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fit of those contemplating the incorporation of such tests in a preventive maintenance program.

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Discussion

G. Fred Lincks (General Electric Company, Pittsfield, Mass.): The authors have made a real contribution in presenting data on protection and maintenance testing of rotating machines. Rotating machines have always posed an exacting problem from the standpoint of lightning and surges since the maximum impulse insulation strength is given as that of the 60-cycle high-potential withstand test. This is very ably covered by the authors. Despite the skimpy margins shown on paper, the protection provided by lightning arresters and capacitors has been excellent. They say that generally the users employ a special arrester designed or selected so as to have lower sparkover and better characteiistics than a standard distribution arrester. It might be well to point out that the American Standard for lightning arresters[1] states that the front of the wave sparkover test for arresters used for rotating machine protection shall have a test voltage with a uniform rate-of-rise to gap sparkover in 10±3 μsec as measured from virtual zero to sparkover. This slower front wave is practicable only because of the use of a capacitor at the machine terminals However, it would be well to point out also that the American Standards require that all arresters, including those used for rotating machine protection, meet the standard duty-cycle and high- and low-current withstand tests specified for arresters of a given class, that is, station, intermediate, or distribution class. Impairment of these discharge capabilities by shortening the gap may detrimentally affect the arrester's ability to meet service requirements.

The authors suggest an even more difficult overvoltage protection problem when they state, "This would indicate that for a 4,800-volt ungrounded system, arresters should have a sparkover of about 10 to 12 kv crest rather than the present 15 or 18 kv." The American Standards specify the minimum power frequency sparkover shall be not less than 1.5 times the rated voltage of the arrester. A 6-kv arrester is required for a 4,800-volt ungrounded circuit and 1.5 times 6 times 1.41, to obtain the crest value, gives 12$^1/_2$ kv. Since an allowance of 25% generally is required on sparkover, the ma mum power frequency (60 cycles per s ond) sparkover would be 15 kv crest.

We can fully appreciate the dilemma posed by the deterioration with time of sulation strength that is difficult to prot even when new. However, complying w the author's proposal would impose an tremely difficult design problem for t arresters. Normal conditions would ha to be reversed so the front of the wave spar over voltage is lower than the 60-cy power-frequency sparkover voltage. A other solution would involve changing t requirements in the American Standar for minimum power frequency sparkov voltage to a lower value than 1.5 times t rated voltage. This would increase t possibility of sparkover of the arrester o power frequency overvoltages which a likely to be sustained long enough to da age or destroy the arrester.

Reference

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H. R. Armstrong and **J. E. Mulavey:** A pointed out by Mr. Lincks, the America Standard for Lightning Arresters does no recognize the special characteristics necessary in rotating machine arresters. It may well be that the present requirement of a duty-cycle test as severe as that necessary for arresters to be installed on open-wire is unrealistic. If it is possible to design an arrester with characteristics more suitable for machine protection than anything now available, and if in so doing, the severe duty cycle tests cannot be met, it seems more desirable to place emphasis on protective ability. The possibility of an occasional arrester failure is much to be preferred to the loss of a machine.

Performance of the Sodium Reactor Experiment

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The Sodium Reactor Experiment (SRE) was designed, built, and is being operated by the Atomics International Divison of North American Aviation, Inc., as part of a program with the US Atomic Energy Commission for the development of the sodium-graphite approach to economic nuclear power. The sodium-graphite type of reactor is particularly attractive because of its inherent safety and the efficiencies which can be achieved with a high-temperature system. These features are associated with the chemical compatibility of the materials used in the core and the heat-transfer system and the high boiling point [1,615 F (degrees Fahrenheit) at atmospheric pressure] of sodium, which permits a low-pressure heat transfer system even at temperatures of interest for modern steam plant technology.

The SRE serves both as an experiment to determine the limits of performance of the original design concept and as a flexible developmental facility, which is used for a vareity of tests and experiments and which is being continually modified to improve performance. The approach to full-power operation of the reactor has purposely proceeded relatively slowly to provide ample time for a variety of tests to be performed and analyzed prior to the operation of the plant at the limits of which it is ultimately capable. This philosophy of operation is consistent with the concept of the SRE as a high-temperature reactor experiment, from which the information needed for the design of a full-scale sodium-graphite reactor is being obtained. It is important to note that future plants designed with information available from the SRE will

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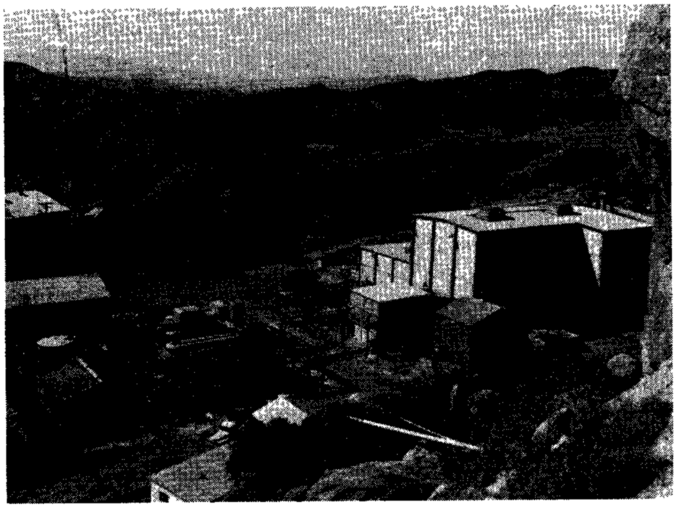


Fig. 1. Completed SRE plant

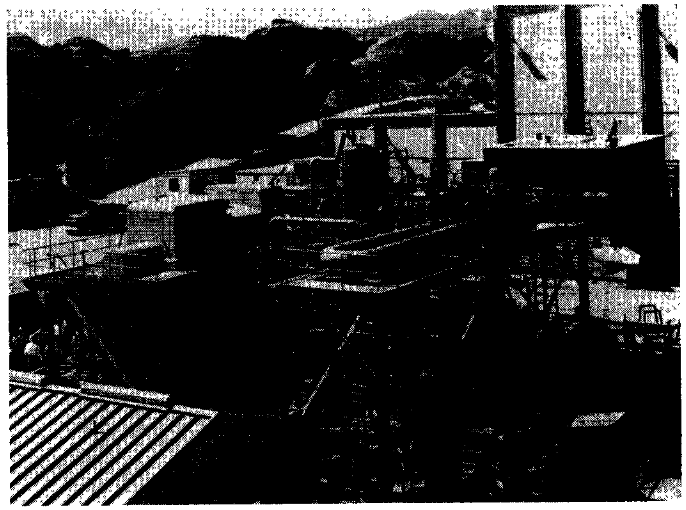


Fig. 2. Complete steam-electric plant

not require the extensive series of pre-operational and low-power tests conducted with the SRE and can, therefore, be started up and put "on line" on a time schedule consistent with conventional power plants.

Detailed descriptions of the SRE have been given in various publications,[1–6] and will not be repeated here. The completed plant is shown in Figs. 1, 2, and 3.

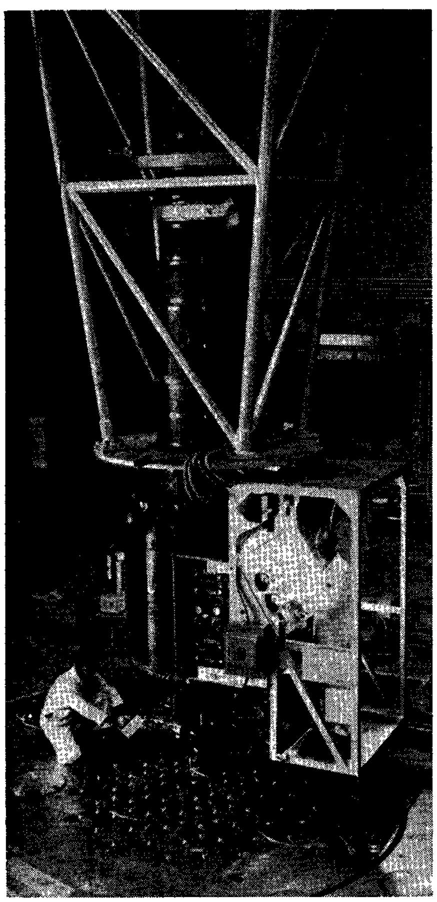


Fig. 3. Reactor loading face and fuel-handling cask

Table I gives the more important design conditions for the plant.

Operation to Date

The SRE reached criticality on April 25, 1957, with a core loading of 32.6 fuel elements (62.2 kilograms of U-235) compared to a theoretical loading of 28.3 fuel elements. During May and June physics tests were conducted which confirmed the theoretical estimates of the flux distribution and the reactivity worth of the various core elements. Low-power runs were made in July with a core loading of 36 fuel elements and again in November with a 43-element loading. The results are given in Table II. The purpose of these runs was to compare the actual performance with the engineering calculations and to uncover, at the earliest possible date, any anomalies in the system. Planned "scram" tests during the runs confirmed the excessive thermal convection flow predicted by reactor simulator studies. Four unexpected results involving the heat-transfer system were observed. These were: 1. excess grid plate leakage (about a factor of four larger than the calculated leakage), 2. a 50% excess log mean temperature differences across the main intermediate heat exchanger, 3. almost complete stratification of the hot and cold sodium in the main intermediate heat exchanger following a scram; 4. production of wet steam in the lower tubes and superheated steam in the upper tubes of the horizontal once through steam generator.

The only factor limiting the reactor to one third of full power at that time was the expected thermal stress in the sodium exit nozzle on the core tank resulting from the large thermal convective flow following a scram. This was eliminated by controlling the flow after scram by means of electromagnetic eddy-current brakes installed in the main primary and secondary coolant loops.

Full-power operation was achieved on May 21, 1958. These results are also given in Table II.

Performance During Full-Power Run

Operation during the power runs has been very stable. Tne neutron flux controller was used to hold the reactor power level constant during most of the later runs. This system performed quite satisfactorily; the regulating drive compensated for temperature variations, poison buildup, and fuel burnup with a minimum of rod motion and no indication of instability or "hunting." Sodium flow was adjusted during the establishment of each set of operating conditions to obtain desired reactor and steam plant temperatures and was maintained constant by automatic-speed-control circuits operating through the Ward-Leonard motor-control system. Feedwater flow was controlled manually. Fig. 4 is a flow schematic of the plant with operating conditions prevailing on May 26, 1958.

The present 950 F maximum allowable coolant temperature is set by the germa-

Table I. Design Operating Conditions

Total heat power, mw	20
Electrical output, kw	6,000
Fuel temperature, F· maximum	1,200
average	750
Coolant outlet temperature, F	960
Coolant inlet temperature, F	500
Main coolant system flow rate, pounds per hour	485,000 (1,080 gallons per minute)
Auxiliary coolant system flow rate, pounds per hour	24,250
Steam temperature, F	825
Steam pressure, pounds per square inch	600
Steam flow rate, pounds per hour	60,000

Table II. Operating Information From the First Four SRE Power Runs

	Run Number 1	2	3	4
Period for run	July 10–15, 1957	July 25–25, 1957	Nov. 7–20, 1957	May 6–28, 1958
Period for electric power generation	July 12–15	July 25–26	Nov. 9–20	May 21–28
Number of hours turbine was "on line"	95	43 5	304 5	170
Kilowatt-hours of electricity produced	59,800	18,400	290,850	691,950
Electric power, maximum, mw	1 7	1	2	5.8
Thermal power, maximum, mw	6 5	4	7.6	21
Thermal efficiency, %	26	25	26	27.6
Main primary sodium flow rate, gallons per minute	1,000	780	1,080	1,300
Main secondary sodium flow rate, gallons per minute	950	690	1,040	1,300
Steam flow rate, pounds per hour	20,500	10,000	27,500	59,000
Reactor inlet temperature, F	481	475	505	576
Reactor outlet temperature, F	631	605	675	944
Steam generator inlet sodium temperature, F	605	530	638	875
Steam generator outlet sodium temperature, F	465	440	473	498
Steam temperature, F	508	590	628	800
Steam pressure, pounds per square inch gage	450	460	470	580

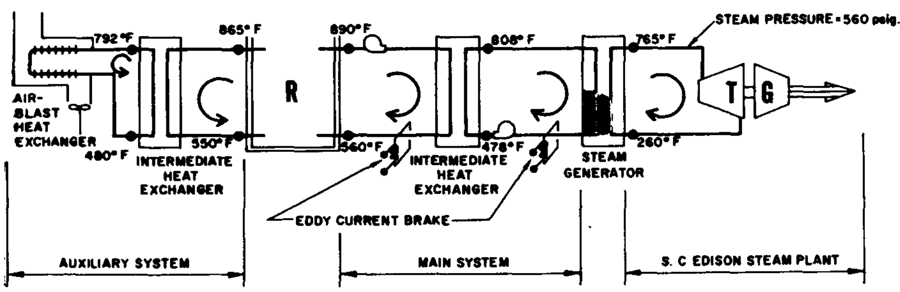


Fig. 4. System temperature at 18 mw thermal

Main primary thermal power = 17.6 mw
Electric power = 5.0 mw
Auxiliary primary thermal power = 0.34 mw
Flow rates, pounds per hour
Main primary sodium = 585,000 (calculated)
Main secondary sodium = 585,000 (calculated)
Auxiliary primary sodium = 11,900
Auxiliary secondary sodium = 11,900
Feedwater = 54,000

tive grain growth in the zirconium which occurs relatively rapidly above this temperature. The effect of excessive grain growth is to reduce the fatigue life of the zirconium cans to approximately 3,000 cycles, based on stressing the zirconium to the yield point. With uniform fuel-channel coolant temperatures the stress cycling derives from the excess thermal convection flow following a scram which cools the coolant tubes more rapidly than it does the can perimeters. Fig. 5 shows these temperatures after a scram. The consequent relative shrinkage of the coolant tube develops a stress at the can head to tube juncture.

By matching the heat flux to the coolant flow rate during the following a scram this stress may be eliminated and the temperature limit raised.

The flow control is achieved by means of eddy-current (electromagnetic) braking devices installed as shown in Fig. 4. The device itself is shown in Fig. 6 and is simply a means of applying a 6,000-gauss d-c induced magnetic field across a flattened section of pipe. The forces from induced electric eddy currents in the liquid metal act to oppose the fluid flow. Varying the magnetic field varies the flow rate. The field is programed automatically, using a thermocouple signal from a fuel-coolant channel to fix the exit sodium temperature. This eliminates thermal stresses throughout the system due to rapid decrease of the sodium outlet temperature. Fig. 7 demonstrates the effectiveness of the brakes in maintaining a reasonably constant reactor outlet temperature following a scram.

Core-Temperature Distribution

Large deviations from the mean temperature result in localized overheating or else require a reduction in the mixed mean sodium temperature and thus a loss in plant efficiency. To prevent this, hydraulic resistance of the interstices between moderator cans was designed to have a low value so that the internal thermal convection in the core would provide uniform moderator can sheath temperatures. That this has worked well is indicated by the fact that the maximum temperature variation among the moderator can sheaths did not exceed 30 F during the full-power run.

Since the zirconium temperature is a limiting operating condition in the SRE the attempt was made to equalize the exit coolant channel temperatures by orificing each channel so that the relative coolant flow rates were directly proportional to the relative heat generation rate.

In the July 1957 power runs the fuel channel orifices were all the same. This furnished an experimental check on the power distribution in the reactor. Fig.

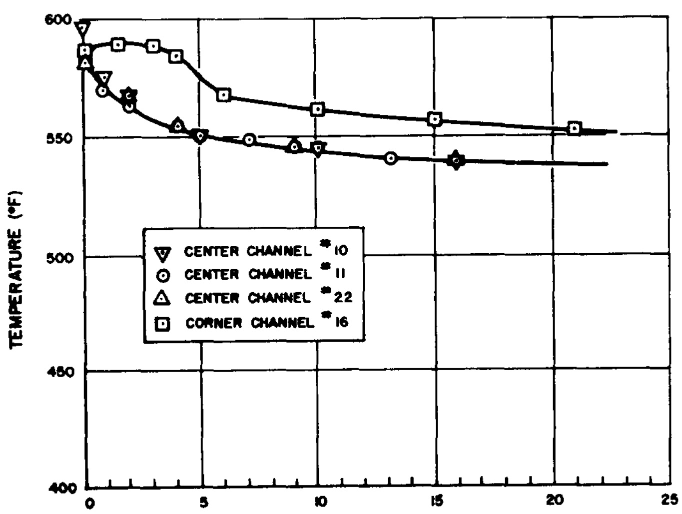
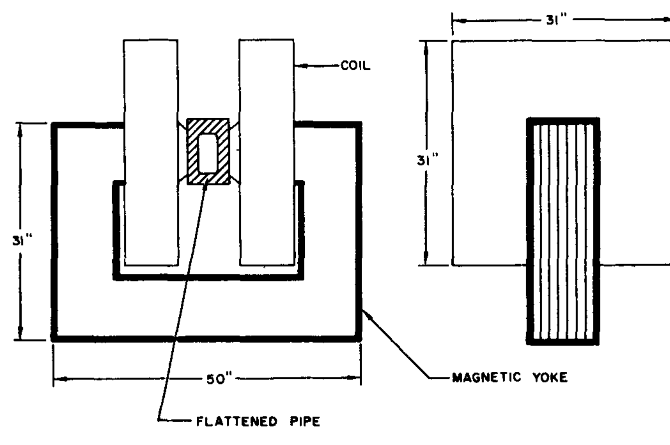


Fig. 5 (left). Moderator can temperature following a scram

Fig. 6 (below). Eddy-current brake



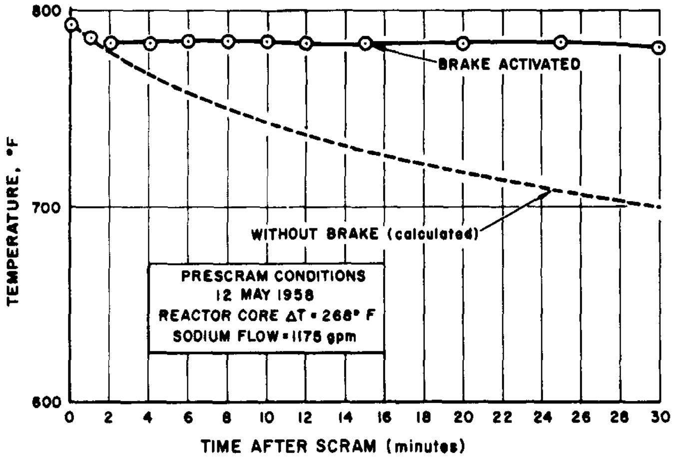


Fig. 7. Brake effect on reactor outlet temperature after scram

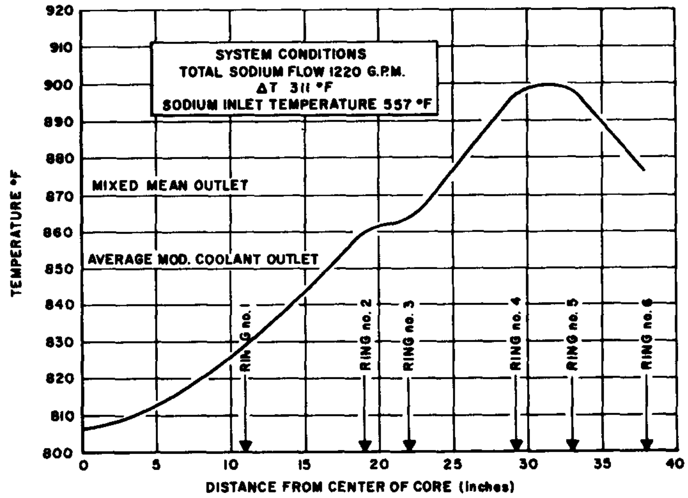


Fig. 9. Radial temperature plot of the core; 43 fuel elements

8 shows the resulting temperature distribution. Orifice plates were then installed to give a flatter temperature distribution. Fig. 9 shows the core-temperature distribution during the May 1958 power run. Analytical effort is continuing to understand all factors affecting core-temperature distributions. Refinements in the calculations will provide orificing to hold the maximum temperature difference among the fuel channels to 30 F. This will permit raising the reactor outlet temperature to 920 F, while still maintaining the maximum zirconium temperature at the 950 F limit.

Intermediate Heat Exchanger

System temperature taken during the first power run indicated that the log mean temperature difference through the main intermediate heat exchanger was 150% of the design value (i.e., 90 F versus 60 F design value). Between power runs, 60 thermocouples were distributed over the length and around the girth of the heat-exchanger shell. These were calibrated in place during the isothermal physics tests to an accuracy of ±2 F. The steady-state temperature distribution along the shell is shown in Fig. 10. The temperature curve is seen to rise uniformly through the straight portions of the exchanger, but it is horizontal around the bend, indicating that no heat transfer is occurring in that section. Examination of the construction photographs disclosed a wide gap between the tube bundle and the shell, and since this section is not baffled the fluid simply bypasses the tubes. Heat-transfer calculations show that the loss of this amount of heat-transfer area accounts for the high log mean temperature difference observed. Transient data taken on this equipment disclosed that rather complete stratification occurs on the shell side of the exchanger. The data are shown in Fig. 11. The resulting stresses have not yet been calculated. The data from test scrams indicate that it may be possible to program the post-scram flow rates (using the eddy-current brakes) to eliminate much of this stratification and the resulting stress.

The design full-power log mean temperature difference was 60 F. The actual measured value at approximately design power level was 90 F. This additional 30 F drop means that the steam temperature is 30 F lower than the design value resulting in a loss of over-all plant thermal efficiency. A startling result of the low flow stratification was the increase of the log mean temperature difference following a scram from 90 F at design power to 190 F at the very low power level prevailing after the reactor has been shut down.

A replacement heat exchanger has been designed, which embodies improvements based on the experimental data. In the interim the present exchanger will be operated without modification.

Preliminary experimental data indicate that similar conditions prevail in the auxiliary intermediate heat exchanger.

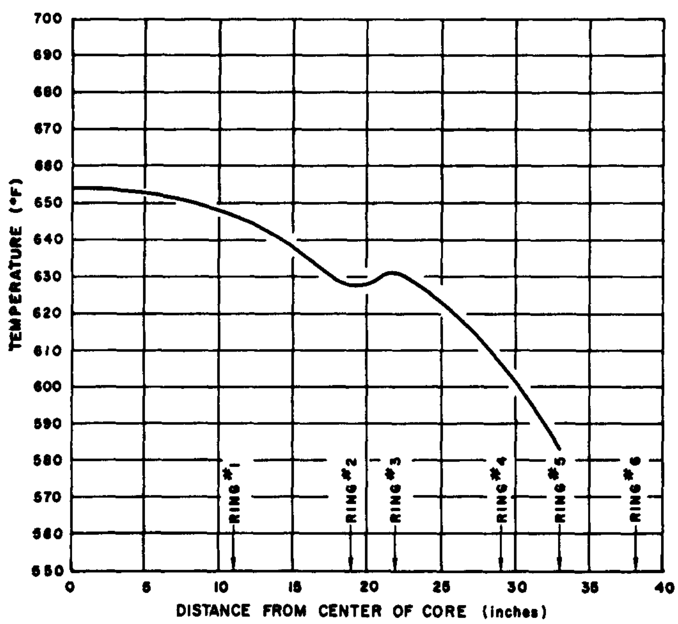


Fig. 8. Radial temperature plot

0.25-inch orifice plate core loading = 36 elements
Ring no. 6 contained seven dummy fuel elements
System conditions:
Total sodium flow = 786 gallons per minute
Δt = 144 F
Sodium inlet temperature = 474 F
Date = July 1957

Steam Generator

The steam generator is a "once-through" boiler with feedwater entering at one end and superheated steam exiting from the other. Fig. 12 is a view of the steam end of the steam generator. To eliminate the possibility of a sodium-water reaction, due to a leak in the steam generator, the tubes are double-walled with a mercury-filled annulus. The mercury system pressure is continuously

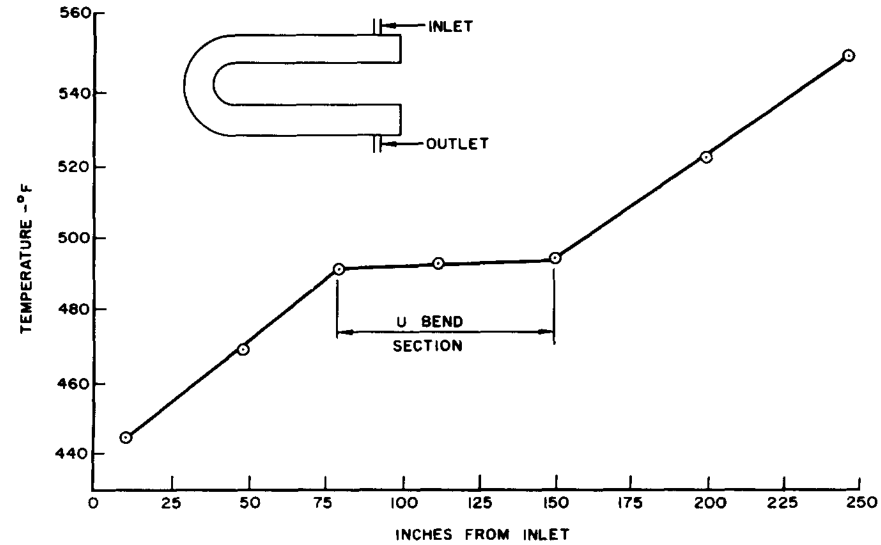


Fig. 10. Heat exchanger shell-side temperature distribution

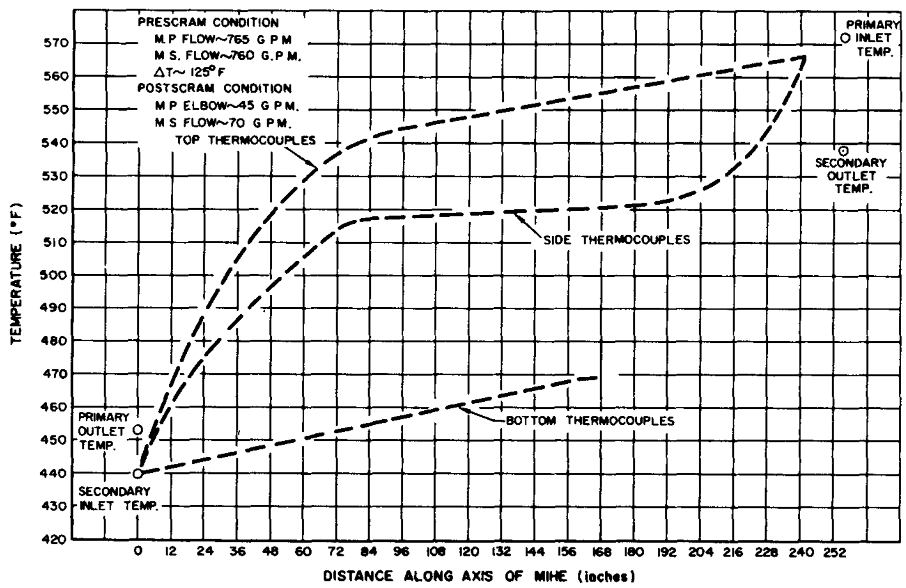


Fig. 11. Stratification in main intermediate heat exchanger following a scram

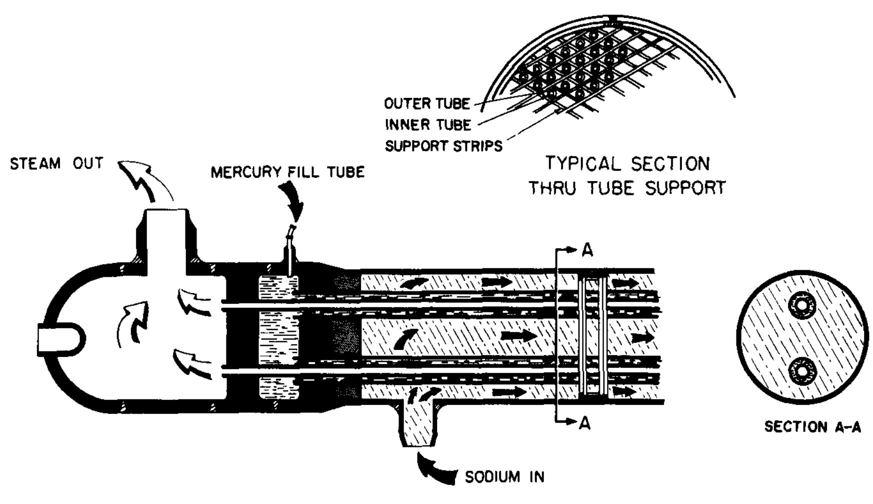


Fig. 12. Cutaway drawing of the steam generator

monitored so that any leak will be immediately detected and corrective action can be taken. The mercury is maintained at a pressure higher than the sodium pressure but lower than the steam pressure. Hence an increasing mercury pressure would indicate a mercury-to-steam leak and a decreasing mercury pressure would indicate a mercury-to-sodium leak. Neither condition presents a significant hazard to the plant or personnel.

In addition to the continuous monitoring of the mercury system pressure, sodium samples are taken from the main secondary sodium system after each power run and are analyzed for mercury content. No leaks have occurred in the steam generator.

The control program of the steam generator is designed to return sodium to the reactor at a constant temperature of 440 F. This is to be accomplished by controlling the rate of feedwater supplied to the horizontal once-through steam generator. During the first power run, it was found that this control worked but that in addition, the steam outlet temperature was very sensitive to feedwater flow. Examination of the data indicated that superheated steam was coming out of the top tubes, but that low-quality steam was coming through the bottom tubes. Later runs showed that this flow unbalance occurs to a certain extent at all steam flows. Feedwater flow during the May power run was limited to minimize the amount of low-quality steam produced in the lower tubes and thus to insure that wet steam would not be admitted to the turbine. As a result the sodium exit temperature could not be reduced to the design value of 440 F but was nearly 500 F. Table II shows the conditions actually met. Orifices have been installed by the Southern California Edison Company in all the steam generator tubes to increase the pressure drop through the steam generator and thus give more equal distribution of flow through top and bottom tubes. This modification enabled the steam generator to meet its design conditions. The temperature difference of the steam from the top and bottom tubes is now 30 F as compared to a 200 F temperature difference previously experienced.

Effect on Hallam Design

The Hallam Nuclear Facility is a nominal 75 mw (megawatts) (net) electrical plant to be constructed at the Sheldon station of the Consumers Public Power District of Nebraska. A conceptual design of the plant using a metallic uranium core was reported in a 1956 paper by Gronemeyer, et al.[7]

Some of the design changes already made because of SRE experience are:

1. Inclusion of convection flow devices for after-scram control of temperatures. The current choice is the throttling valve, since a valve is required for blocking purposes.

2. Use of stainless-steel moderator cladding, which permits higher local temperatures than does zirconium.

3. A conceptual design has been made of adjustable orifices for fuel elements for better control of the fuel-channel outlet temperatures.

4. Emphasis on low-flow performance and consideration of stratification problems has been included in the specifications for the intermediate heat exchangers and steam generators. The exchangers for Hallam may be vertical instead of horizontal, to prevent stratification.

In 1959, a 20-thermal-mw prototype of the Hallam steam generators is planned for installation at SRE. The Hallam plant control system will also be mocked up, and the SRE operated as a load-following plant to check out the entire system.

Conclusions

The excellent performance of the SRE has shown that sodium-cooled reactors are technically feasible and that the sodium-graphite approach to economic nuclear power is indeed a very promising one. As has been pointed out in this paper, the reactor has operated at design power level even though both outlet and inlet temperatures have been restricted by component limitations. Present indications are that the reactor can easily operate at 20% above its design power level.

It should again be emphasized that the SRE is an experimental plant and the startup and operating schedules to date reflect the philosophy stated at the beginning of this paper. The Hallam Nuclear Power Facility project is making full use of the experience and information gathered at the SRE. As a result of improvements in the design and operating procedures already made, it is expected that the Hallam Nuclear Power Facility will be put into operation on a schedule approximating that of a conventional power plant. There is no reason to expect more difficulties in the startup of a sodium-graphite-reactor-powered steam plant than might be encountered in an oil- or coal-fired plant of comparable rating.

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Computing Iron Losses in Fractional-Horsepower Induction Motor Design

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THE GREAT REDUCTION in the cost of a computation from the days of an engineer calculating single-phase induction motor performance with a slide rule and empirical charts, to the present day of high-speed digital computers, means that an evaluation of the design procedures and precision of the equivalent circuits is in order. Approximations that were formerly optimum between cost and accuracy for iron losses are no longer optimum. This paper presents an equivalent circuit and a calculation procedure for computing iron losses in single-phase induction motors which is judged to be an optimum for the situation of a standard punching being designed into many different motors using high-speed digital computers. With the possible exception of the effects of skew on the iron losses, no new theory is involved.

Paper 58-1318, recommended by the AIEE Rotating Machinery Committee and approved by the AIEE Technical Operations Department for presentation at the AIEE Fall General Meeting, Pittsburgh, Pa , October 26–31, 1958, and re-presented at the Winter General Meeting, New York, N. Y., February 1–6, 1959 Manuscript submitted July 28, 1958; made available for printing January 22, 1959.

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Nomenclature

a = turn ratio of start to main winding fundamental effective turns
I_{mp}, I_{mn} = positive and negative sequence stator currents respectively, referred to the main winding
I_{2p}, I_{2n} = positive and negative sequence rotor currents respectively, referred to main winding turns and axis
I_m, I_a = main and auxiliary currents respectively
N_o = synchronous speed, rpm
r_m = fundamental frequency iron loss resistor as if flux were uniformly distributed across the stack
$r_{2\alpha}$ = fundamental frequency iron loss resistor due to axial distortion from rotor currents
$r_{1\epsilon}$ = fundamental frequency iron loss resistor due to end fluxes from stator currents
$r_{m\delta}$, $r_{2\delta}$, $r_{e\delta}$ = slot-order frequency iron loss resistors due to stator mmfs, (magnetomotive force), rotor mmfs, and harmonic permeances, respectively
r_1, r_{1a} = resistances of main and auxiliary winding respectively.
${}_pr_2$, ${}_nr_2$ = resistance of rotor referred to main winding for positive and negative fields respectively
S = slip for positive sequence field
V_L = line voltage
V_{mp}, V_{mn} = total induced voltage in main winding due to positive and negative fields respectively
$V_{\delta p}$, $V_{\delta n}$ = induced voltage in main winding due to positive and negative air-gap fields respectively
W_{iF}, W_{iH} = iron loss due to fundamental and slot-order frequencies respectively
x_m = magnetizing reactance referred to main winding
x_l = total leakage reactance of main winding
x_{la} = total leakage reactance of auxiliary winding
$x_{1\sigma}$, $x_{2\sigma}$ = slot reactance referred to the main winding turns
$x_{1\delta}$, $x_{2\delta}$ = differential reactance referred to the main winding turns
$x_{1\alpha}$, $x_{2\alpha}$ = skew reactance referred to the main winding turns