CERTS Meeting

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Development of an Attribute Preserving Network Equivalent

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Overview

- Overall objective
  - To develop equivalent models that preserve desired properties of the full model
- Current focus
  - To create line limit preserving equivalents of interconnection level power systems
  - To assign limits to equivalent lines so that total transfer capability (TTC) in equivalent system matches that of original system as much as possible

Background

- For decades power system network models have been equivalenced using the approach originally presented by J.B. Ward in 1949 AIEE paper "Equivalent Circuits for Power-Flow Studies"
  - Paper's single reference is to 1939 book by Gabriel Kron, so this also known as Kron's reduction
- System buses are partitioned into a study system (s) to be retained and an equivalent system (e) to be eliminated; buses in study system that connect to the equivalent are known as boundary buses

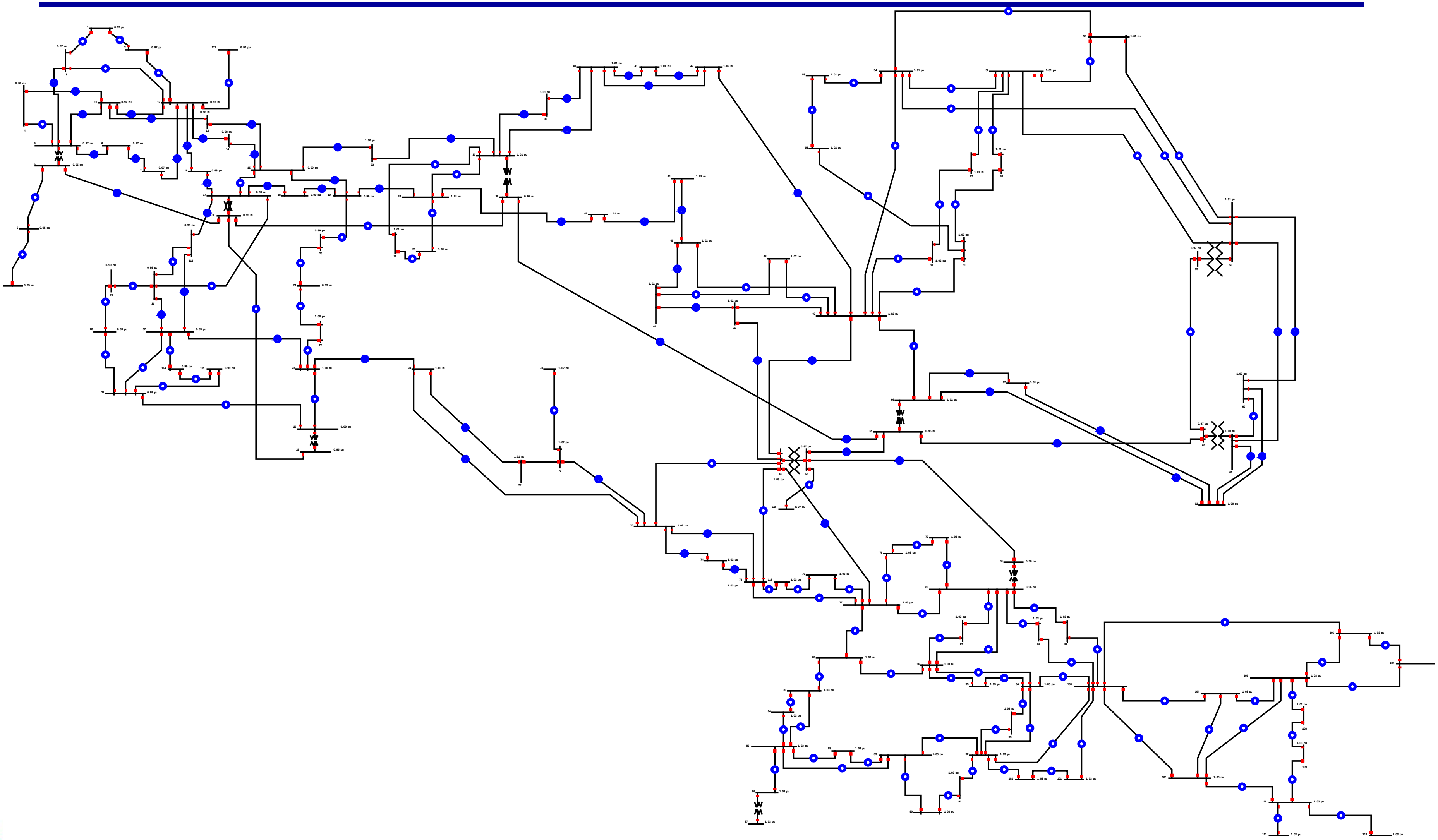
Ward Equivalents

- Equivalent is created by doing a partial factorization of the Ybus
  - Actual lines of eliminated buses are replaced by equivalent lines joining its first neighbor buses
  - Computationally efficient
  - Standard algorithms do not retain limits
- Our algorithm does this reduction, setting limits on the equivalent lines to match the total transfer capability (TTC) of the original network

Summary of Accomplishments

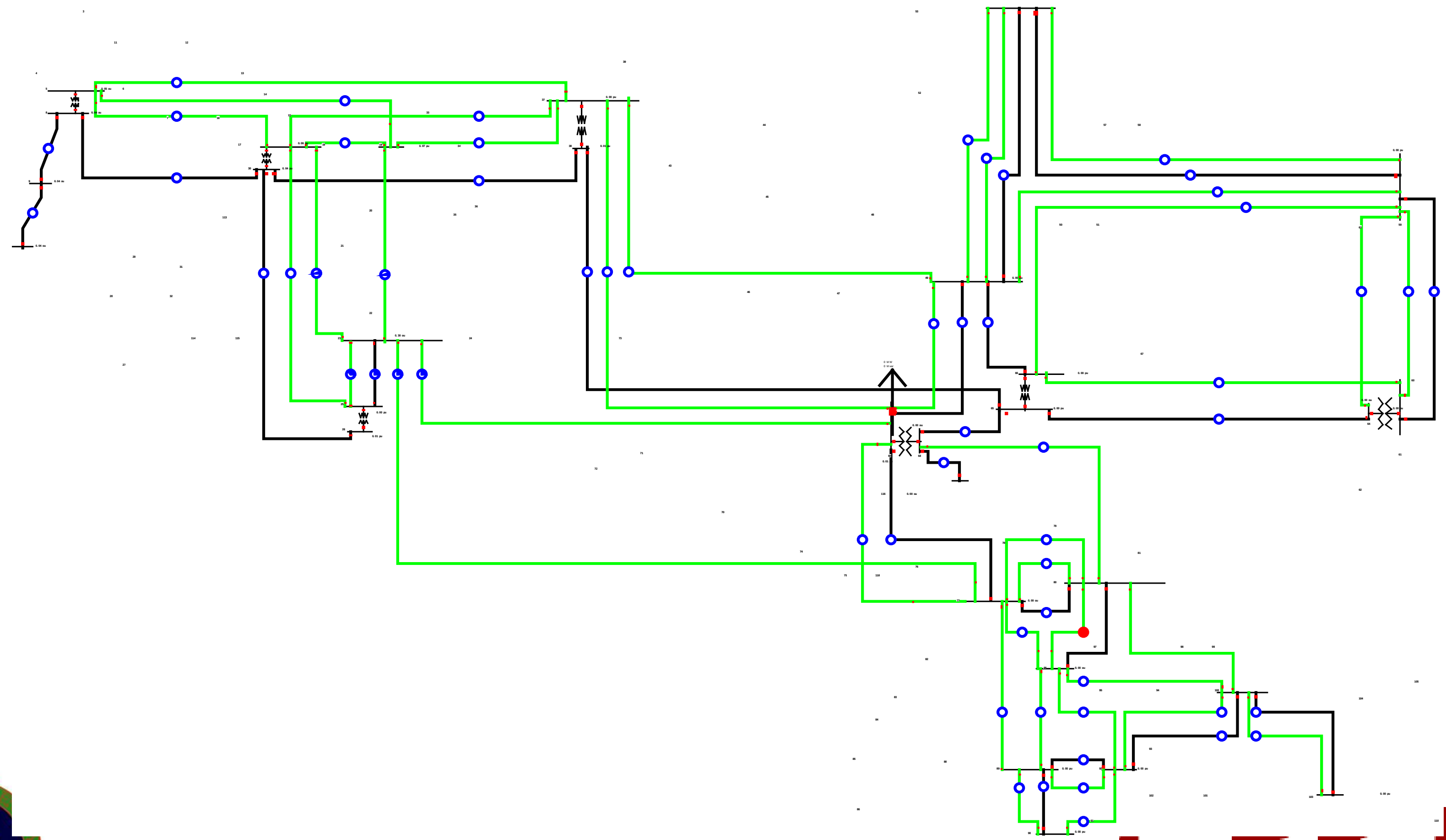
- Developed an improved algorithm for calculating the equivalent line limits
  - Algorithm is described in-depth in paper we submitted in July 2013 to IEEE Transactions on Power Systems
  - Prototype of algorithm presented *IEEE Power and Energy Conference at Illinois (PECI)*: W. Jang, S. Mohapatra, T. J. Overbye and H. Zhu, “Line Limit Preserving Power System Equivalent,” in *Proc. 2013 PECI, Feb.* 2013.
- Improved Algorithm has been applied to larger systems

IEEE 118-bus System



Equivalent 30-bus System

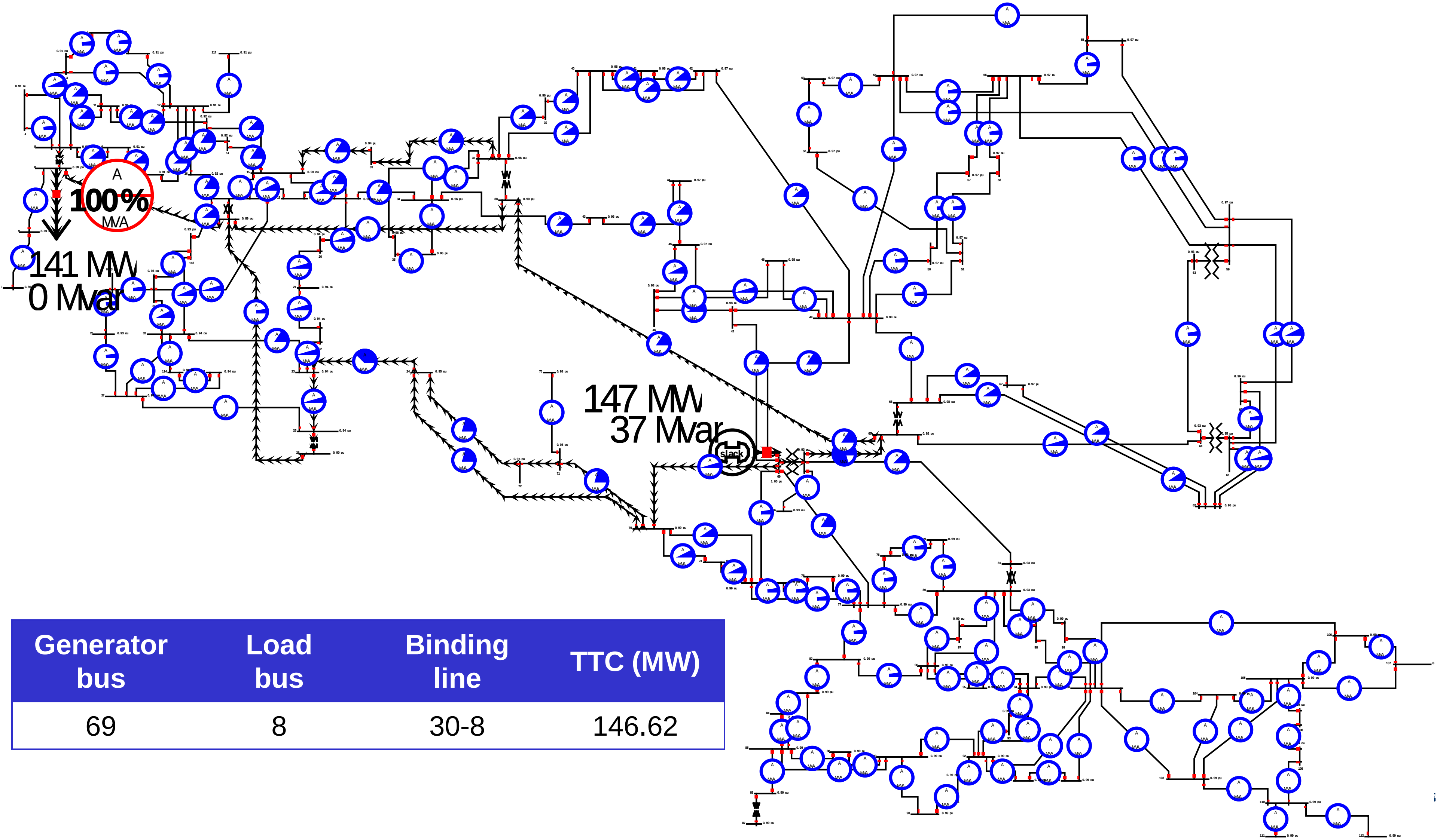
Black lines represent fully retained lines between buses from the original case. Green lines correspond to equivalent lines, now with limits



Criteria for Algorithm

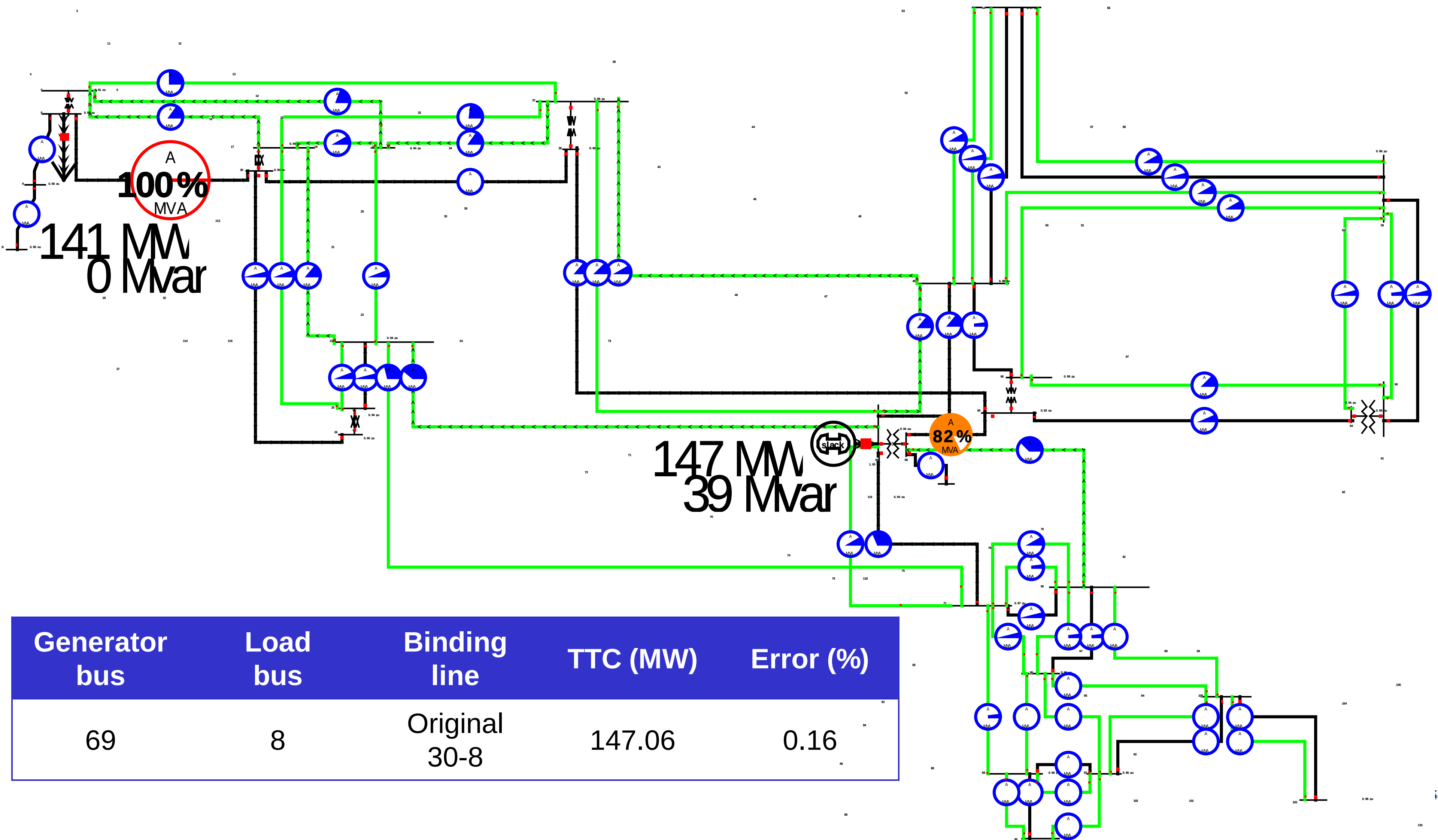
- Criteria for limit preserving equivalents
  - Total transfer capability (TTC) of the reduced system matches that of the full system
- Verification
  - Comparison of TTC between a pair of buses that are distant, at least more than one bus in between, in the equivalent system and that of the same buses in the original system

Verification - Ex 1



Generator bus	Load bus	Binding line	TTC (MW)
69	8	30-8	146.62

Verification - Example 1

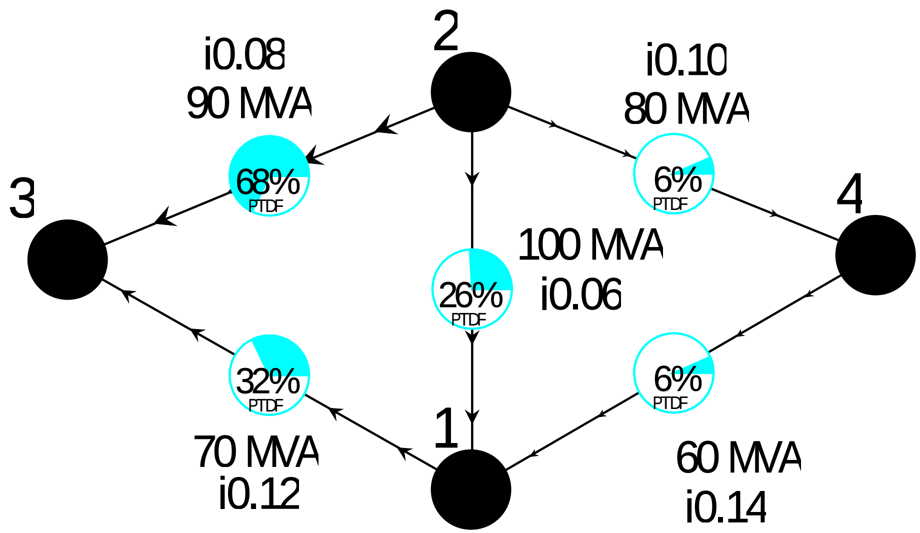


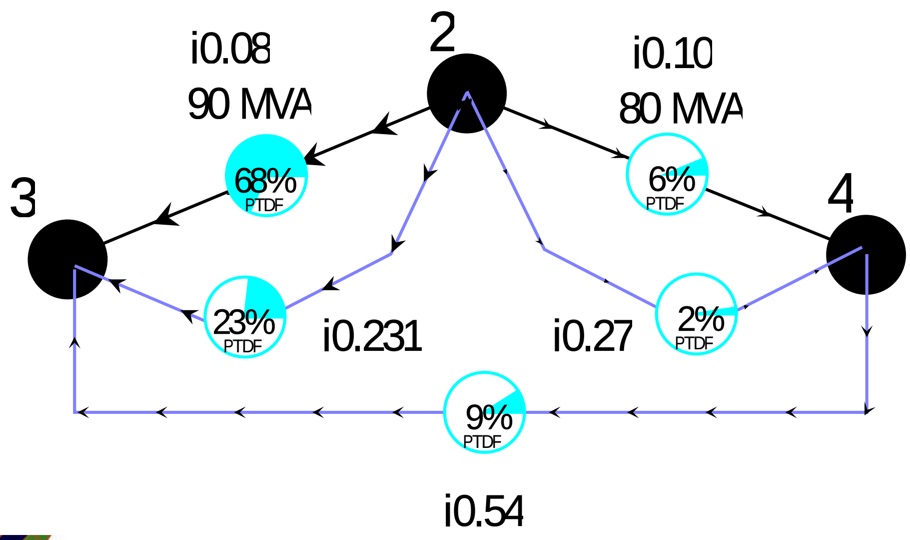
Generator bus	Load bus	Binding line	TTC (MW)	Error (%)
69	8	Original 30-8	147.06	0.16

Algorithm Overview

- Sequentially for each bus being equivalenced
  1. Calculate the PTDFs between the first neighbor buses
  2. Using these PTDFs, determine the TTC between the first neighbor buses, just considering the limits on the lines that are being removed
     - Limits on the other lines do not need to be considered since these lines are being retained (at least until the next bus is considered).
  3. Select limits for the new equivalent lines so that the TTCs of the reduced system match that of the original system.
  4. Combine limits/impedances on parallel equivalent lines

Four Bus Example (Bus 2-3 PTDFs Shown)





With removing bus 1, three equivalent lines will be added between the other three buses. The original TTCs are

2-3: 216.7 MW (1-3 binding)

2-4: 171.7 MW (1-4 binding)

3-4: 144.9 MW (1-4 binding)

For 2-3 direction for new equivalent line limits we require

1) Lim23 >= 216.7*0.234= 50.7MW
2) Lim24 >= 216.7*0.024= 5.2MW
3) Lim34 >= 216.7*0.088=19.1MW

Similar constraints for the other directions

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General Solution Procedure

- Matrix representation of the constraints: each entry shows the PTDF x MPT: limit must be largest entry in each row, and each column needs a binding limit. Solution is shown below, but sometimes no solution exists

Directions

	2-3	2-4	3-4
Eqv Line 2-3	50.7 MW	4.8 MW	29.8 MW
Eqv Line 2-4	5.2 MW	41.4 MW	31.4 MW
Eqv Line 3-4	19.1 MW	18.7 MW	28.5 MW

No Solution Example

- No solution example was created by reducing one of the limits in the previous example
- Previous work determined an overestimate and an underestimate of the solution; but this gave a limit range, which could grow quite large during the sequential solution

Directions

	2-3	2-4	3-4
Eqv Line 2-3	50.7 MW	4.8 MW	29.8 MW
Eqv Line 2-4	5.2 MW	41.4 MW	31.4 MW
Eqv Line 3-4	19.1 MW	18.7 MW	28.5 MW

Overestimate Limits: Easy

- Results can be overestimated by just satisfying the inequality constraints (i.e., pick largest entry in each row). But this leaves directions that are not binding. The limits err on being too high, allowing for larger power transfers.

Directions (Modified System Data)

	2-3	**2-4**	**3-4**
Eqv Line 2-3	**50.7 MW**	1.6 MW	9.9 MW
Eqv Line 2-4	5.2 MW	**13.8 MW**	10.5 MW
Eqv Line 3-4	**19.1 MW**	6.2 MW	9.5 MW

Underestimate Limits: Requires a Limit Violation Cost Function

- Insure all the equality constraints are satisfied, which keeps the flow in every direction to be no more than its original TTC. But because some of the inequality constraints would be in violation, these limits under-estimate the TTC in at least some directions
- Solution is motivated by defining a “limit violation cost” for each matrix entry, which is the sum of violations for all entries in the row.

General Solution Procedure: Limit Violation Cost

Directions (Modified System Data)

	2-3	2-4	3-4
Eqv Line 2-3	**50.7 MW**	1.6 MW	9.9 MW
Eqv Line 2-4	5.2 MW	**13.8 MW**	10.5 MW
Eqv Line 3-4	**19.1 MW**	6.2 MW	9.5 MW

Directions: Limit Violation Costs

	2-3	2-4	3-4
Eqv Line 2-3	0	57.4	40.8
Eqv Line 2-4	13.9	0	3.3
Eqv Line 3-4	0	16.2	9.6

Example: For the first row, the 2-3 entry is 0 because it involves no limit violations; the 2-4 entry is 57.4 = (50.7 – 1.6) + (9.8 – 1.6), while 3-4 is 40.8 = (50.7 – 9.9)

Hungarian Algorithm

- Problem was solved using the Hungarian algorithm (assignment problem), which picks one entry from each row and one from each column

Directions: Limit Violation Costs

	2-3	**2-4**	**3-4**
Eqv Line 2-3	**0**	57.4	40.8
Eqv Line 2-4	13.9	**0**	3.3
Eqv Line 3-4	0	16.2	**9.6**

For the second approach the new limits would 50.7 MW for the line between 2-3, 13.8 MW for 2-4 and 9.5 MW for line 3-4. This is compared with 50.7, 13.8 and 19.1 for the first approach.

Need for Improvement

- For most buses a solution exists so there is no need for a new algorithm for these buses
- But for the buses without a solution, just bracketing the limits could eventually result in wide limit ranges.
- Hungarian algorithm is discrete: the selected limits are one of the matrix entries
- Needed determination of the “optimal” limit for the line

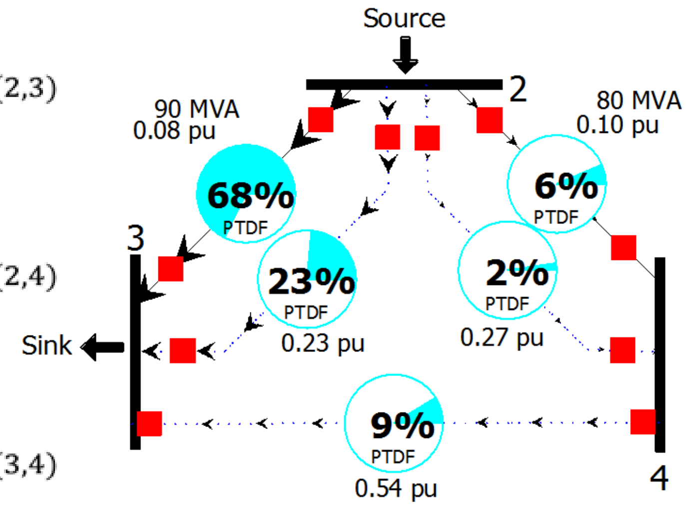
Modified Problem Formulation

- For equivalent system

$$min\left\{\frac{eq.limit_{2,3}}{eq.PTDF_{2,3}^{(2,3)}},\frac{eq.limit_{2,4}}{eq.PTDF_{2,4}^{(2,3)}},\frac{eq.limit_{3,4}}{eq.PTDF_{3,4}^{(2,3)}}\right\}=TTC^{(2,3)}$$

$$min\left\{\frac{eq.limit_{2,3}}{eq.PTDF_{2,3}^{(2,4)}},\frac{eq.limit_{2,4}}{eq.PTDF_{2,4}^{(2,4)}},\frac{eq.limit_{3,4}}{eq.PTDF_{3,4}^{(2,4)}}\right\}=TTC^{(2,4)}$$

$$min\left\{\frac{eq.limit_{2,3}}{eq.PTDF_{2,3}^{(3,4)}},\frac{eq.limit_{2,4}}{eq.PTDF_{2,4}^{(3,4)}},\frac{eq.limit_{3,4}}{eq.PTDF_{3,4}^{(3,4)}}\right\}=TTC^{(3,4)}$$



Equivalent system with PTDF 2-3

- For both sides, each TTC is divided and 1 is subtracted for normalized TTC mismatch

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Modified Problem Formulation

- $$\underset{\tilde{F}_{\tilde{l}}, m^{(i,j)}}{minimize} \left(m^{(2,3)}\right)^2 + \left(m^{(2,4)}\right)^2 + \left(m^{(3,4)}\right)^2$$

$$\text{s.t. } m^{(2,3)} = min\left\{\frac{\tilde{F}_{2,3}}{\psi_{2,3}^{(2,3)}}, \frac{\tilde{F}_{2,4}}{\psi_{2,4}^{(2,3)}}, \frac{\tilde{F}_{3,4}}{\psi_{3,4}^{(2,3)}}\right\} - 1$$

$$m^{(2,4)} = min\left\{\frac{\tilde{F}_{2,3}}{\psi_{2,3}^{(2,4)}}, \frac{\tilde{F}_{2,4)}}{\psi_{2,4}^{(2,4)}}, \frac{\tilde{F}_{3,4}}{\psi_{3,4}^{(2,4)}}\right\} - 1$$

$$m^{(3,4)} = min\left\{\frac{\tilde{F}_{2,3}}{\psi_{2,3}^{(3,4)}}, \frac{\tilde{F}_{2,4}}{\psi_{2,4}^{(3,4)}}, \frac{\tilde{F}_{3,4}}{\psi_{3,4}^{(3,4)}}\right\} - 1$$

  - $\tilde{F}_{\tilde{l}}: limit\ of\ eq.line\ \tilde{l}$
  - $\psi_{\tilde{l}}^{(i,j)}: eq.PTDF_{\tilde{l}}^{(i,j)} \times TTC^{(i,j)}$

  – Under estimate: all $m \leq 0$

  – Over estimate: all $m \geq 0$

Solution with Quadratic Programming (QP)

- For each line, one of inequalities is picked as an equality
  - Selected equality constraints are substituted into 6 inequalities
  - 6 inequalities are now specified with only m
  - If (1,1,1) is the combination of equality constraints

$$\underset{\tilde{F}_{\tilde{l}},m^{(i,j)}}{minimize}\left(m^{(2,3)}\right)^2+\left(m^{(2,4)}\right)^2+\left(m^{(3,4)}\right)^2$$

s.t. $\tilde{F}_{2,3}=(m^{(2,3)}+1)\ \psi_{2,3}^{(2,3)},\ \ \tilde{F}_{2,4}=(m^{(2,3)}+1)\ \psi_{2,4}^{(2,3)},\ \ \tilde{F}_{3,4}=(m^{(2,3)}+1)\ \psi_{3,4}^{(2,3)}$

$\tilde{F}_{2,3}\geq(m^{(2,4)}+1)\ \psi_{2,3}^{(2,4)},\ \ \tilde{F}_{2,4}\geq(m^{(2,3)}+1)\ \psi_{2,4}^{(2,4)},\ \ \tilde{F}_{3,4}\geq(m^{(2,3)}+1)\ \psi_{3,4}^{(2,4)}$

$\tilde{F}_{2,3}\geq(m^{(3,4)}+1)\ \psi_{2,3}^{(3,4)},\ \ \tilde{F}_{2,4}\geq(m^{(2,3)}+1)\ \psi_{2,4}^{(3,4)},\ \ \tilde{F}_{3,4}\geq(m^{(2,3)}+1)\ \psi_{3,4}^{(3,4)}$

Solution of QP

- Exclude the set of limits that violates inequality conditions for m
- Choose the combination with the minimum value of objective function and corresponding line limits
- That is, we find the point with the minimum distance from the origin, where all mismatches are zero, in the feasible region for each estimate

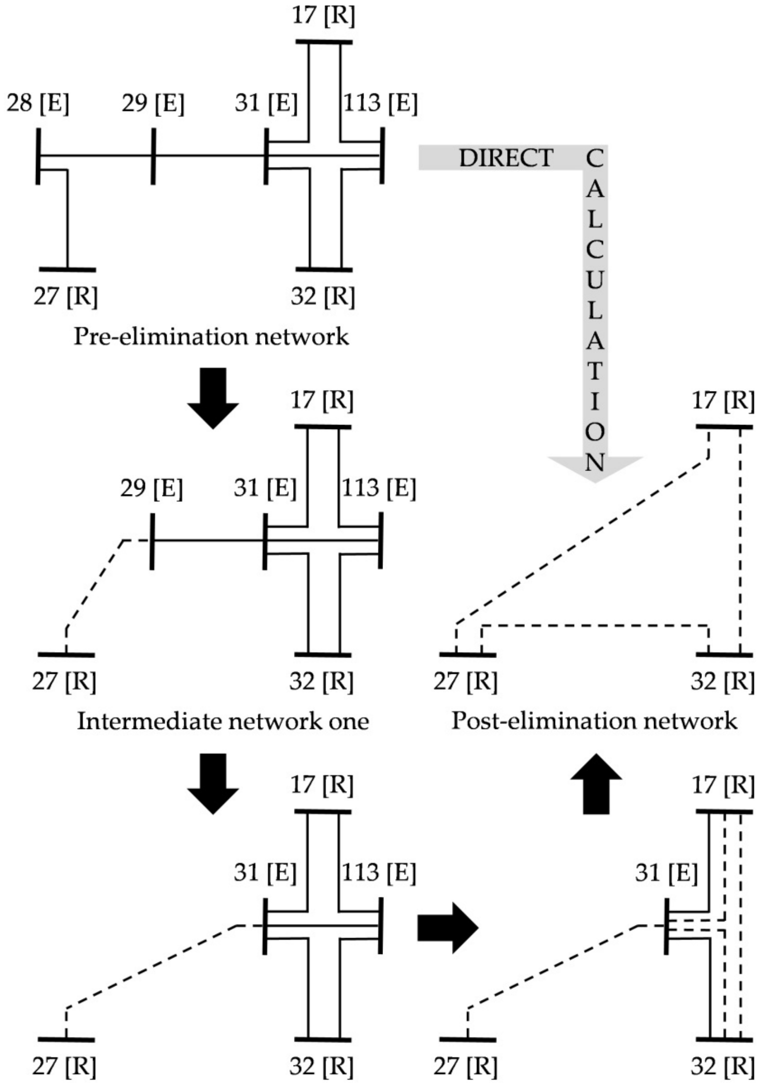
Results of 4-bus System

Results Comparison: Original Approach vs. QP

	Transaction	Original Approach		QP	
		Eq. line limit (MW)	Norm. TTC mismatch (%)	Eq. line limit (MW)	Norm. TTC mismatch (%)
Over Estimate	(2, 3)	50.8	0.0	50.8	0.0
	(2, 4)	13.8	0.0	13.8	0.0
	(3, 4)	19.2	31.5	19.2	31.5
Best Estimate	(2, 3)	N/A	N/A	50.8	0.0
	(2, 4)	N/A	N/A	11.7	-15.2
	(3, 4)	N/A	N/A	19.2	11.5
Under estimate	(2, 3)	50.8	-50.4	50.8	0.0
	(2, 4)	13.8	0.0	10.5	-24.0
	(3, 4)	9.5	0.0	19.2	0.0

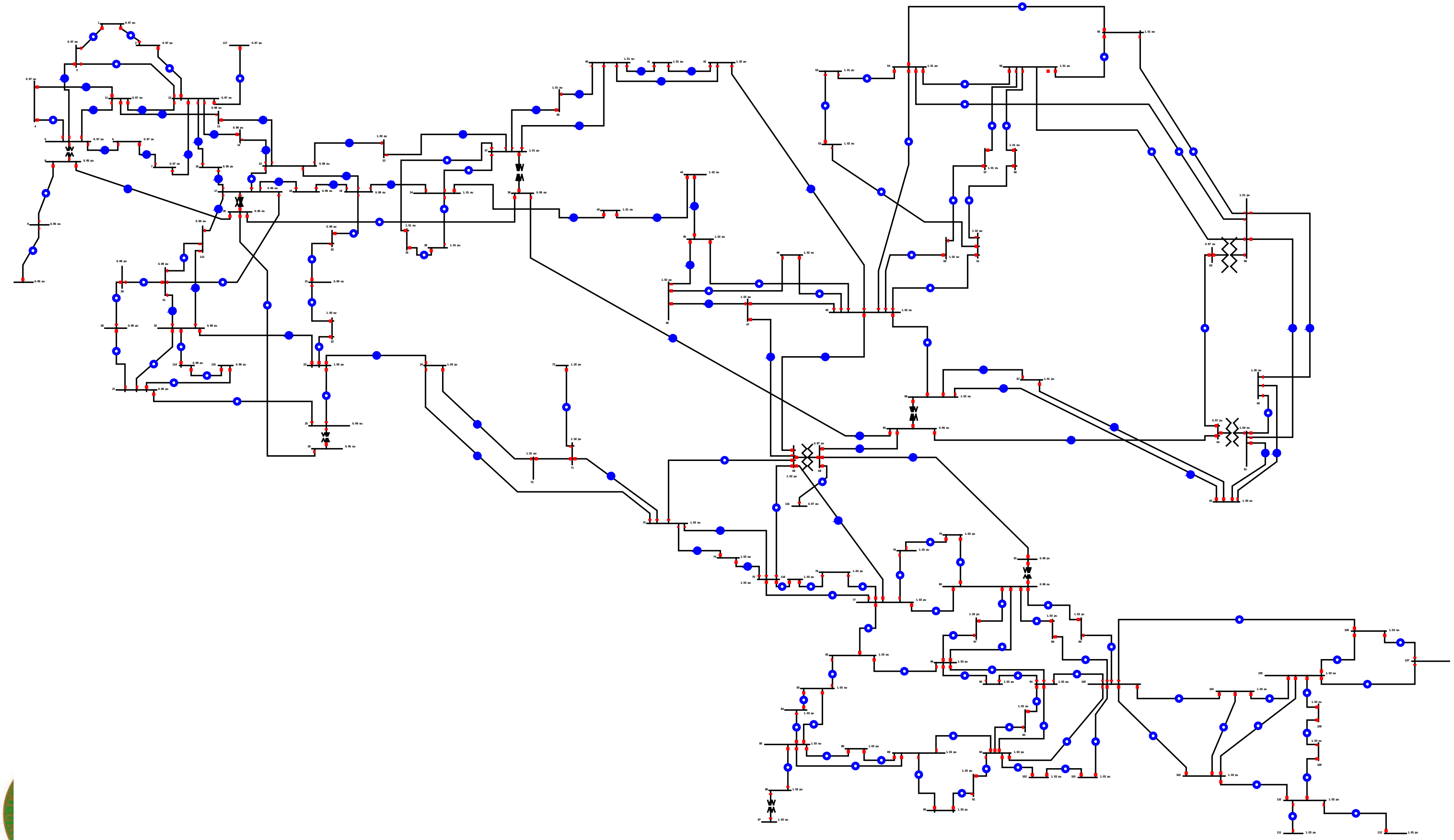
Direct Method for Group Elimination

- Blocks of buses can be directly equivalenced, with limits calculated, if there are few boundary buses
- Advantages
  - Faster simulation
  - Remove elimination order dependency
  - More accurate than sequential method
- Disadvantage
  - Does not work well if there are lots of boundary buses



Group (bus 28,29,113, and 31) elimination from IEEE 118-bus system

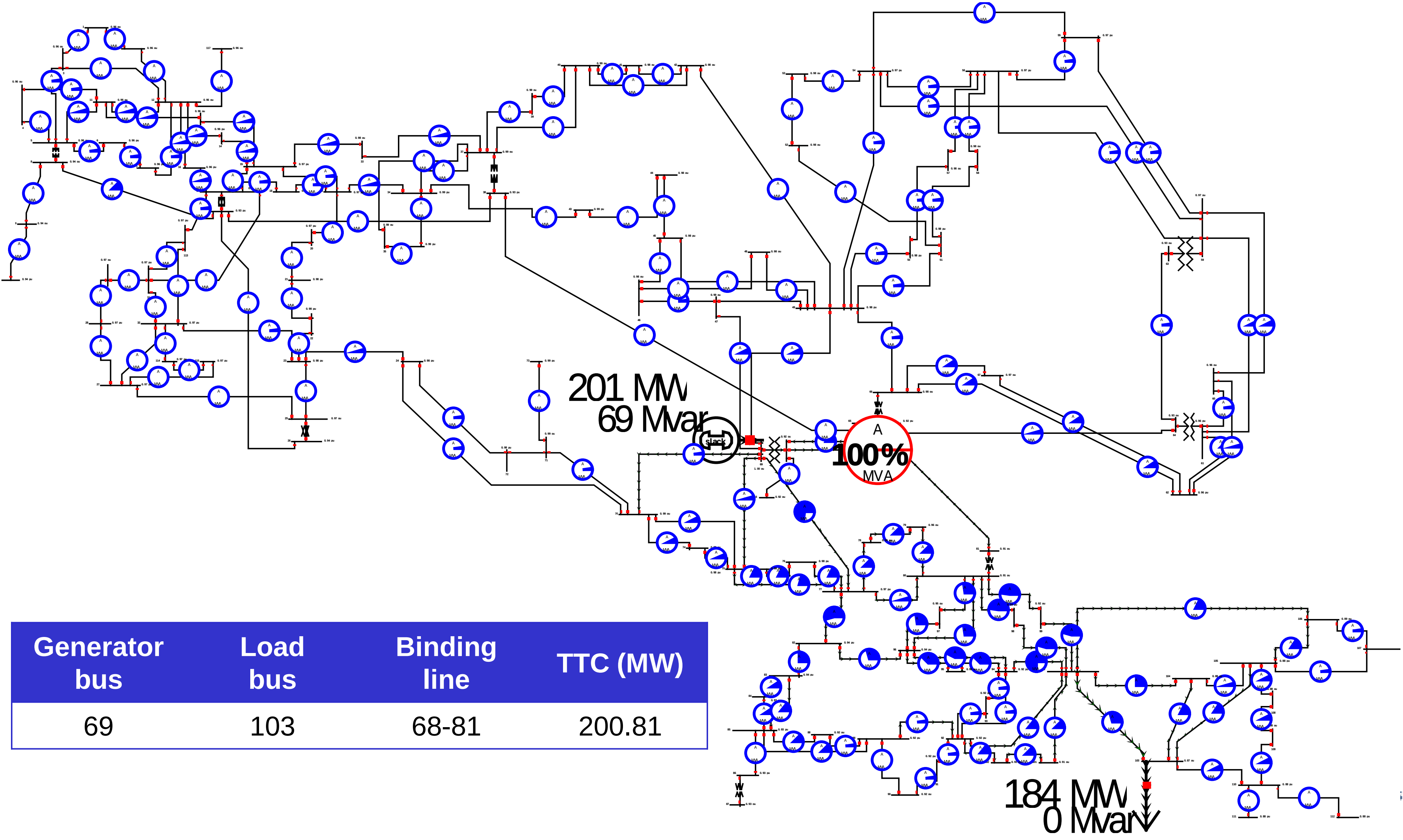
IEEE 118-bus System Results



Criteria for Algorithm (Again)

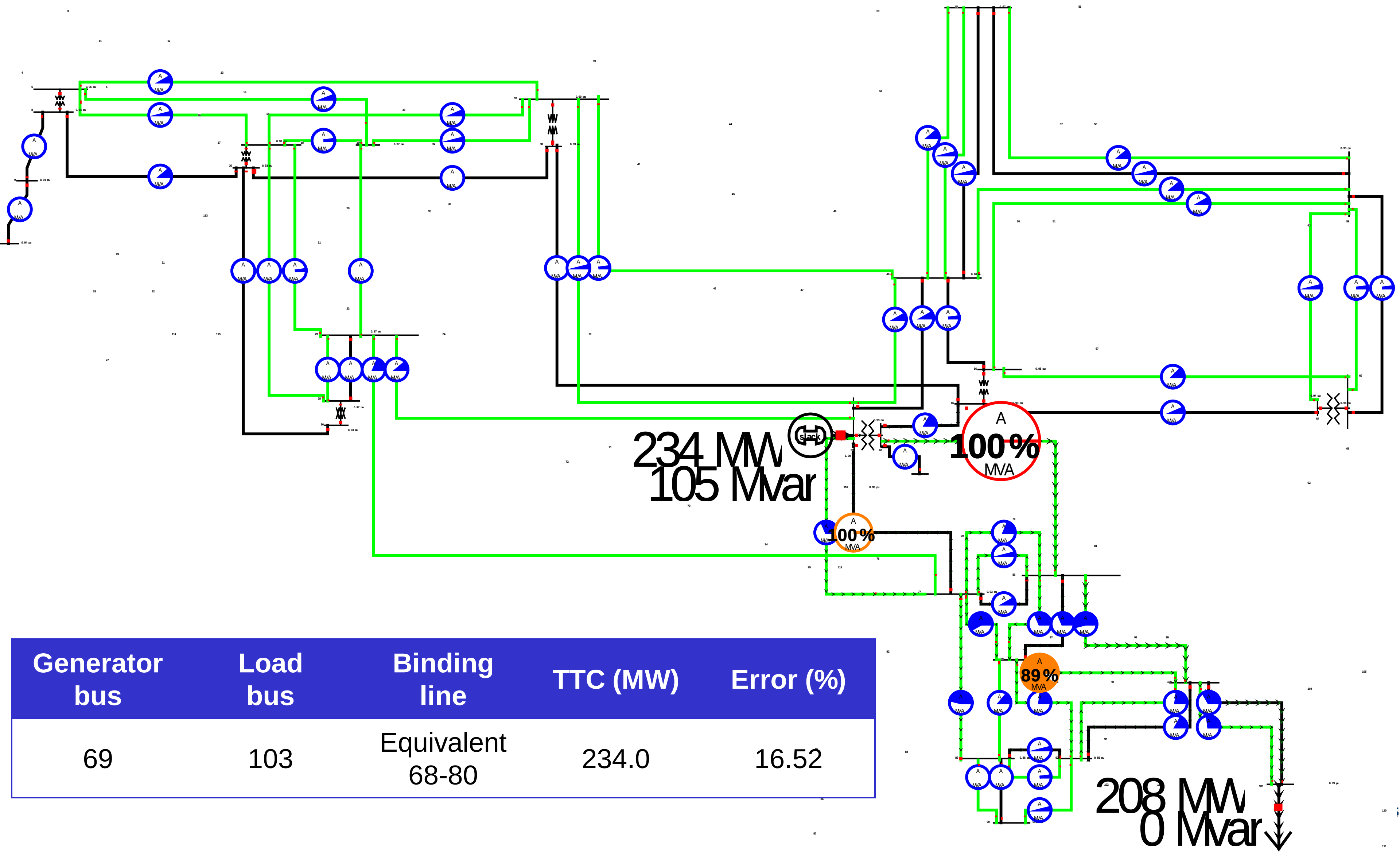
- Criteria for limit preserving equivalents
  - Total transfer capability (TTC) of the reduced system matches that of the full system
- Verification
  - Comparison of TTC between a pair of buses that are distant, at least more than one bus in between, in the equivalent system and that of the same buses in the original system

Verification - Example 2



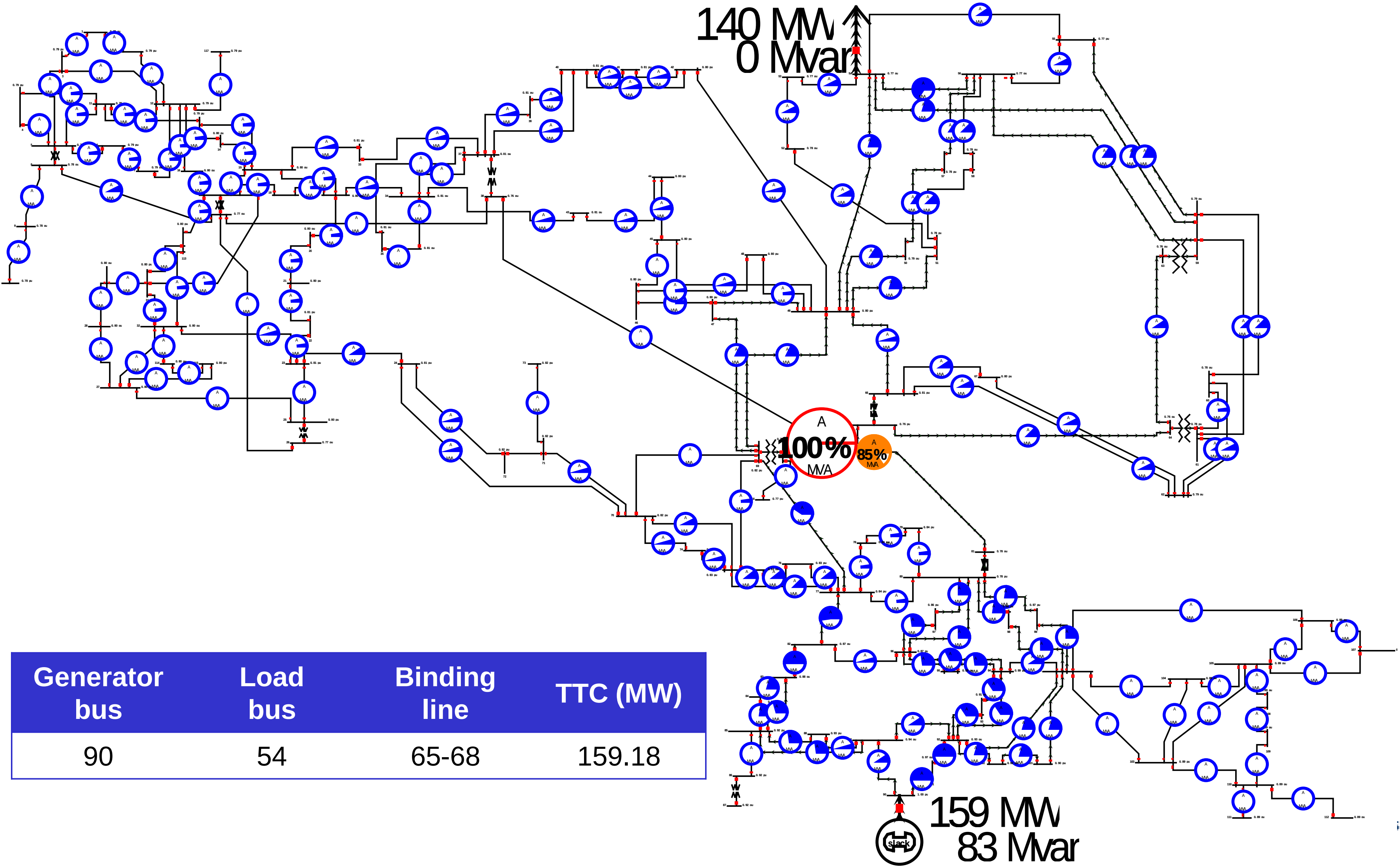
Generator bus	Load bus	Binding line	TTC (MW)
69	103	68-81	200.81

Verification - Ex 2



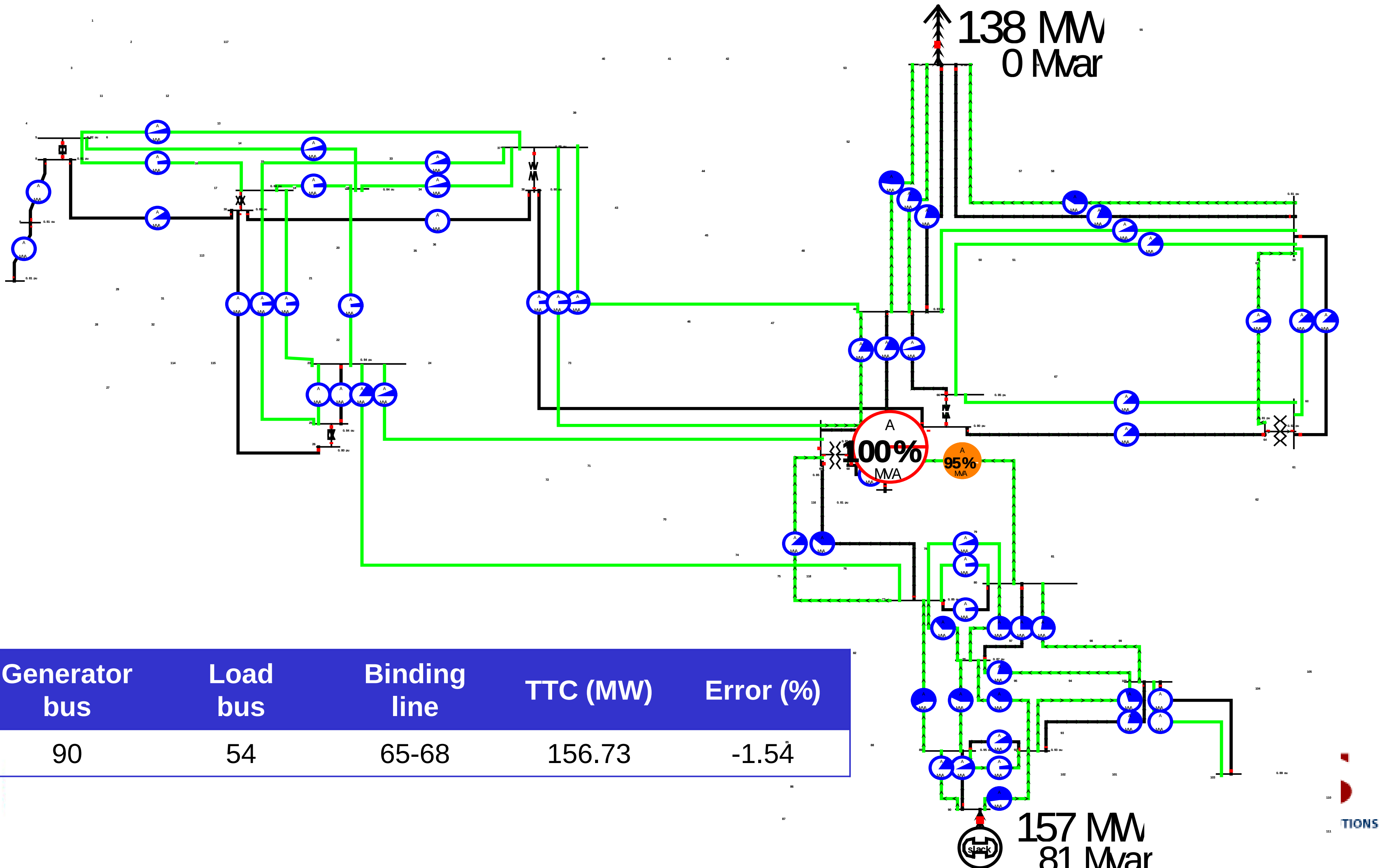
Generator bus	Load bus	Binding line	TTC (MW)	Error (%)
69	103	Equivalent 68-80	234.0	16.52

Verification - Example 3



Generator bus	Load bus	Binding line	TTC (MW)
90	54	65-68	159.18

Verification - Ex 3



Generator bus	Load bus	Binding line	TTC (MW)	Error (%)
90	54	65-68	156.73	-1.54

Summary of Simulation

- Simulation condition
  - 88 buses are selected to be eliminated and are divided into groups
    - Ones with through flow less than 160 MW
  - Each group consists of maximal adjacent buses
    - Elimination order in each group is based on Tinney scheme 2
  - 20 mutually independent groups
    - Elimination of a bus from one group has no effect on that from another group
    - Parallel calculation is possible

Summary of Simulation

- Simulation results
  - Exact solution case
    - 16 groups
    - 100% TTC match between the original and the equivalent
  - Non-exact solution case

Buses of each group	First neighbor buses of group	Rams norm. TTC mismatch for group (%)		
		Upper estimate	Best estimate	Lower estimate
34, 35, 36, 43, 44, 45, 46, 48	19, 37, 49, 69	0.38	0.27	0.38
50, 51, 52, 53, 55, 56, 57, 58	49, 54, 59	96.85	30.37	32.97
60, 62, 67	59, 61, 66	4.43	3.01	4.11
1, 2, 3, 4, 6, 7, 11, 12, 13, 14, 15, 16, 33, 117	5, 17, 19, 37	27.42	19.00	24.49

Direct Method for Sub-group Elimination

Results comparison: sequential vs. direct

	Transaction	MH		QP	
		Sequential	Direct	Sequential	Direct
		Eq. Line limit (MW)	Eq. Line limit (MW)	Eq. Line limit (MW)	Eq. Line limit (MW)
Over Estimate	(17,27)	63.5	44.7	63.5	44.7
	(17,32)	170.8	121.8	170.8	121.8
	(27,32)	54.6	53.2	54.6	53.2
	Rams norm. TTC mismatch of entire group	39.4 %	5.5 %	39.4 %	5.5 %
Best Estimate	(17,27)	N/A	N/A	56.7	42.6
	(17,32)	N/A	N/A	140.5	121.8
	(27,32)	N/A	N/A	54.6	50.7
	Rams norm. TTC mismatch of entire group	N/A	N/A	18.2 %	3.7 %
Under estimate	(17,27)	56.7	40.8	56.7	40.8
	(17,32)	124.3	121.8	124.3	121.8
	(27,32)	54.6	53.2	54.6	48.6
	Rams norm. TTC mismatch of entire group	7.6 %	5.0 %	7.6 %	5.0 %

Computational Aspects

- Assume an n bus system, in which m buses are being reduced. Let F_i be the number of first neighbor buses for bus i (a number that will vary during the simulation). Algorithm will be applied sequentially at m buses. For each step we must
  - Calculate $(F_i)^2/2$ PTDFs
  - With sparse vector methods each PTDF has computational order equivalent to the depth of the factorization path, close to ln(n)
- QP solution is quite costly, so improvements are needed here

Future Work

- Reduced computational time, perhaps through the use of heuristics for minimizing the number of directions
- Incorporation of bus injections from gen and load
  - TTC reduces to available transfer capability (ATC) to meet existing transmission commitments
  - Our key concern is to prevent operating point dependence
- Additional testing on larger systems

Questions?