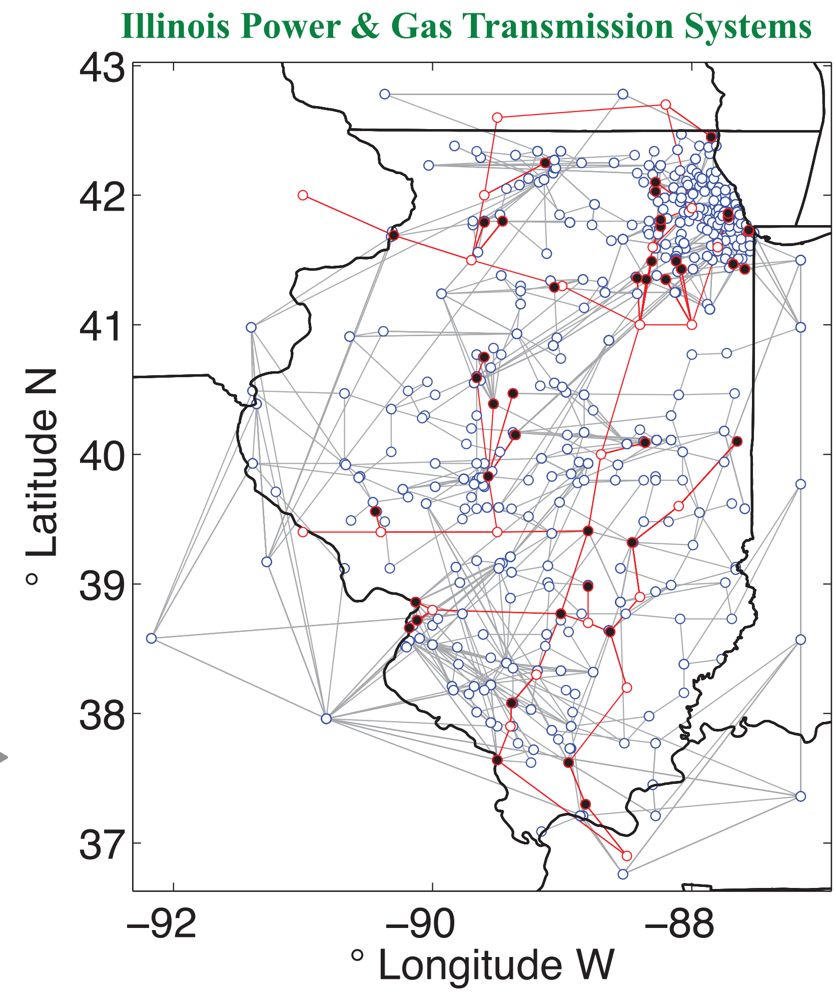
$$\min \sum_{\omega\in\Omega} f_\omega(x, u, x_\omega, u_\omega)$$

$$\text{s.t. } g_\omega(x, u, x_\omega, u_\omega) \geq 0, \omega \in \Omega$$

$$\begin{bmatrix} K_0 & B_1^T & B_2^T & \dots & B_\Omega^T \\ B_1 & K_1 & & & \\ B_2 & & K_2 & & \\ \vdots & & & \ddots & \\ B_\Omega & & & & K_\Omega \end{bmatrix} \quad K_\omega = \begin{bmatrix} \Phi_0 & C_1^\top & \cdots & C_n^\top \\ C_1 & \Phi_1 & & \\ \vdots & & \ddots & \\ C_n & & & \Phi_n \end{bmatrix} \quad \Phi_\omega = \left[\begin{array}{cc|c} J_x & & J_u \\ H_{xx} & J_x^T & H_{xu} \\ \hline H_{ux} & J_u^T & H_{uu} \end{array}\right]$$

Gas-Electric Co-Dispatch





Emerging Structures: Stochastic-PDE-Network

Transport Equations for link $\ell \in \mathcal{L} := \mathcal{L}_p \cup \mathcal{L}_a$

$$\frac{\partial p_\ell}{\partial t} + \frac{1}{A_\ell}\frac{p_\ell}{\rho_\ell}\frac{\partial f_\ell}{\partial x} = 0$$

$$\frac{1}{A_\ell}\frac{\partial f_\ell}{\partial t} + \frac{\partial p_\ell}{\partial x} + \frac{8\lambda_\ell}{\pi^2 D_\ell^5}\frac{f_\ell|f_\ell|}{\rho_\ell} = 0$$

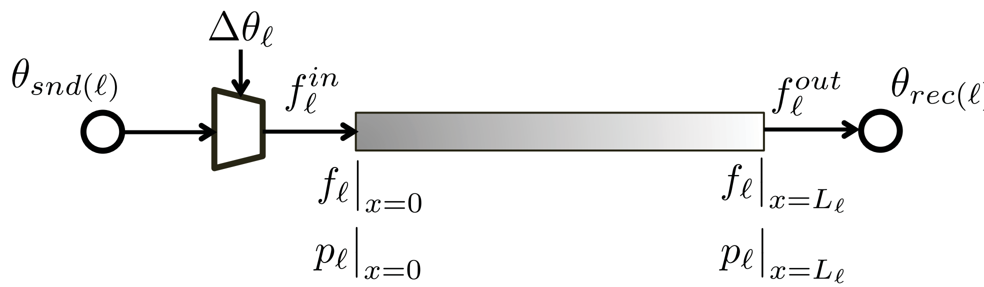
$$f_\ell\Big|_{x=0} = f_\ell^{in}$$

$$f_\ell\Big|_{x=L_\ell} = f_\ell^{out}$$

$$p_\ell\Big|_{x=L_\ell} = \theta_{rec(\ell)}$$

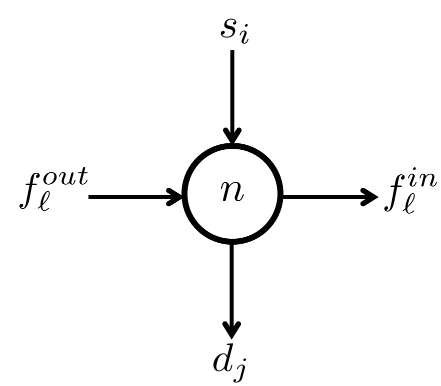
$$p_\ell\Big|_{x=0} = \theta_{snd(\ell)},\ \ell \in \mathcal{L}_p$$

$$p_\ell\Big|_{x=0} = \theta_{snd(\ell)} + \Delta\theta_\ell,\ \ell \in \mathcal{L}_a$$



Conservation at node $n \in \mathcal{N}$

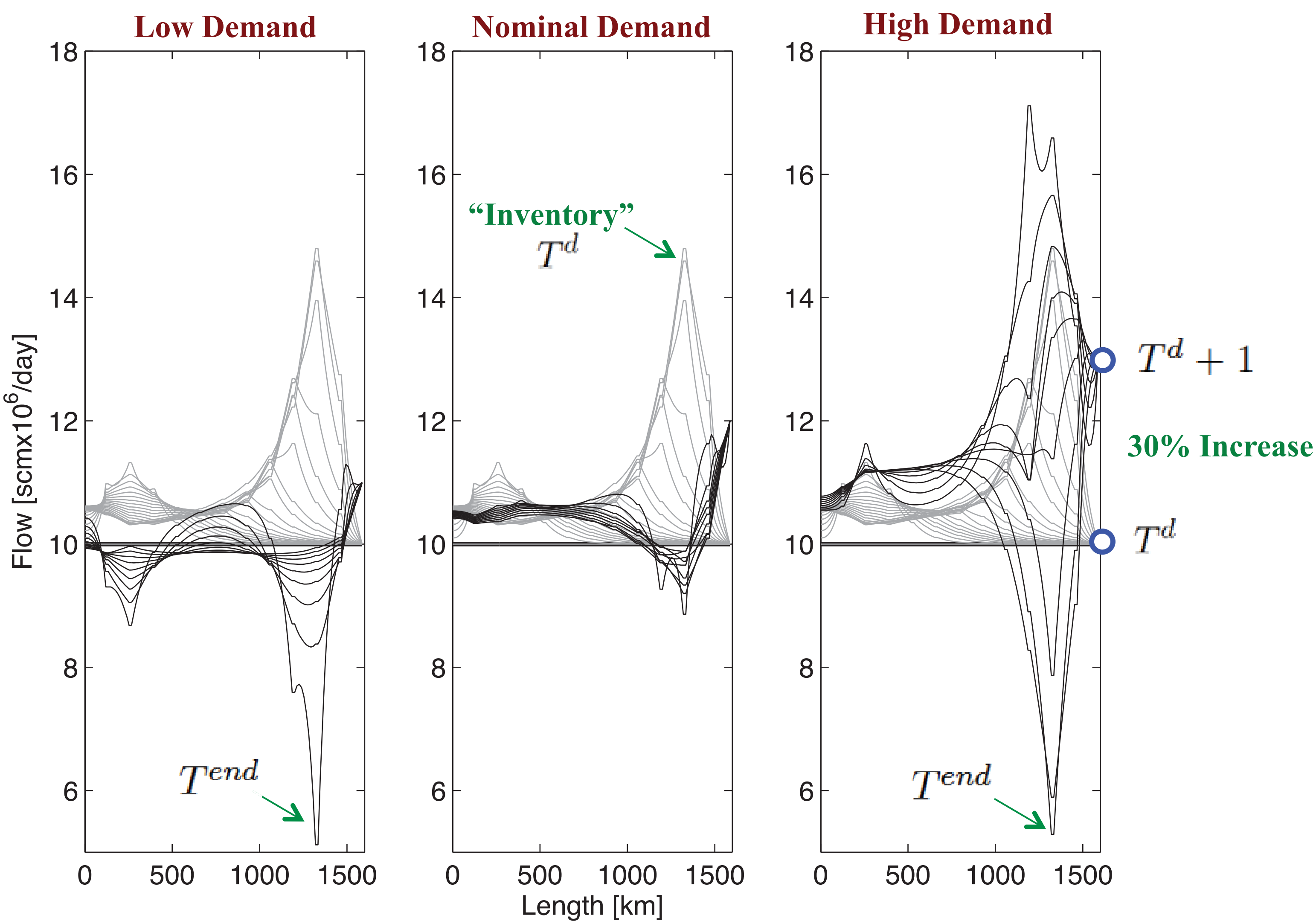
$$\sum_{\ell:rec(\ell)=n} f_\ell^{out} + \sum_{i:sup(i)=n} s_i - \sum_{\ell:snd(\ell)=n} f_\ell^{in} - \sum_{j:dem(j)=n} d_j = 0$$



Compression Power for link $\ell \in \mathcal{L}_A$

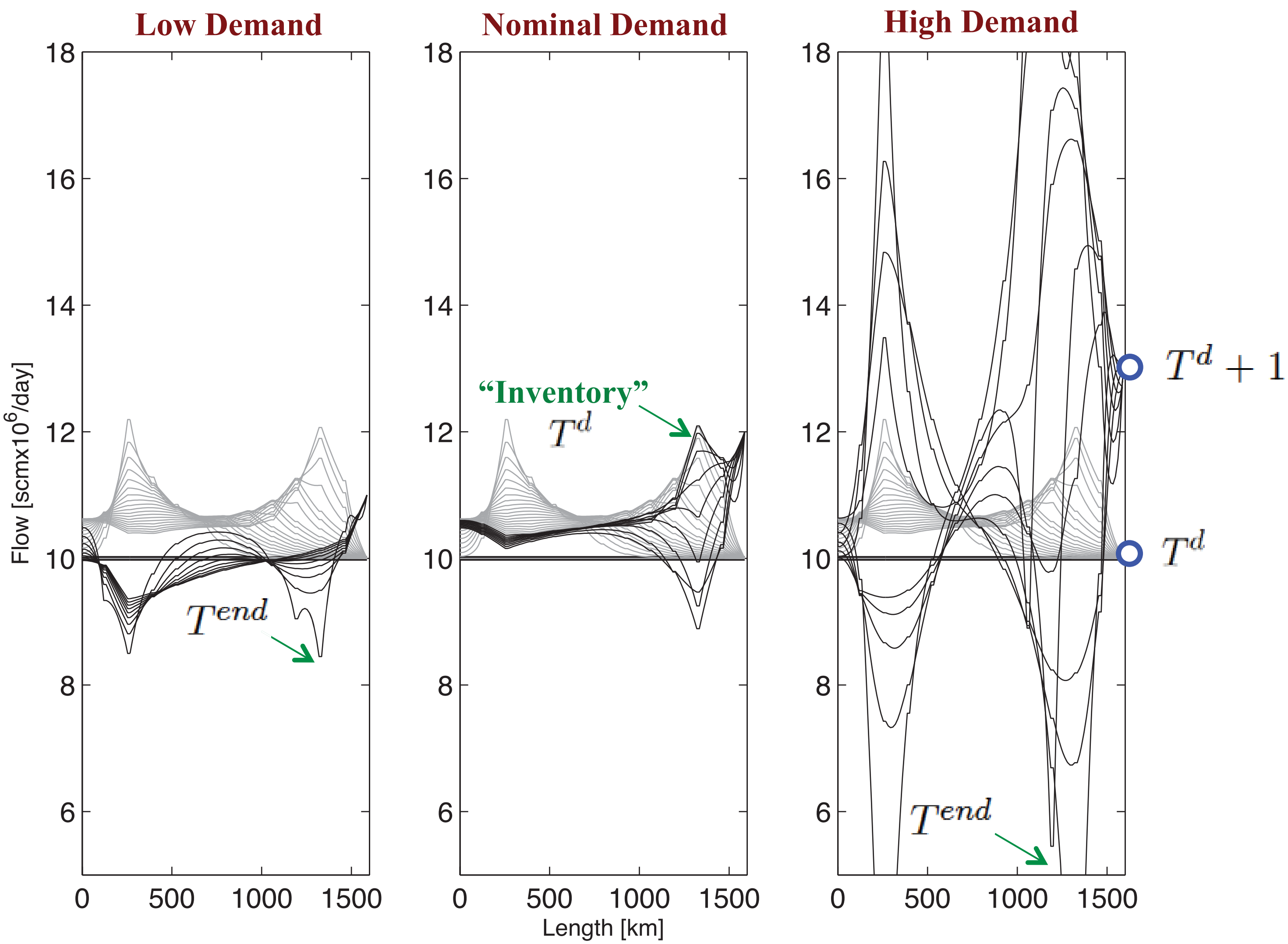
$$P_\ell = f_\ell^{in} c_p T \left(\left(\frac{\theta_{snd(\ell)} + \Delta\theta_\ell}{\theta_{snd(\ell)}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right)$$

Stochastic Gas Line-Pack Dispatch

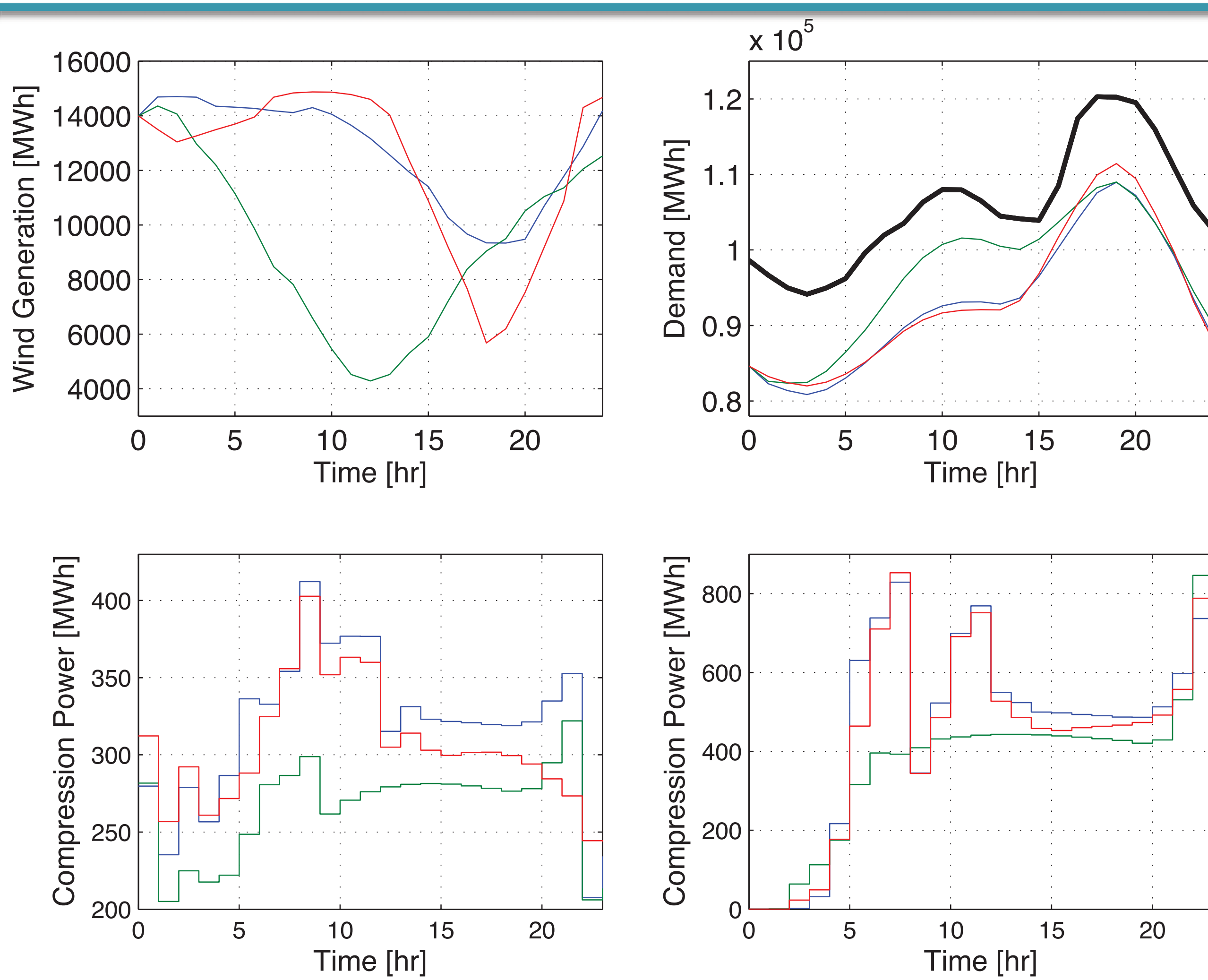


Finding: **Line-Pack Withstands Abrupt Changes of Power Plant Demand (30% in an Hour)**

Deterministic Gas Line-Pack Dispatch



Integrated Gas+Electric Dispatch



Finding: Sacrificing 0.2% of Electric Dispatch Cost Results in <u>50% Savings</u> in Gas Compression Power

Scalability Nonconvex PIPS : Gas Dispatch

Argonne's Fusion Cluster
320 Nodes
12.5 TB Memory

Exploiting Stochastic Structure

No.Sce	n	Obj	Iters	Time(hh:mm:ss)	MPI Proc.
96	1,930,752	1.39×10^2	42	01:13:16	8
96	1,930,752	1.39×10^2	42	00:38:18	16
96	1,930,752	1.39×10^2	42	00:24:55	24
96	1,930,752	1.39×10^2	42	00:19:23	32
96	1,930,752	1.39×10^2	42	00:12:42	48
96	1,930,752	1.39×10^2	42	00:06:48	96

$$\begin{bmatrix} K_0 & B_1^T & B_2^T & \dots & B_\Omega^T \\ B_1 & K_1 & & & \\ B_2 & & K_2 & & \\ \vdots & & & \ddots & \\ B_\Omega & & & & K_\Omega \end{bmatrix}$$

$$K_\omega = \begin{bmatrix} H_\omega & J_\omega^T \\ J_\omega & \end{bmatrix}$$

Exploiting Stochastic + Reduced-Space Structure

No.Sce	n	Obj	Iters	Time(hh:mm:ss)	MPI Proc.
96	1,930,752	1.39×10^2	42	00:29:54	8
96	1,930,752	1.39×10^2	42	00:14:45	16
96	1,930,752	1.39×10^2	42	00:10:00	24
96	1,930,752	1.39×10^2	42	00:07:36	32
96	1,930,752	1.39×10^2	42	00:05:14	48
96	1,930,752	1.39×10^2	42	00:02:54	96

$$\begin{bmatrix} K_0 & B_1^T & B_2^T & \dots & B_\Omega^T \\ B_1 & K_1 & & & \\ B_2 & & K_2 & & \\ \vdots & & & \ddots & \\ B_\Omega & & & & K_\Omega \end{bmatrix}$$

$$K_\omega = \left[\begin{array}{cc|c} J_x & & J_u \\ H_{xx} & J_x^T & H_{xu} \\ \hline H_{ux} & J_u^T & H_{uu} \end{array}\right]$$

Concluding Remarks

- **Collaborators:**
  - University of Chicago, Virginia Tech, Università della Svizzera Italiana (Switzerland)
  - Levitan & Associates

- **On-Going Work:**
  - Stochastic Global MINLP Framework
  - Gas+Electricity Markets

- **Publications:**

- C. G. Petra, O. Schenk, M. Anitescu. **Real-time Stochastic Optimization of Complex Energy Systems on High Performance Computers**. Computing in Science & Engineering (CiSE), 2014.
- Zavala, V. M.; Anitescu, M. and Birge, J. **A Stochastic Electricity Market Clearing Formulation with Consistent Pricing Properties**. Operations Research, Under Review, 2014.
- C. G. Petra, V. M. Zavala, E. Nino, and M. Anitescu. **Economic Impacts of Wind Covariance Estimation on Power Grid Operations**, IEEE Transactions on Power Systems, Under Review 2014.
- Zavala, V. M. **Stochastic Optimal Control Model for Natural Gas Network Operations**. Computers & Chemical Engineering, 64(1), pp. 103-113, 2014.
- C. G. Petra, et.al. **An Augmented Incomplete Factorization Approach for Computing the Schur complement in Stochastic Optimization.** SIAM Journal on Scientific Computing, 2013.