GJBX-61(76)

A BRIEF REVIEW OF THE BASIS FOR, AND THE PROCEDURES CURRENTLY UTILIZED IN, GROSS GAMMA-RAY LOG CALIBRATION

BENDIX FIELD ENGINEERING CORPORATION
Grand Junction Operations
Grand Junction, Colorado 81501

October 1976

PREPARED FOR THE U. S. ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION
GRAND JUNCTION OFFICE, UNDER CONTRACT NO. E(05-1)-1664

GJBX-61(76)

A BRIEF REVIEW OF THE BASIS FOR , AND THE PROCEDURES CURRENTLY UTILIZED IN, GROSS GAMMA-RAY LOG CALIBRATION

John R. Duray

BENDIX FIELD ENGINEERING CORPORATION
Grand Junction Operations
Grand Junction, Colorado 81501

October 1976

Prepared for the U. S. Energy Research and Development Administration
Grand Junction Office, Under Contract No. E(05-1)-1664

TABLE OF CONTENTS

I. Introduction

The purpose of this note is to bring together the various assumptions that are stated in the literature or are tacitly understood and which form the basis for the well known relationship between the grade-thickness product and the total or gross gamma ray response of a borehole probe. As this relation pertains to calibration, certain conditions have been established by the Energy Research and Development Administration (ERDA) in order to assure uniformity among the various gross gamma ray probes. That these conditions and their implications are sometimes misunderstood is an understatement. An example illustrating the analysis of digital calibration data is presented in the hope that such misunderstanding may be cleared up and at the same time demonstrate the efficacy of a few common integration techniques.

II. Basis for Quantitative Analysis

The quantitative interpretation of gross gamma-ray logs is based on the relation [1,2]

$$\overline{G}_\gamma T = kA$$

where $\overline{G}_\gamma$ is the average radiometric grade expressed in percent equivalent (%e) U_3O_8 by weight, T is the thickness of the uranium bearing zone, A is the area under the gamma-ray response curve corrected for resolving time losses, and k is a constant of proportionality referred to as the k-factor. This relation has been established both empirically[1] and theoretically[2] by making certain assumptions and imposing a set of standard conditions.

The assumptions made for a characteristic response, interpretable by the above relation are:

1.) The effective volume that the probe passively samples (referred to as the effective sample volume) is essentially spherical; that portion on and near the axis of the probe being sufficiently small such that the departure from sphericity is negligible.

2.) Any zone, either barren or mineralized, is completely and uniformly filled with a homogeneous material that extends beyond the effective sample volume of the probe. Zone isotropy implies that the density, the equivalent atomic number, and the linear gamma-ray absorption coefficient are constant throughout a zone. (The presence of other matter such as rock, borehole fluid, casing, probe housing, etc., is neglected.)

3.) The boundary between different zones, such as mineralized and barren, is well defined.

4.) The probe moves along the borehole with constant velocity and samples its environment uniformly in time (hence the spatial sampling interval is constant).

5.) The probe does not respond to the mineralized zone when the distance between the mineralized zone and the probe, comprised of barren material, is greater than one effective sample volume radius.

6.) The thickness of a mineralized zone is at least one effective sample volume diameter.[3]

7.) The detector is taken as a point detector (i.e., a detector without extent).

While this completes the basic assumptions on which the relation $\bar{G}_\gamma T = kA$ is predicted, item 2.) above, neglects conditions which are encountered in actual logging. Therefore, a requirement is imposed which relates any gamma-ray log to an accepted standard. This requirement, generally stated, specifies a set of standard conditions, under which all sampling intervals are to be made. These conditions[2], defined by ERDA (nee AEC), may require corrections to the logging parameters if they are not met. The standard conditions are:

A) A borehole diameter of 4.5 inches,
B) The medium filling the borehole is air,
C) No borehole casing,
D) Effective interstitial fluid in the mineralized zone is 12% by weight,
E) The ratio of the true mean uranium grade to the mean radiometric (equivalent) grade is unity. (Parent and daughters are in secular equilibrium.),

continuing with specific applicability to the GJO recommended calibration procedure,[4]

F) The logging speed should be 5 ft/min (1 inch/sec),
G) The sampling interval should be at most 0.5 feet for digital systems (or 10 cm for metric based digital systems),
H) The ratemeter time constant should be about one second,
I) The abscissa of the chart recorder should be five feet per inch (or one meter per two centimeters) of chart paper,
J) The hole is logged from the bottom to the surface with the probe in physical contact with the wall of the hole.

These standard conditions are rarely, if ever, met in field logging operations. However, they can be maintained for the calibration procedure.

III. Test Pit Calibration - General

When industry logging units come to the ERDA-GJO facility for the purpose of calibrating a total gamma-ray probe, they are interested in determining (or checking) four logging parameters. These parameters are:

a) The electronic resolving time τ,
b) The k-factor,
c) The water-hole size correction factor,
d) The casing correction.

The resolving time (or "dead" time) is the minimum time necessary to completely process an event (gamma-ray) and be ready to accept and process another event. The k-factor relates the response of the probe to the grade-thickness product ($\overline{G}_\gamma T$). The water-hole size factor relates water filled holes of various diameters to the standard (4.5 inches) dry hole. The dependence of hole size in air is small. The casing factor relates casing thickness to that of the standard (uncased) borehole.

More often than not, the first two parameters (τ and k) are frequently checked while the latter two are performed once per probe and only repeated if a significant variation in the k-factor is found. Therefore, only the determination of τ and k will be pursued herein.

IV. Resolving Time Algorithm

The resolving time τ is determined using an approach developed by Crew and Berkoff[5] from their detailed "two pit concept". Each output datum n_i, measured within the i^{th} one-half foot (standard) interval must be corrected to include those counts not recorded in that interval because the electronic system was busy. Letting N_i represent the true count rate for the i^{th} interval, then

$$N_i = \frac{n_i}{1-n_i\tau}$$

relates the observed count rate n_i to the true count rate through the resolving time τ of the system. This relationship is valid when the process sampled is a true random process and the resolving time is not a function of the counting rate. A typical (characteristic) response curve is shown in Figure 1 and indicates both the observed and the true count rate versus probe depth.

The approach currently used utilizes the data obtained from logging two test pits of different grades; one referred to as the low grade pit and the other the high grade pit, having grade-thickness products designated by $(\overline{G}_\gamma T)_\ell$ and $(\overline{G}_\gamma T)_h$ respectively. Taking the ratio of the relations governing the response to the low and high grade pits gives

$$\frac{(\overline{G}_\gamma T)_\ell}{(\overline{G}_\gamma T)_h} = \frac{A_\ell}{A_h}$$

where A denotes the area under the designated response curve. Approximating the area by a rectangle (see Table I for comparative values) that is $\overline{N}$ high by T wide where $\overline{N}$ is the average over the statistically constant portion of the peak (referred to as the peak plateau) and T is the width of the zone defined[3] as the full width at half maximum. Substituting for the area $A \cong \overline{N}T$, the above becomes

$$\frac{(\overline{G}_\gamma)_\ell}{(\overline{G}_\gamma)_h} = \frac{\overline{N}_\ell}{\overline{N}_h} .$$

Representing the low to high grade ratio by R, i.e.,

$$R \equiv \frac{(\overline{G}_\gamma)_\ell}{(\overline{G}_\gamma)_h}$$

and substituting for $\overline{N}_\ell$ and $\overline{N}_h$, using

$$\overline{N}_\ell = \frac{\overline{n}_\ell}{1-\overline{n}_\ell\tau} \quad \text{and} \quad \overline{N}_h = \frac{\overline{n}_h}{1-\overline{n}_h\tau}$$

gives

$$R = \frac{\overline{n}_\ell}{1-\overline{n}_\ell\tau} \; \frac{1-\overline{n}_h\tau}{\overline{n}_h}$$

where $\overline{n}_\ell$ is the average count rate over the statistically constant portion of the peak as determined from the observed low grade log while $\overline{n}_h$, in the same way, corresponds to the high grade log. Solving the above for the resolving time

$$\tau = \frac{\overline{n}_\ell-\overline{n}_h\ R}{\overline{n}_\ell\overline{n}_h\ (1-R)}$$

gives the desired relation.

V. Current Determination of Resolving Time

Currently four test pits are logged and used to determine the resolving time: N3, U1, U2, and U3. From this data, five ratios are computed: U2/U1, U3/U1, N3/U1, U3/U2, and N3/U2. Each of these five combinations determines a τ as outlined above, viz., the average over the statistically constant portion of the peak is computed for each of the two pits and combined with the corresponding R which is determined from the grade data given in the test pit specification sheets. Finally, the average of the five values for the resolving time is computed and used in further analysis.

VI. Calculation of the k-factor for a Symmetric Digital Response - An Example

The characteristic response curve obtained from the N3 test pit is used in conjunction with the average resolving time to determine the k-factor. First, each measured point of the response curve is corrected for finite resolving time losses and tabulated on a work sheet (Form BFE-1025) as, for example, is shown in Figure 2 and plotted in Figure 1. In this example, N3 was sampled in 5 cm intervals (labeled I) which are tabulated in the first (fourth) column, the measured data (n) in the second (fifth) column, and the resolving time corrected data (N) in the third (sixth) column. Although the data does not conform to the ERDA standard of 10 cm intervals it will be treated without this restriction for now and adjusted later.

First, notice in Figure 1 that the response to N3 is not symmetric. For the moment, assume that the response is symmetric; then the area under the curve would be determined by the trapezoidal "rule" which, for equal increments, reduces to a sum over all the corrected counts from constant background on one side to a constant background on the other side of the peak response[3]. Multiplying the sum by the (constant) summation interval gives the area in units of counts-meters. Thus in Figure 1, summing from 0.325 m to 3.325 m inclusive gives

$$\sum_{i = 0.35\ m}^{3.30\ m} N_i = 140{,}375 \text{ counts/sec}$$

and for the constant 5 cm (0.05 m) interval gives an area

$$A = (140{,}375 \text{ counts/sec})(0.05 \text{ m}) = 7018.75 \text{ m - counts/sec.}$$

Converting this result to the ERDA standard of 10 cm intervals (or 0.5 ft in the British system of units) and following the current convention of expressing the area in units of counts when normalized to the standard interval, yields

$$A_{std} = \frac{7018.75 \text{ m - counts/sec}}{0.10 \text{ m}} = 70187.5 \text{ counts/sec.}$$

Although the result of this example is incorrect (because the response is not symmetric), it exemplifies the procedure employed when the response is symmetric. Note that there is a slight difficulty in picking the limits of summation. Most likely though this uncertainty will not significantly affect the final result. The non-zero contribution to the tail is assumed to be due to naturally occurring activity.

The area, in units of counts for a standard interval, is used to compute the k-factor. Using for N3, $\overline{G_\gamma T}$ = 0.9976 %eU_3O_8-feet (0.3041 %eU_3O_8-meters) gives

$$k = \frac{(\bar{G}_\gamma T)_{N3}}{A_{std}} = \frac{0.3041 \text{ \%eU}_3\text{0}_8\text{-meters}}{70187.5 \text{ counts/sec}}$$

$$= 4.33 \times 10^{-6} \text{ \%eU}_3\text{0}_8\text{-meter/counts/sec}$$

Note the units of k. Had we not normalized the area under the response curve to the ERDA standard 10 cm interval but expressed the area under the curve in its proper two dimensional units (i.e., meter - counts/sec), then the units of k would be $\%eU_3O_8$/counts/sec and the value of k itself would be 4.33×10^{-5} $\%eU_3O_8$/counts/sec.

Another example which brings out this feature of expressing the area under the curve in units of one dimension can be given by considering an application of the rectangular approximation to compute the area. The area of a rectangle is the product of two adjacent sides: Estimating the area of the response curve then, the area is $A \approx \bar{N}T$ where $\bar{N}$ is the average count rate over the statistically constant (flat) portion of the peak expressed in counts/sec and T, a length, is the full width at half maximum of the response curve i.e., the width of the response taken between the half maximum points (see Figure 1 where two corners of the rectangle are indicated). Dimensionally the area A is given in units of count rate-length. If one converts the rectangular approximation to a standard interval, say in the British system, then the normalized area would be $A_{std} \approx \bar{N}T/0.5$ and would have units of count rate only because the one half foot in the denominator is a standard interval. In other words, there are T/0.5 or 2 T standard half foot intervals in the full width at half maximum. The normalized area A_{std} is the product of $\bar{N}$ and 2 T; the latter is dimensionless.

VII. Current Procedure for an Asymmetric Digital Response

The response curve from N3, as noted earlier, is not symmetric, therefore, it must be treated differently from the example given. After determining the average τ and correcting each measured datum (which collectively define the response curve) for finite resolving time losses, the average over the peak plateau is computed and the half maximum found. The peak response is summed (trapezoidal method) within the limits defined by the full width at half maximum. An appropriate correction is made at each limit of the summation because the half maximum points do not usually coincide with a boundary of a summation interval (e.g., a 5 cm interval in the example previously given). The tails of the response curve are determined by summing only the one side from the region of constant background to the half maximum point (again a correction may be necessary for the upper limit or half maximum point). This result is doubled and is taken as the total area for both tail regions. The tail that is summed corresponds to the upper part of the N3 test pit (the limits are 0.325 m and 1.375 m in the example shown in Figure 1). The total tail area and the area defined by the half maximum limits are summed to give the total area which is then used to compute the k-factor. An adjustment may be necessary, as in the example, to convert the area into units of count rate for an ERDA standard interval.

VIII. Comparative Numerical Techniques

Table I gives the results of various numerical integration techniques. The most accurate is the parabolic method. All other results are normalized to this result for comparison purposes. The current technique used by the Test and Evaluation Department is the trapezoidal method. The final two entries both compute the area using one-half of the response curve. By integrating only over half the curve, the anomaly found in the lower barren region of N3 is neglected. The basis for doing this is the presumption that the response curve from the test pit should be symmetric. The last entry, the rectangular approximation (used earlier in obtaining the two pit algorithm) gives a result which is low partly because by its use, a portion of the tail-background region is automatically excluded. Note however, that the thickness computed by this method agrees quite well with the value quoted on the N3 specification sheet of T=4.19 feet (to within less than 1%). This supports the approximation made in the derivation of the two pit algorithm. The relative accuracy in determining the thickness T using only half the curve is a factor of three better than using either integration method. The rectangular method does not require the resolving time correction be applied to each datum, it only need be applied to the final result. Two corners of the rectangle are indicated in Figure 1. The k-factor is computed and tabulated in the final column of Table I.

This comparison of numerical techniques is intended for the understanding of the user making deadtime and k-factor determinations from digital data with the aid of a desk calculator. There are of course computer programs which may employ techniques different from those outline herein. Nevertheless, the procedure and techniques discussed above will provide a rapid check of the more elaborate calculation.

REFERENCES

1. R. F. Droullard and P. H. Dodd, "Gamma-Ray Logging Techniques in Uranium Evaluation", Proc. 2nd U.N. Int. Conf. Peaceful Uses of Atomic Energy (Geneva, 1958) 3, 46 (1958).

2. J. H. Scott, P. H. Dodd, R. F. Droullard, and P. J. Meedra, "Quantitative Interpretation of Gamma-Ray Logs", Geophysics 26, 182 (1962).

3. P. H. Dodd and D. H. Eschliman, "Borehole Logging Techniques for Uranium Exploration and Evaluation", Uranium Prospecting Handbook, Proc. NATO Adv. Study Inst. Method Prosp. for U Min. (London, 1971) 244 (1972).

4. D. H. Eschliman, "Procedure for Industry Logging Units", Lucius Pitkin Internal Report, un-numbered.

5. M. E. Crew and E. W. Berkoff, "TWOPIT, A Differential Approach to Calibration of Gamma-ray Logging Equipment", Log Analyst, 11 no. 6, 26, (1970)

TABLE I Comparison of numerical integration techniques. Symbols are defined in the text.

Method	$A^{\dagger}$ (counts/sec)	Normalized Area	T (feet)	k ($\times 10^{-6}$ %eU_3O_8-meters/counts/sec)
Parabolic	69051.6	1.0	4.291	4.404
Trapezoidal	68981.0	0.9989	---	4.408
Parabolic	67019.4*	0.9705	4.160*	4.537
Rectangular	65920.5*	0.9546	4.156*	4.613

†The area is normalized to the ERDA standard 10 cm interval

*Within factor of two (2) of calculated value

FIGURE CAPTIONS

Figure 1 Log of N3 Test Borehole. Plot of typical total gamma-ray log of ERDA-GJO test pit N3. The dots are the measured digital response and the triangles correspond to the resolving time corrected digital response. A curve has been drawn through the data to aid the eye. The limits of the rectangular integration method are indicated on either side of the measured response. See text for details.

Figure 2 Work sheet (Form BFE-1025) used to determine calibration parameters. The interval I (columns one and four) is in units of meters, the measured response (n) is in column two (five) and the resolving time corrected data (N) is in column three (six). The data are plotted in Figure 1.

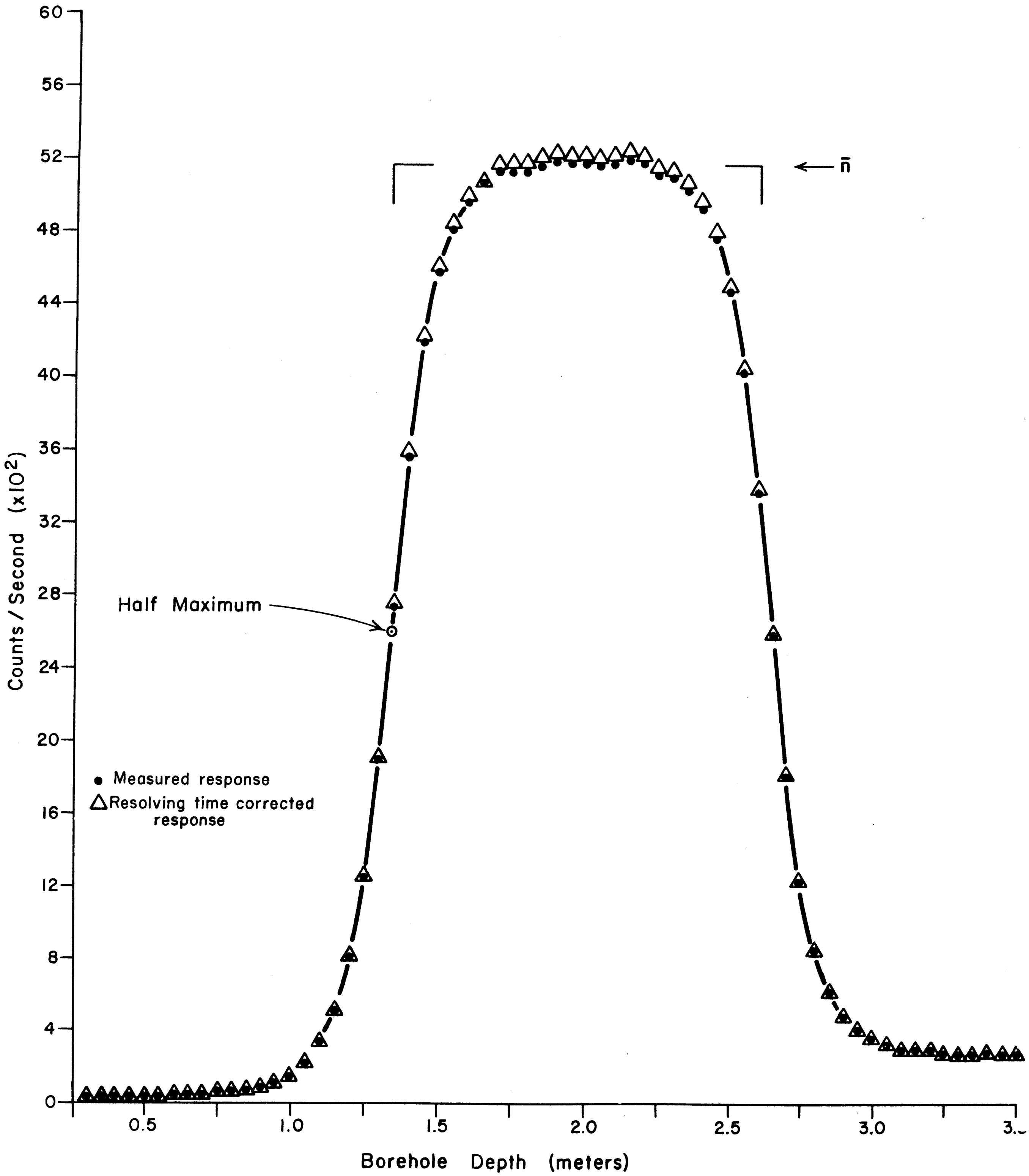

Figure 1. Log of N3 Test Borehole

Bendix Field Engineering Corporation
Grand Junction Operations
P.O.Box 1569
Grand Junction, Colorado 81501

Page ____ of ____

GAMMA RAY LOG INTERPRETATION WORK SHEET

BFE-1025

Company ______ Log Operator ______ Unit No. ______
Address ______ Interpreter ______ Probe No. ______
City ______ State ______ Water Factor ______ Ratemeter No. ______
Zip ______ K Factor ______ Equipment Manufacturer ______
Date Logged ______ Other Factors ______
Date Interpreted ______
Company Contact ______

Inches	n	N	Inches	n	N
E_1			E_1		
E_2			E_2		
$E_1 + E_2 =$			$E_1 + E_2 =$		
$E_1 + E_2 \times 1.38 =$			$E_1 + E_2 \times 1.38 =$		
I 0.35	30	30	I 1.85	5170	5207
0.4	30	30	1.9	5190	5227
0.45	30	30	1.95	5180	5217
0.5	30	30	2.0	5180	5217
0.55	30	30	2.05	5170	5207
0.6	40	40	2.1	5180	5217
0.65	40	40	2.15	5200	5238
0.7	40	40	2.2	5180	5217
0.75	50	50	2.25	5130	5167
0.8	50	50	2.3	5110	5146
0.85	60	60	2.35	5040	5075
0.9	80	80	2.4	4940	4974
0.95	100	100	2.45	4770	4802
1.0	140	140	2.5	4480	4508
1.05	210	210	2.55	4030	4053
1.1	330	330	2.6	3370	3386
1.15	510	510	2.65	2580	2589
1.2	800	801	2.7	1810	1814
1.25	1250	1252	2.75	1230	1232
1.3	1900	1905	2.8	850	851
1.35	2740	2750	2.85	620	620
1.4	3570	3588	2.9	490	490
1.45	4200	4224	2.95	410	410
1.5	4580	4601	3.0	360	360
1.55	4810	4842	3.05	330	330
1.6	4960	4994	3.1	310	310
1.65	5040	5045	3.15	290	290
1.7	5130	5167	3.2	290	290
1.75	5140	5177	3.25	280	280
1.8	5150	5187	3.3	280	280

APPARENT DEADTIME	Probe No ____	Probe No ____
U_1/U_2		
U_1/U_3		
U_1/N_3		
U_2/U_3		
U_2/N_3		
U_3/N_3		
Mean	1.38 usec	

WATER FACTORS MODEL	Probe No. ____	Probe No ____
0-2.25"		
0-4.5"		
0-6.5"		
0-8.5"		
N-3		
U-3		
U-2		
U-1		

Comments:

Figure 2