CERTS Meeting

Development of Attribute Preserving Network Equivalents

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Overview

- Objective - develop attribute preserving power system network equivalents
  - preserve the essence of a model for some purpose
- Desirable properties include…
  - Economic analysis of electric power systems including transfer capacity
  - Transient stability response
  - LMP characteristics
  - **Application Dependent!!**
- Present focus is on developing equivalents that preserve the line limits of the original system

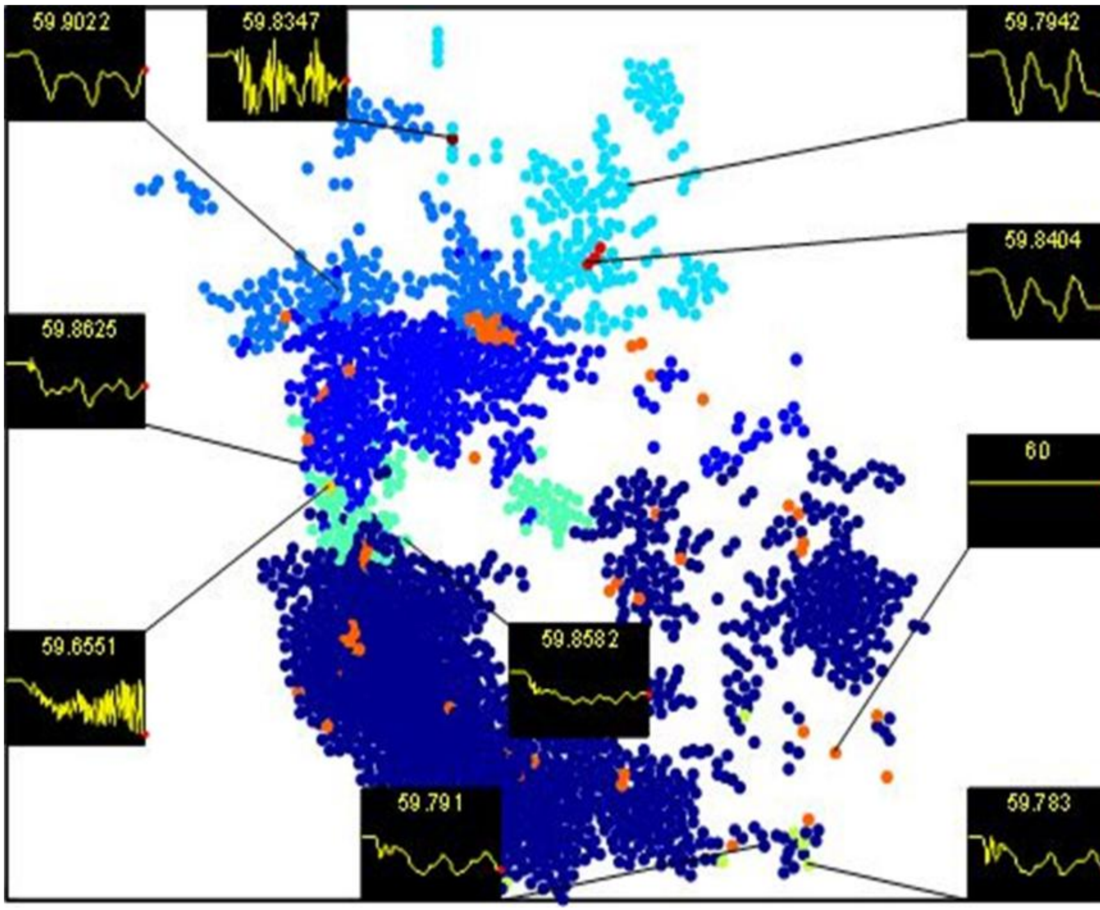
Applications of Previous Work

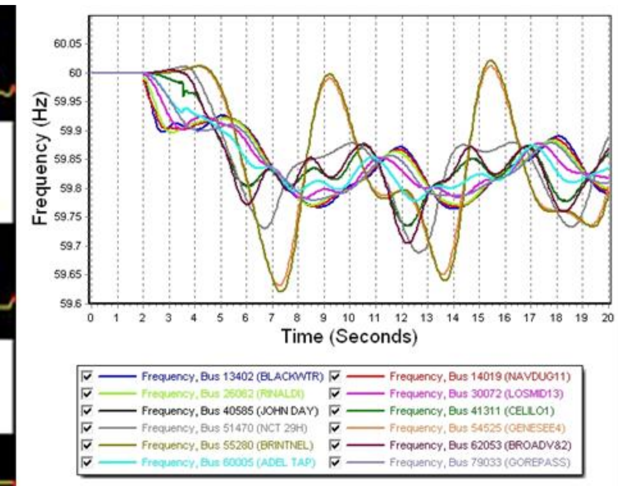
- Previous work in this task has focused on the application of data analysis methods for grouping power system buses
  - Clustering techniques have been key
- Kate Davis (Rogers), who was previously supported by CERTS, received her PhD in October 2011
  - Thesis title is "Data-enhanced applications for power system analysis;" Available online at https://www.ideals.illinois.edu/handle/2142/29740

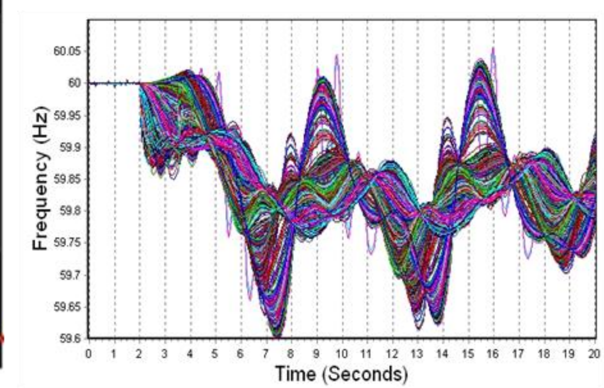
Applications of Previous Work

- B.C. Lesieutre, K.M. Rogers, T.J. Overbye, A.R. Borden, "A Sensitivity Approach to Detection of Local Market Power Potential," *IEEE Trans. on Power Systems*, November, 2011, pp. 1980-1988.
- S. Dutta, T.J. Overbye, "Optimal Wind Farm Collector System Topology Design Considering Total Trenching Length," *IEEE Trans. on Sustainable Energy*, July 2012, pp. 339-348.
- S. Dutta, T.J. Overbye, "Information Processing and Visualization of Power System Wide Area Time Varying Data," Submitted June 2012 to *IEEE. Trans. Power Systems*.

Clustering Application: Transient Stability Result Visualization







Preserving Transmission Line Limits

- For decades power system network models have been equivalenced using the approach originally presented by J.B. Ward in 1949 AIEE paper "Equivalent Circuits for Power-Flow Studies"
  - Paper's single reference is to 1939 book by Gabriel Kron, so this is also known as Kron's reduction
- System buses are partitioned into a study system (s) to be retained and an equivalent system (e) to be eliminated; buses in study system that connect to the equivalent are known as boundary buses

Ward Equivalents

- No impact on study, non-boundary buses

$$\begin{bmatrix} I_s \\ I_e \end{bmatrix} = \begin{bmatrix} Y_{ss} & Y_{se} \\ Y_{es} & Y_{ee} \end{bmatrix} \begin{bmatrix} V_s \\ V_e \end{bmatrix}$$

$$\left(I_s - Y_{se} Y_{ee}^{-1} I_e\right) = \left(Y_{ss} - Y_{se} Y_{ee}^{-1} Y_{es}\right) V_s$$

- Equivalent is created by doing a partial factorization of the Ybus
  – Computationally efficient

Retaining Line Limits

- As each equivalent bus is removed during the solution, equivalent lines are added joining each of its first neighbors
  - these equivalent lines have no associated limits
- Gist of our approach is to sequentially assign limits to these equivalent lines that preserve some desired attribute of the original network
- This is an iterative process that occurs as each bus is equivalenced. So as bus k, between i and j, is eliminated, then

$$Y_{ij}^{'} = Y_{ij} - \frac{Y_{ik} Y_{kj}}{Y_{kk}}$$

First term is admittance of any existing line (which may be zero), second term is for the new, equivalent line

Desired Characteristic: Bus to Bus Transfer Capacity

- Desire is to have the maximum power transfer (MPT) between retained buses match that for these buses in the original system
- Value is determined from PTDFs, ignoring loading

$$\text{MPT}_{ij} = \min\left(\frac{\text{Line Limit}_l}{\left|\text{PTDF}_{ij,l}\right|}\right) \text{ for } l=1,2..L$$

- Idea is to do this sequentially, for all the first neighbors of bus k, as it is being eliminated

PTDF Characteristics

- The lossless (dc power flow) PTDFs can be determined from a factored B matrix using a fast-forward, full-backward substitution
  - Fast-backward can be used if just a few are needed as will be the case here
- PTDFs on study system lines are not affected by the equivalencing process, and those on the new equivalent lines can be easily calculated from the original system

Algorithm Overview

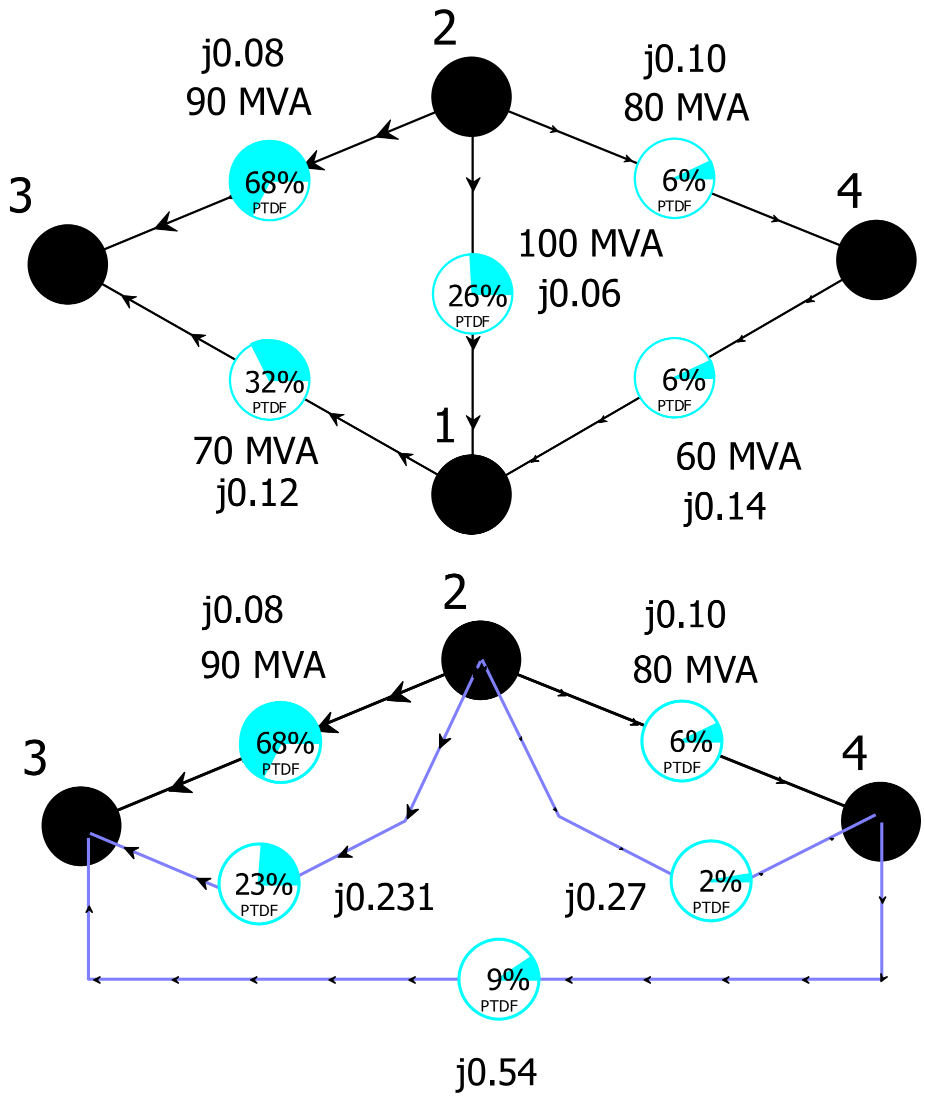
- Sequentially for each bus being equivalenced
  1. Calculate the PTDFs between the first neighbor buses
  2. Using these PTDFs, determine the MPTs between the first neighbor buses, just considering the limits on the lines that are being removed
     - Limits on the other lines do not need to be considered since these lines are being retained (at least until the next bus is considered).
  3. Select limits for the new equivalent lines so that the MPTs of the reduced system match that of the original system.
  4. Combine limits/impedances on parallel, equivalent lines

Combining Parallel Lines

- One consequence of this algorithm is the creation of lots of parallel, equivalent lines. Parallel line equivalencing is trivial, with the new limit just determined by determining which line in the parallel bundle is binding. That is, for n lines in parallel, each with impedance Z_i and total impedance Z_{combo}

$$Lim_{combo} = min\left(Lim_1 \frac{Z_1}{Z_{combo}}, Lim_2 \frac{Z_2}{Z_{combo}}, \ldots, Lim_n \frac{Z_n}{Z_{combo}}\right)$$

Four Bus Example (Bus 2-3 PTDFs Shown)



With removing bus 1, three equivalent lines will be added between the other three buses. The original MPTs are

2-3: 216.7 MW (1-3 binding)

2-4: 171.7 MW (1-4 binding)

3-4: 144.9 MW (1-4 binding)

For 2-3 direction for new equivalent line limits we require

1) Lim23 >= 216.7*0.234= 50.7MW
2) Lim24 >= 216.7*0.024= 5.2MW
3) Lim34 >= 216.7*0.088=19.1MW

Similar constraints for the other directions

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Four Bus Example, cont.

- To determine the new line limits we need to satisfy inequality and equality constraints

For the 2-3 direction for the new equivalent line limits we require
1) Lim23 >= 216.7*0.234= 50.7MW
2) Lim24 >= 216.7*0.024= 5.2MW
3) Lim34 >= 216.7*0.088=19.1MW
And one must be an equality!

For the 2-4 direction for the new equivalent line limits we require
1) Lim23 >= 171.7*0.028 = 4.8 MW
2) Lim24 >= 171.7*0.241= 41.4 MW
3) Lim34 >= 171.7*0.109 =18.7 MW
And one must be an equality!

For the 3-4 direction for the new equivalent line limits we require
1) Lim23 >= 144.9*0.206= 29.8 MW
2) Lim24 >= 144.9*0.217= 31.4 MW
3) Lim34 >= 144.9*0.197=28.5 MW
And one must be an equality!

Often times the solution will be trivial, just picking the largest in each row. Here the answer is Lim23=50.7 MW, Lim24=41.4 MW and Lim34=28.5 MW

General Solution Procedure

- Sometimes no solution exists. Then the best we use two approaches to bound the solution.
- To better understand, define matrix view, with entries showing PTDFs x MPT, hence they give the minimum limit needed to allow for the original MPTs

Directions

	2-3	2-4	3-4
Eqv Line 2-3	50.7 MW	4.8 MW	29.8 MW
Eqv Line 2-4	5.2 MW	41.4 MW	31.4 MW
Eqv Line 3-4	19.1 MW	18.7 MW	28.5 MW

First Approach: Overestimate

- In the first approach we satisfy all of the inequality constraints. This is a solution which overestimates the transfer capacity – the largest in each row
  - Equality constraints for at least one direction may not be satisfied, overestimating the flow

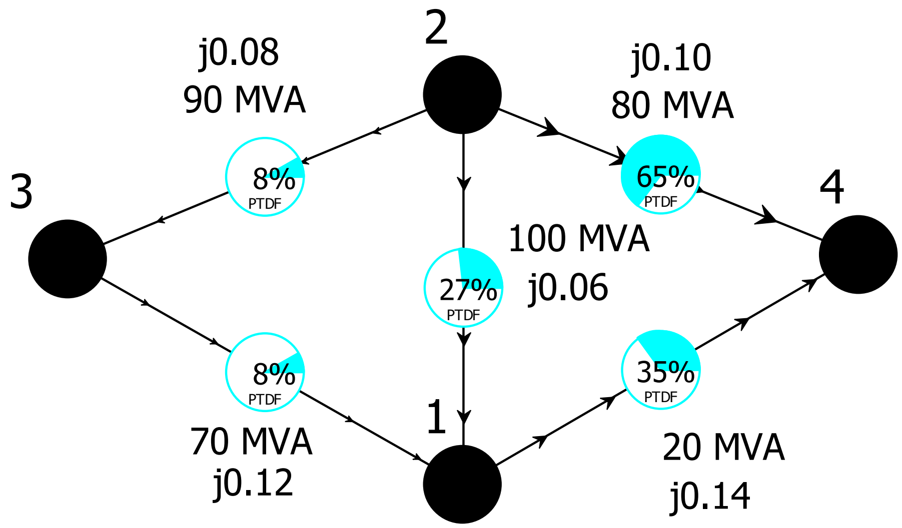
Directions (Original System Data)

	2-3	**2-4**	**3-4**
Eqv Line 2-3	**50.7 MW**	4.8 MW	29.8 MW
Eqv Line 2-4	5.2 MW	**41.4 MW**	31.4 MW
Eqv Line 3-4	19.1 MW	18.7 MW	**28.5 MW**

No Solution Example

- Original four bus case, except the limit on line 1-4 has been reduced to 20 MVA

Figure shows PTDFs from 2 to 4



MPTs using reduced limit

2-3: 216.7 MW (1-3 binding)
2-4: 57.2 MW (1-4 binding)
3-4: 48.3 MW (1-4 binding)

No Solution Example

- New limits that need to be satisfied

For the 2-3 direction for the new equivalent line limits we require
1) Lim23 >= 216.7*0.234= 50.7MW
2) Lim24 >= 216.7*0.024= 5.2MW
3) Lim34 >= 216.7*0.088=19.1MW
And one must be an equality!

For the 2-4 direction for the new equivalent line limits we require
1) Lim23 >= 57.2*0.028 = 1.6 MW
2) Lim24 >= 57.2*0.241= 13.8 MW
3) Lim34 >= 57.2*0.109 = 6.2 MW
And one must be an equality!

For the 3-4 direction for the new equivalent line limits we require
1) Lim23 >= 48.3*0.206= 9.9 MW
2) Lim24 >= 48.3*0.217= 10.5 MW
3) Lim34 >= 48.3*0.197=9.5 MW
And one must be an equality!

All cannot be true simultaneously!

No Solution Example: First Approach, Overestimate

- In the first approach we satisfy all of the inequality constraints. But here this means one of the equality constraints is not satisfied

Directions (Modified System Data)

	2-3	2-4	3-4
Eqv Line 2-3	**50.7 MW**	1.6 MW	9.9 MW
Eqv Line 2-4	5.2 MW	**13.8 MW**	10.5 MW
Eqv Line 3-4	**19.1 MW**	6.2 MW	9.5 MW

Allowable flow in direction 3-4 is overestimated since none of the entries in its column are enforced. Overestimated flow is 13.8/10.5 = 131% of the actual value.

General Solution Procedure, cont.

- In the second approach we insure all the equality constraints are satisfied, which insures that the flow in every direction is no more than its original MPT. But because some of the inequality constraints would be in violation, these limits under-estimate the MPT in at least some directions
- Solution is motivated by defining a “limit violation cost” for each matrix entry, which is the sum of violations for all entries in the row (other norms could be used!)

General Solution Procedure: Limit Violation Cost

Directions (Modified System Data)

	2-3	2-4	3-4
Eqv Line 2-3	**50.7 MW**	1.6 MW	9.9 MW
Eqv Line 2-4	5.2 MW	**13.8 MW**	10.5 MW
Eqv Line 3-4	**19.1 MW**	6.2 MW	9.5 MW

Directions: Limit Violation Costs

	2-3	2-4	3-4
Eqv Line 2-3	0	57.4	40.8
Eqv Line 2-4	13.9	0	3.3
Eqv Line 3-4	0	16.2	9.6

Example: For the first row, the 2-3 entry is 0 because it involves no limit violations; the 2-4 entry is 57.4 = (50.7 – 1.6) + (9.9 – 1.6), while 3-4 is 40.8 = (50.7 – 9.9)

Second Approach: Underestimate

- The gist of the second approach is to pick just one entry from each row and just one from each column that minimizes the sum of the limit violation costs.

Directions: Limit Violation Costs

	2-3	**2-4**	**3-4**
Eqv Line 2-3	0	57.4	40.8
Eqv Line 2-4	13.9	0	3.3
Eqv Line 3-4	0	16.2	9.6

This is the Assignment Problem, which can be stated as the optimal assignment of n tasks to n people. The Assignment Problem was solved in the late 1950's using what is known as the Hungarian Algorithm (also known as Munkres' Assignment Algorithm).

Second Approach: Underestimate

Directions: Limit Violation Costs

	2-3	2-4	3-4
Eqv Line 2-3	**0**	57.4	40.8
Eqv Line 2-4	13.9	**0**	3.3
Eqv Line 3-4	0	16.2	**9.6**

For the second approach the new limits would be 50.7 MW for the line between 2-3, 13.8 MW for 2-4 and 9.5 MW for line 3-4.
This is compared with 50.7, 13.8 and 19.1 for the first approach.

Computational Aspects

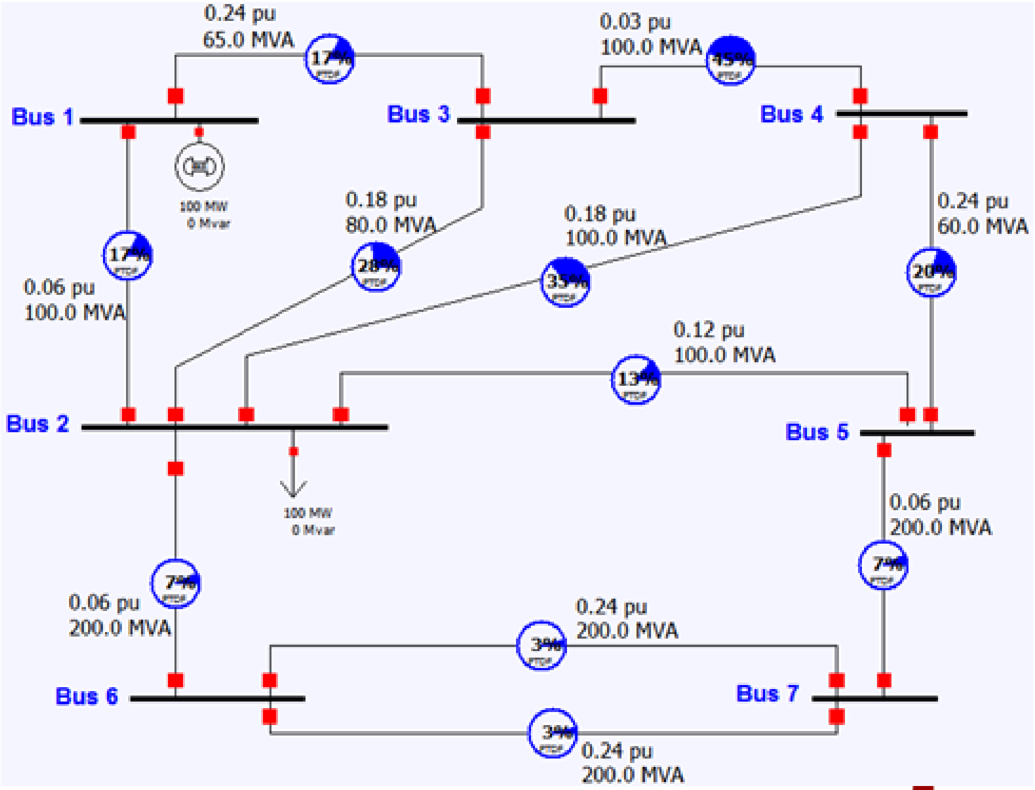
- Assume an n bus system, in which m buses are being reduced. Let F_i be the number of first neighbor buses for bus i (a number that will vary during the simulation). Algorithm will be applied sequentially at m buses. For each step we must
  - Calculate $(F_i)^2/2$ PTDFs
  - With sparse vector methods each PTDF has computational order equivalent to the depth of the factorization path, close to ln(n)
- Overall we expect this to be computationally tractable even for large systems, on the order of m $(F_i)^2$ ln(n)

Computational Aspects, Cont.

- An area of concern is the growth in the first neighbor buses as the system is being equivalenced. However, it has long been recognized that many of these new equivalent lines have quite high impedance and hence they are ignored
  - Limits and hence PTDFs will not need to be calculated for these lines.

Example: Modified 7-bus system

- The limit on line 1-2 has been reduced to 100 MVA from 150 MVA. Reduced by removing buses 3, 5 & 2
- Eliminating bus 3 first



Ex: modified 7-bus system
Exact solution

Determination of equivalent line limits

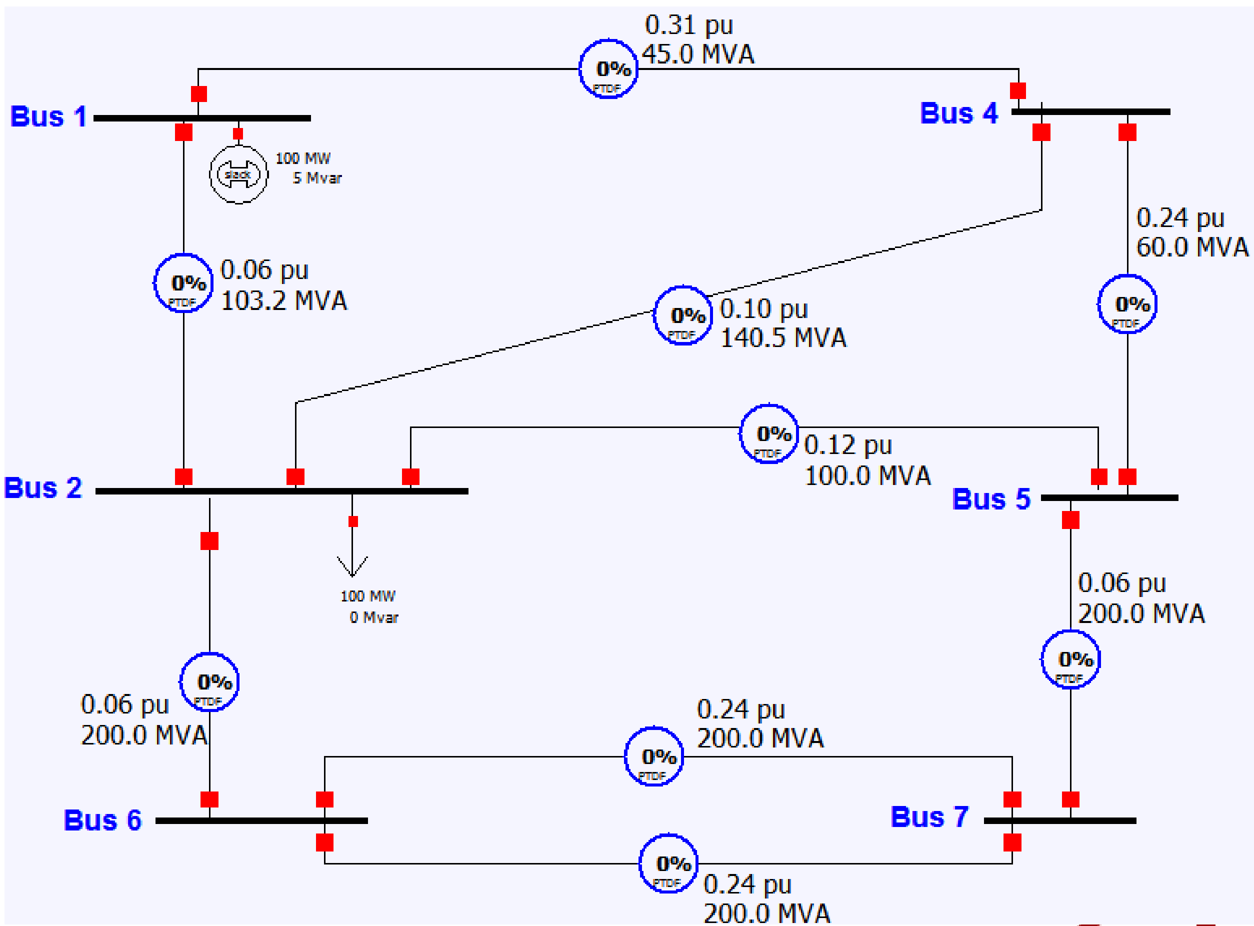
	1-2	1-4	2-4
Eqv Line 1-2	**3.2 MW**	3.2 MW	1.2 MW
Eqv Line 1-4	15.5 MW	**45.0 MW**	38.7 MW
Eqv Line 2-4	5.1 MW	34.2 MW	**61.3 MW**

MPT comparison between 7-bus system and 6-bus

	1-2	1-4	2-4
Reduced 7-bus	118.7 MW (line 1-2 binding)	148.2 MW (line 1-2 binding)	223.7 MW (line 3-4 binding)
Reduced 7-bus	118.7 MW (line 1-2 binding)	148.2 MW (Eq line 1-2 binding)	223.7 MW (Eq line 2-4 binding)
Error rate (%)	0.0	0.0	0.0

Ex: modified 7-bus system

- Equivalent 6-bus system, eliminating bus 5



Ex: modified 7-bus system Exact solution

Determination of equivalent line limits

	2-4	2-7	4-7
Eqv Line 2-4	**17.0 MW**	3.6 MW	10.6 MW
Eqv Line 2-7	12.7 MW	**96.4 MW**	56.5 MW
Eqv Line 4-7	27.6 MW	41.1 MW	**49.4 MW**

MPT comparison between 6-bus system and 5-bus

	2-4	2-7	4-7
Reduced 6-bus	223.7 MW (line 2-4 binding)	250.1 MW (line 2-5 binding)	171.8 MW (line 4-5 binding)
Reduced 5-bus	223.7 MW (line 2-4 binding)	250.1 MW (Eq line 2-7 binding)	171.8 MW (Eq line 4-7 binding)
Error rate (%)	0.0	0.0	0.0

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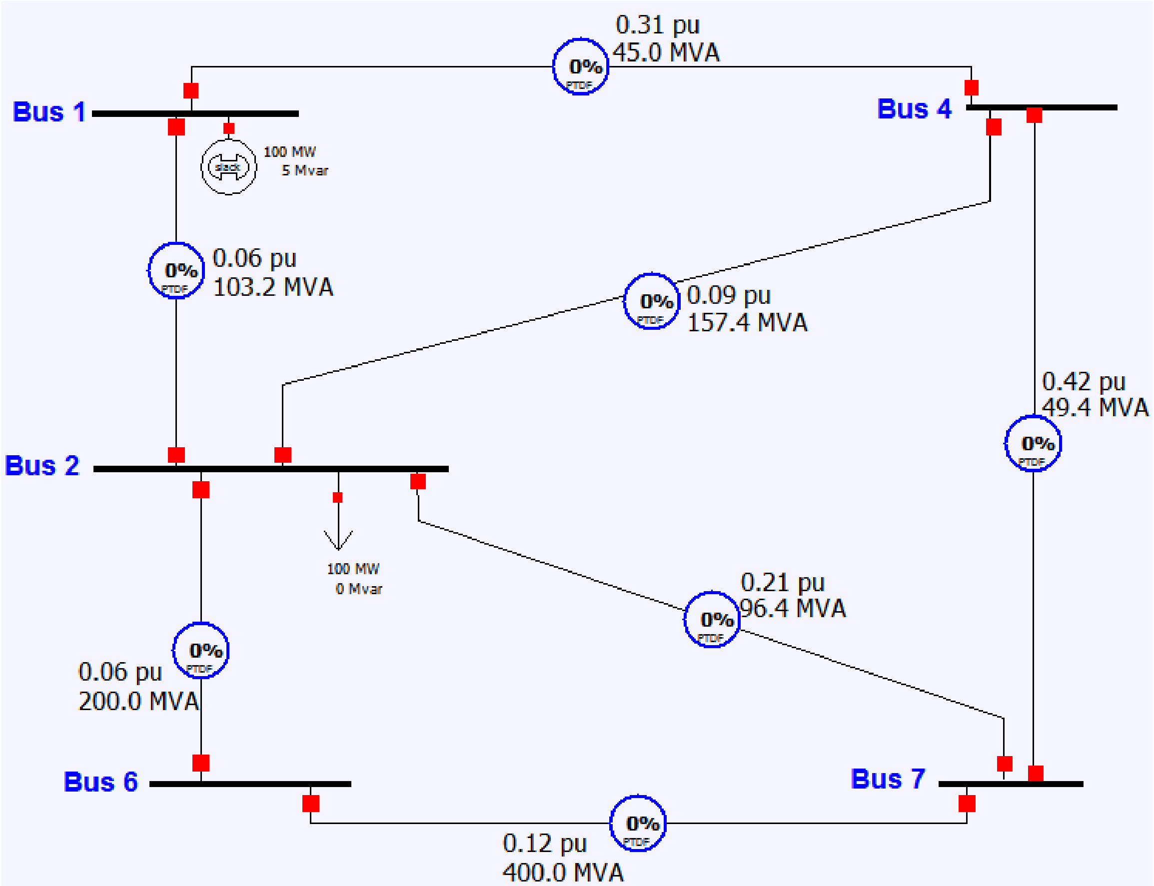
Ex: modified 7-bus system

- Equivalent 5-bus system, eliminating bus 2



Ex: modified 7-bus system
First approach

Determination of equivalent line limits

	1-4	1-6	1-7	4-6	4-7	6-7
Eqv Line 1-4	**53.4 MW**	20.2 MW	23.8 MW	0.1 MW	28.6 MW	9.6 MW
Eqv Line 1-6	37.5 MW	68.2 MW	53.4 MW	**75.0 MW**	30.9 MW	50.4 MW
Eqv Line 1-7	12.4 MW	14.9 MW	26.0 MW	9.6 MW	21.9 MW	**32.6 MW**
Eqv Line 4-6	27.6 MW	24.3 MW	11.2 MW	**92.6 MW**	47.6 MW	41.7 MW
Eqv Line 4-7	6.8 MW	4.0 MW	10.1 MW	18.9 MW	**22.0 MW**	18.3 MW
Eqv Line 6-7	1.6 MW	4.5 MW	10.4 MW	11.4 MW	12.6 MW	**45.5 MW**

Ex: modified 7-bus system First approach to Overestimate

MPT comparison between 5-bus system and 4-bus

	1-4	1-6	1-7	4-6	4-7	6-7
Reduced 5-bus	148.2 MW (line 1-4 binding)	120.2 MW (line 1-2 binding)	123.3 MW (line 1-2 binding)	238.6 MW (line 2-4 binding)	171.8 MW (line 4-7 binding)	375.1 MW (line 2-7 binding)
Reduced 4-bus	148.2 MW (line 1-4 binding)	132.2 MW (Eq line 1-6 binding)	154.9 MW (Eq line 1-7 binding)	238.6 MW (Eq line 1-6 binding)	171.8 MW (line 4-7 binding)	375.1 MW (Eq line 6-7 binding)
Error rate (%)	0.0	10.0	25.6	0.0	0.0	0.0

Ex: modified 7-bus system

Second approach to Underestimate

Determination of equivalent line limits

	1-4	1-6	1-7	4-6	4-7	6-7
Eqv Line 1-4	**53.4 MW**	20.2 MW	23.8 MW	0.1 MW	28.6 MW	9.6 MW
Eqv Line 1-6	37.5 MW	68.2 MW	53.4 MW	**75.0 MW**	30.9 MW	50.4 MW
Eqv Line 1-7	12.4 MW	14.9 MW	26.0 MW	9.6 MW	21.9 MW	**32.6 MW**
Eqv Line 4-6	27.6 MW	24.3 MW	11.2 MW	**92.6 MW**	47.6 MW	41.7 MW
Eqv Line 4-7	6.8 MW	4.0 MW	10.1 MW	18.9 MW	**22.0 MW**	18.3 MW
Eqv Line 6-7	1.6 MW	4.5 MW	10.4 MW	11.4 MW	12.6 MW	**45.5 MW**

Limit violation costs

	1-4	1-6	1-7	4-6	4-7	6-7
Eqv Line 1-4	**0.0**	45.4	34.3	135.0	24.7	87.5
Eqv Line 1-6	96.9	6.8	36.4	**0.0**	129.8	45.5
Eqv Line 1-7	45.9	35.9	6.6	59.5	14.9	**0.0**
Eqv Line 4-6	99.0	112.5	177.8	**0.0**	45.0	56.9
Eqv Line 4-7	41.9	56.2	28.9	3.1	**0.0**	4.3
Eqv Line 6-7	76.5	62.1	38.4	35.3	32.9	**0.0**

Ex: modified 7-bus system Second approach

- Find minimum sum of limit violation costs using Hungarian algorithm

	1-4	1-6	1-7	4-6	4-7	6-7	
Eqv Line 1-4	**0.0**	38.6	27.7	135.0	24.7	87.5	0.0
Eqv Line 1-6	96.9	**0.0**	29.7	0.0	129.8	45.5	0.0
Eqv Line 1-7	45.9	29.1	**0.0**	59.5	14.9	0.0	0.0
Eqv Line 4-6	99.0	105.7	171.2	**0.0**	45.0	56.9	0.0
Eqv Line 4-7	41.9	49.4	22.3	3.1	**0.0**	4.3	0.0
Eqv Line 6-7	76.5	55.3	31.8	35.3	32.9	**0.0**	0.0
	0.0	6.8	6.6	0.0	0.0	0.0	13.4

Ex: modified 7-bus system
Second approach

Determination of equivalent line limits

	1-4	1-6	1-7	4-6	4-7	6-7
Eqv Line 1-4	**53.4 MW**	20.2 MW	23.8 MW	0.1 MW	28.6 MW	9.6 MW
Eqv Line 1-6	37.5 MW	**68.2 MW**	53.4 MW	**75.0 MW**	30.9 MW	50.4 MW
Eqv Line 1-7	12.4 MW	14.9 MW	**26.0 MW**	9.6 MW	21.9 MW	**32.6 MW**
Eqv Line 4-6	27.6 MW	24.3 MW	11.2 MW	**92.6 MW**	47.6 MW	41.7 MW
Eqv Line 4-7	6.8 MW	4.0 MW	10.1 MW	18.9 MW	**22.0 MW**	18.3 MW
Eqv Line 6-7	1.6 MW	4.5 MW	10.4 MW	11.4 MW	12.6 MW	**45.5 MW**

Ex: modified 7-bus system Second approach

MPT comparison between 5-bus system and 4-bus

	1-4	1-6	1-7	4-6	4-7	6-7
Reduced 5-bus	148.2 MW (line 1-4 binding)	120.2 MW (line 1-2 binding)	123.3 MW (line 1-2 binding)	238.6 MW (line 2-4 binding)	171.8 MW (line 4-7 binding)	375.1 MW (line 2-7 binding)
Reduced 4-bus	148.2 MW (line 1-4 binding)	120.2 MW (Eq line 1-6 binding)	123.3 MW (Eq line 1-7 binding)	217.0 MW (Eq line 1-6 binding)	171.8 MW (line 4-7 binding)	298.7 MW (Eq line 1-7 binding)
Error rate (%)	0.0	0.0	0.0	-9.1	0.0	-20.4

Ex: modified 7-bus system

MPT comparison between original and reduced 4-bus with 1st approach

	1-4	1-6	1-7	4-6	4-7	6-7
Original 7-bus	148.2 MW (line 1-2 binding)	120.2 MW (line 1-2 binding)	123.3 MW (line 1-2 binding)	238.6 MW (line 3-4 binding)	171.8 MW (line 4-5 binding)	375.1 MW (line 2-5 binding)
Reduced 4-bus	148.2 MW (line 1-4 binding)	132.2 MW (line 1-6 binding)	154.8 MW (line 1-7 binding)	238.6 MW (line 1-6 binding)	171.8 MW (line 4-7 binding)	375.1 MW (line 6-7 binding)
Error rate (%)	0.0	10.0	25.6	0.0	0.0	0.0

MPT comparison between original and reduced 4-bus with 2nd approach

	1-4	1-6	1-7	4-6	4-7	6-7
Original 7-bus	148.2 MW (line 1-2 binding)	120.2 MW (line 1-2 binding)	123.3 MW (line 1-2 binding)	238.6 MW (line 3-4 binding)	171.8 MW (line 4-5 binding)	375.1 MW (line 2-5 binding)
Reduced 4-bus	148.2 MW (line 1-4 binding)	120.2 MW (line 1-6 binding)	123.3 MW (line 1-7 binding)	217.0 MW (line 1-6 binding)	171.8 MW (line 4-7 binding)	375.1 MW (line 1-7 binding)
Error rate (%)	0.0	0.0	0.0	-9.1	0.0	-20.4

Summary and Future Work

- Developed conditions for determining when exact limits can be determined for equivalent system lines
- In the event exact limits cannot be determined, developed a method for determining limits that bracket the maximum transfer capacity for the equivalent lines
- Algorithm needs to be demonstrated on larger systems. This will allow us to gain additional engineering insight into the problem and see the width of the limit ranges.
- Integrate in impact of bus loading

Questions?