Commercialization of the Super Optimal Power Flow (SuperOPF) Framework (Phase II)

Hsiao-Dong Chiang, Bin Wang, Patrick Causgrove, Ray Zimmerman

Presenter: Hsiao-Dong Chiang

Super-OPF (for operation)

Super-OPF Method

Input Data → Stage I: OPF Constraint Analyzer → Stage II: Simple OPF w/o Thermal Limits → Stage III: Homotopy OPF w/ Active Thermal Limits → Stage IV: Sensitivity Analyzer for Discretization → OPF Result

$$
\begin{aligned}
\min \quad & f(V,\theta,t,\phi,b,P^G,Q^G) \\
s.t. \quad & P_i(V,\theta,t,\phi,b) + P_i^L - P_i^G = 0, \; i = 1,\cdots,n_B \\
& Q_i(V,\theta,t,\phi,b) + Q_i^L - Q_i^G = 0, \; i = 1,\cdots,n_B \\
& \underline{V}_i \le V_i \le \overline{V}_i, \; i = 1,\cdots,n_B \\
& \underline{t}_i \le t_i \le \overline{t}_i, \; i = 1,\cdots,n_T \\
& \underline{\phi}_i \le \phi_i \le \overline{\phi}_i, \; i = 1,\cdots,n_P \\
& \underline{b}_i \le b_i \le \overline{b}_i, \; i = 1,\cdots,n_S \\
& \underline{P}_j^G \le P_j^G \le \overline{P}_j^G, \; j = 1,\cdots,n_G \\
& \underline{Q}_j^G \le Q_j^G \le \overline{Q}_j^G, \; j = 1,\cdots,n_G
\end{aligned}
$$

OPF without thermal constraints

$$
\begin{aligned}
\min \quad & f(V,\theta,t,\phi,b,P^G,Q^G;\lambda) \\
s.t. \quad & P_i(V,\theta,t,\phi,b) + P_i^L - P_i^G = 0, \; i = 1,\cdots,n_B \\
& Q_i(V,\theta,t,\phi,b) + Q_i^L - Q_i^G = 0, \; i = 1,\cdots,n_B \\
& S_{ij}(V,\theta,t,\phi,b) \le \overline{S}_{ij} + \lambda\tilde{S}_{ij}, \; (i,j) \in \mathcal{L}_\alpha \\
& S_{ji}(V,\theta,t,\phi,b) \le \overline{S}_{ij} + \lambda\tilde{S}_{ij}, \; (i,j) \in \mathcal{L}_\alpha \\
& \underline{V}_i \le V_i \le \overline{V}_i, \; i = 1,\cdots,n_B \\
& \underline{t}_i \le t_i \le \overline{t}_i, \; i = 1,\cdots,n_T \\
& \underline{\phi}_i \le \phi_i \le \overline{\phi}_i, \; i = 1,\cdots,n_P \\
& \underline{b}_i \le b_i \le \overline{b}_i, \; i = 1,\cdots,n_S \\
& \underline{P}_j^G \le P_j^G \le \overline{P}_j^G, \; j = 1,\cdots,n_G \\
& \underline{Q}_j^G \le Q_j^G \le \overline{Q}_j^G, \; j = 1,\cdots,n_G
\end{aligned}
$$

OPF with active thermal constraints

$$
\begin{aligned}
S_y^f &= \frac{\partial f}{\partial y} - \left(\frac{\partial g}{\partial y}\right)^T \left[\left(\frac{\partial g}{\partial x}\right)^T\right]^{-1} \frac{\partial f}{\partial x} \\
S_y^h &= \frac{\partial h}{\partial y} - \frac{\partial h}{\partial x}\left(\frac{\partial g}{\partial x}\right)^{-1} \frac{\partial g}{\partial y}
\end{aligned}
$$

$$
\begin{aligned}
\Delta f_i^+ &= S_{d_i}^f (d_i^{j_i+1} - d_i^{j_i}) \\
\Delta f_i^- &= S_{d_i}^f (d_i^{j_i-1} - d_i^{j_i}) \\
\Delta h_{k_i}^+ &= S_{d_i}^{h_k} (d_i^{j_i+1} - d_i^{j_i}), \; \forall k = 1,\cdots,n_h \\
\Delta f_{k_i}^- &= S_{d_i}^{h_k} (d_i^{j_i+1} - d_i^{j_i}), \; \forall k = 1,\cdots,n_h
\end{aligned}
$$

$$
\begin{aligned}
\eta_i^+ &= w_f \Delta f_i^+ + \textstyle\sum_{k=1}^{n_h} w_h \max[0, h_k(\hat{x},\hat{y}) + \Delta h_{k_i}^+] \\
\eta_i^- &= w_f \Delta f_i^- + \textstyle\sum_{k=1}^{n_h} w_h \max[0, h_k(\hat{x},\hat{y}) + \Delta h_{k_i}^-]
\end{aligned}
$$

$$
\begin{aligned}
& y_i^{j_i} \leftarrow y_i^{j_i+1} \text{ if: } \eta_i^+ \le \eta_i^- \text{ and } \eta_i^+ \le \eta_{th} \text{ or} \\
& y_i^{j_i} \leftarrow y_i^{j_i-1} \text{ if: } \eta_i^- < \eta_i^+ \text{ and } \eta_i^- \le \eta_{th}.
\end{aligned}
$$

Sensitivity based adjustment

Results: Efficiency and Robustness (Analytical Jacobian matrices)

Effects of constraint analysis

Base case
Without constraint analysis
• Converged in 217 iterations • CPU time: 177 seconds • OPF loss: 3251.284MW
With constraint analysis
• Converged in 191 iterations • CPU time: 143 seconds • OPF loss: 3251.353MW

Robustness of our method

Loading Condition	One-Staged Scheme	Multi-Staged Scheme
1	Succeeded	Succeeded
2	Succeeded	Succeeded
3	Succeeded	Succeeded
4	Succeeded	Succeeded
5	Failed	Succeeded
6	Failed	Succeeded
7	Failed	Succeeded
8	Failed	Succeeded
9	Failed	Succeeded
10	Failed	Succeeded

Supported Objective Functions

- System Real Power Loss

$$f(x) = \sum_{(i,j)\in L} g_{ij}\left(V_i^2 - 2V_iV_j\cos\left(\theta_i - \theta_j\right) + V_j^2\right)$$

- System Reactive Power Loss

$$f(x) = -\sum_{(i,j)\in L} b_{ij}\left(V_i^2 - 2V_iV_j\cos\left(\theta_i - \theta_j\right) + V_j^2\right)$$

- System Real Power Generation

$$f(x) = \sum_{i=1}^{n_G} P_{Gi}$$

- System Reactive Power Generation

$$f(x) = \sum_{i=1}^{n_G} Q_{Gi}$$

- System Generation Cost

$$f(x) = \sum_{i=1}^{n_G} C_i(P_{Gi})$$

where $C_i(P_{Gi})$ is the cost function for *i*-th generator, which can be a polynomial or piece-wise linear function.

Supported Constraint Functions

- Power flow equality constraints

$$P_{Gi} - P_{Di} - V_i \sum_{j \in \aleph_i} V_j \left(G_{ij} cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right) = 0$$

$$i = 1, \cdots, n_B$$

$$Q_{Gi} - Q_{Di} - V_i \sum_{j \in \aleph_i} V_j \left(G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \right) = 0$$

- Thermal-limit constraints

$$|P_{ij}| = \left| V_i V_j \left(G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right) - V_i^2 G_{ij} \right| \leq \bar{P}_{ij}, (i, j) \in L$$

- Interface-flow limit constraints

$\underline{F_k} \leq F_k = \sum_{i,j} d_{ij} P_{ij} \leq \overline{F_k}$, $(i, j) \in \beth_k$, where $d_{ij} = \pm 1$ is the flow direction.

- All variables' lower and upper bounds

$$\underline{x_i} \leq x_i \leq \overline{x_i}, i = 1, \cdots, n_x.$$

Supported Optimization Variables

- Voltage magnitudes and phase angles
- Real and reactive power generations
- Transformer tap ratios (continuous or discrete)
- Phase shifters (continuous or discrete)
- Switchable shunts (continuous or discrete)

Adaptive Homotopy-guided Primal-Dual Interior Point OPF Solver

Homotopy-based Methodology (continuation method + adaptive step-size)

IPM methods consists of three basic modules:

- Newton method solving nonlinear equations
- Lagrange's method for optimization with equality constraints.
- barrier method for optimization with inequalities.

Super-OPF Dimensions

PJM 13183-Bus System

Stage I: OPF Constraint Analyzer → Stage II: Simple OPF w/o Thermal Limits → Stage III: Homotopy OPF w/ Active Thermal Limits → Stage IV: Sensitivity Analyzer for Discretization

Input Data

OPF Result

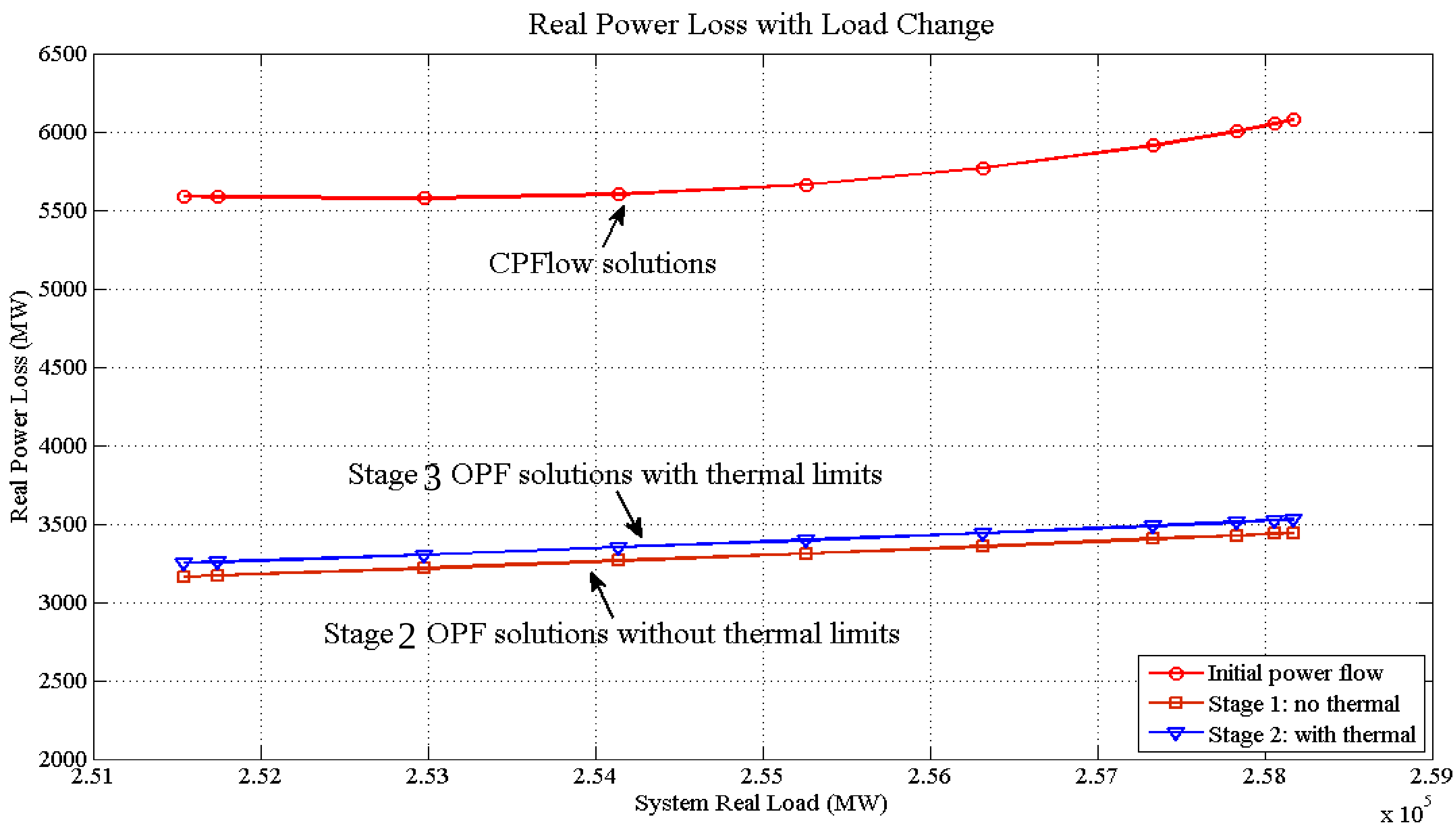
PJM System:
- Buses: 13183
- Loads: 9691
- Generators: 2304
- Branches: 18168
- Transformers: 1410
- Switched shunts: 1404

OPF Dimensions:
- Dimension of x: 31134
- Nonlinear equality constraints: 26366
- Nonlinear inequality constraints: 0
- Total equality constraints: 26367
- Total inequality constraints: 35902

OPF Dimensions:
- Dimension of x: 31134
- Nonlinear equality constraints: 26366
- Nonlinear inequality constraints: varying (<100)
- Total equality constraints: 26367
- Total inequality constraints: >35902 (varying)

PJM System:
- Continuous variables: 28320
- Discrete variables: 2814

Results: Real Power Loss



Super-OPF Dimensions

CAISO 6534-Bus System

Input Data

Stage I: OPF Constraint Analyzer

Stage II: Simple OPF w/o Thermal Limits

Stage III: Homotopy OPF w/ Active Thermal Limits

Stage IV: Sensitivity Analyzer for Discretization

OPF Result

CAISO System:
- Buses: 6534
- Loads: 2901
- Generators: 1903
- Branches: 8295
- Transformers: 294
- Switched shunts: 520

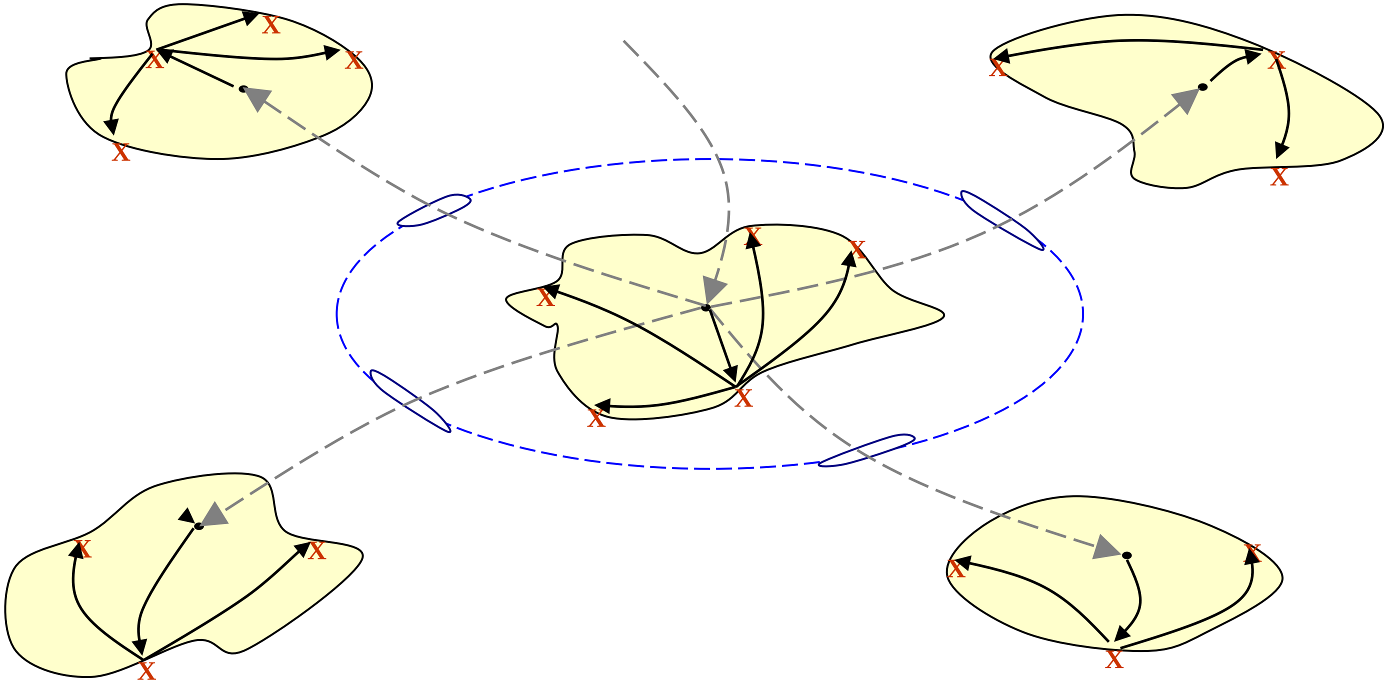
OPF Dimensions:
- Dimension of x: 12868
- Nonlinear equality constraints: 12084
- Nonlinear inequality constraints: 0
- Total equality constraints: 12085
- Total inequality constraints: 13652

OPF Dimensions:
- Dimension of x: 31134
- Nonlinear equality constraints: 12084
- Nonlinear inequality constraints: varying (<100)
- Total equality constraints: 12085
- Total inequality constraints: >13652(varying)

CAISO System:
- Continuous variables: 12054
- Discrete variables: 814

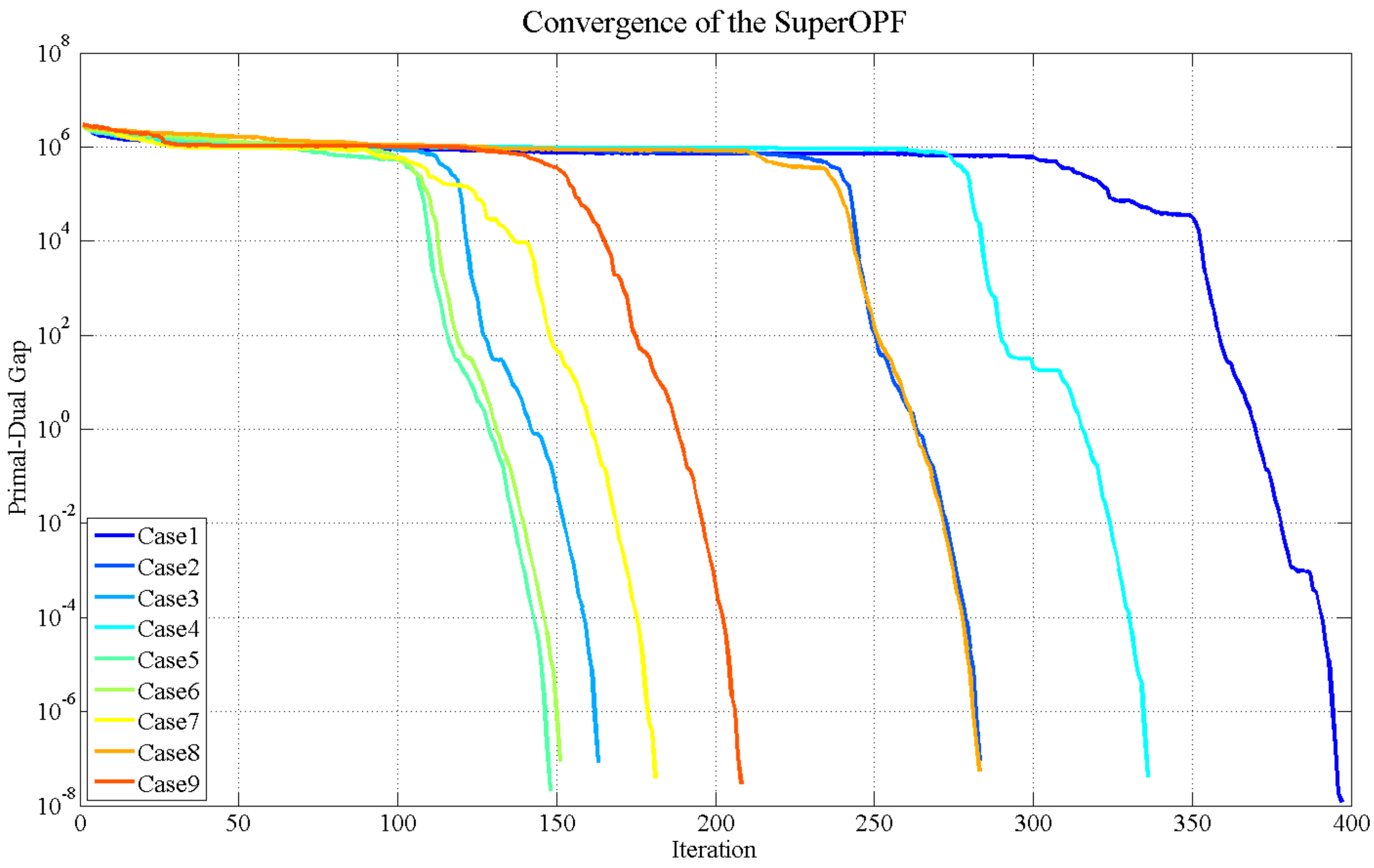
Multiple Optimal Solutions

(1) There are multiple feasible components
(2) Multiple local optimal solutions in each feasible component



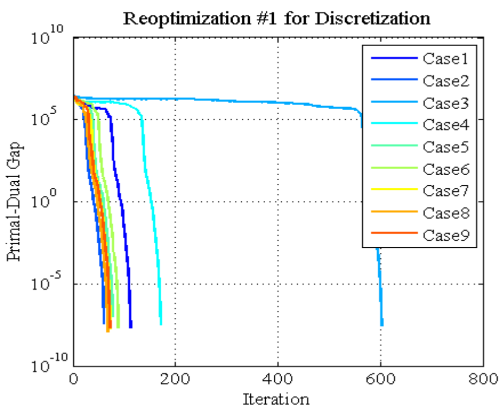
Convergence of Super-OPF on PJM System

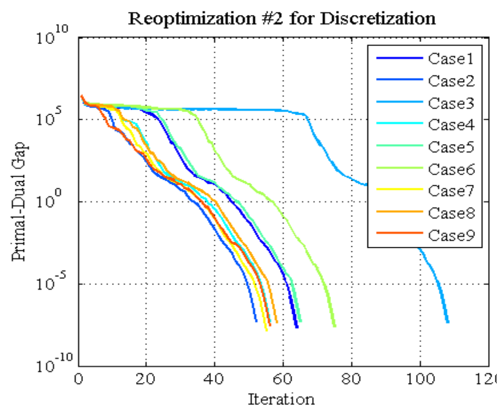
System loss minimization (continuous OPF)

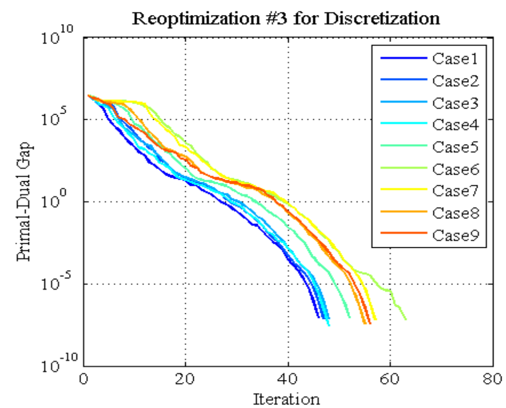


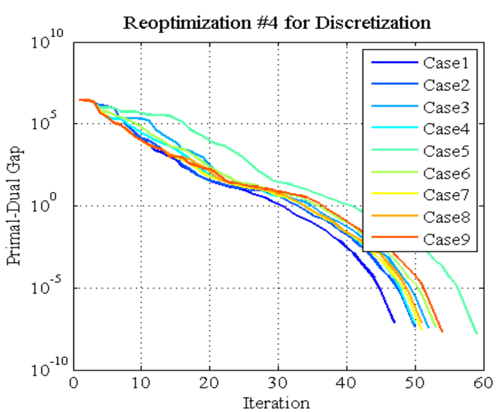
Convergence of Super-OPF on PJM System

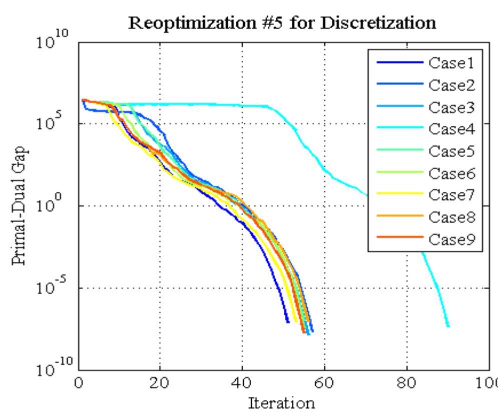
System loss minimization (re-optimization after discretization)





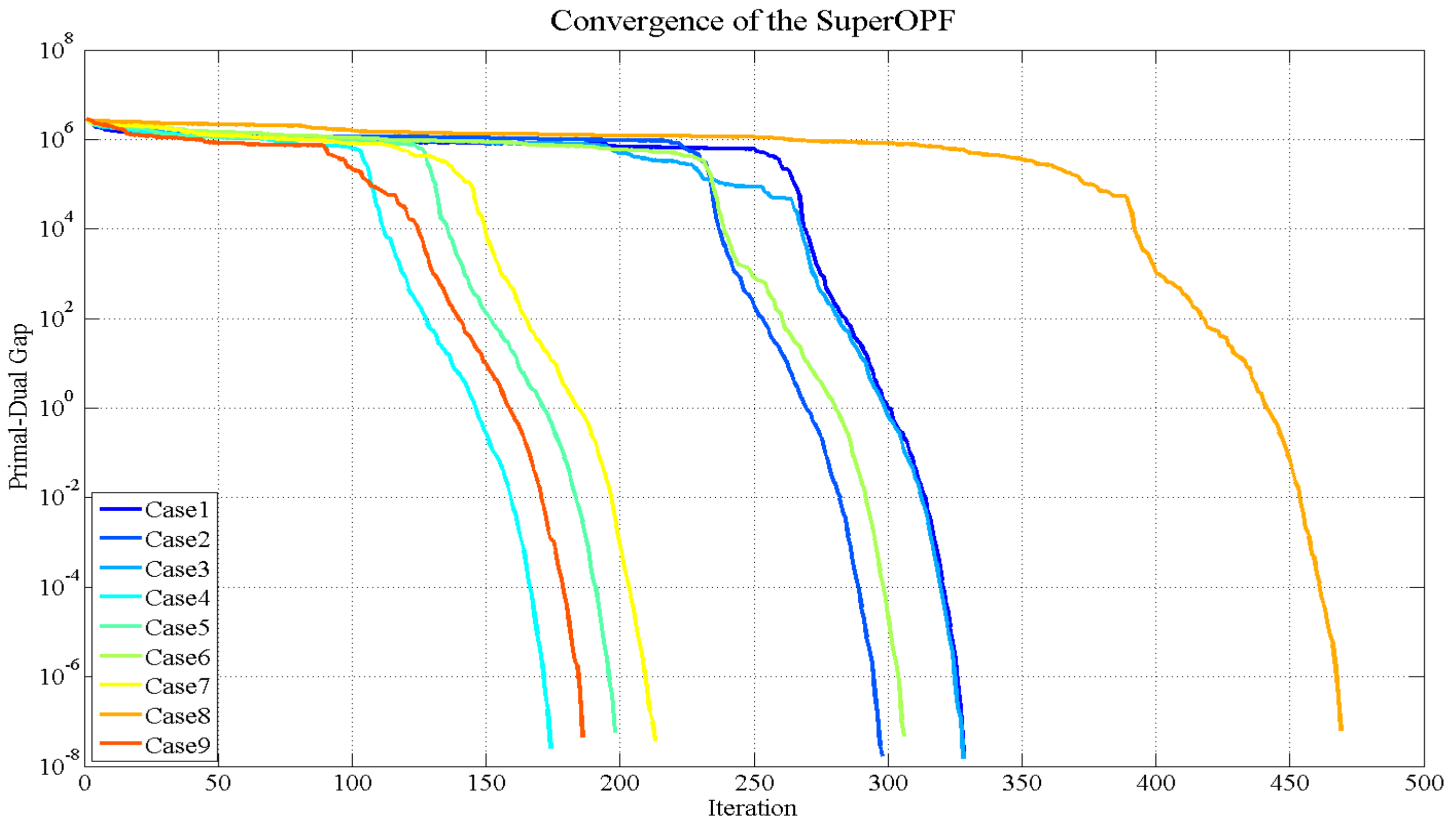






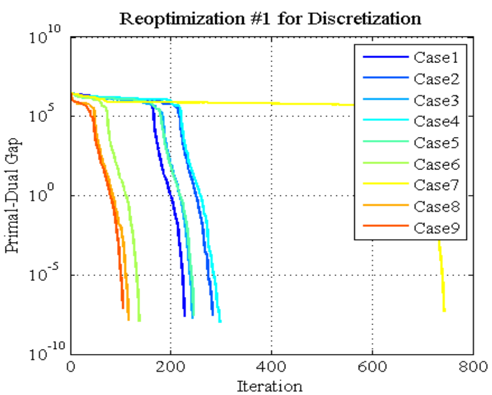
Convergence of Super-OPF on PJM System

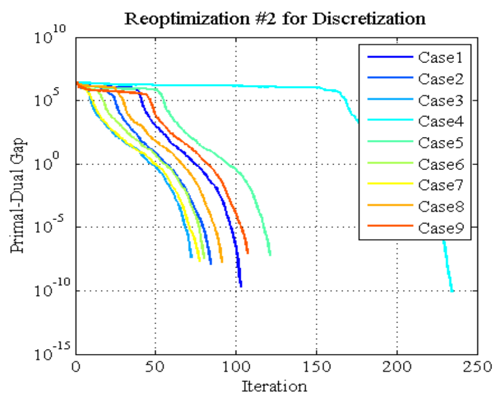
Generation cost minimization (continuous OPF)

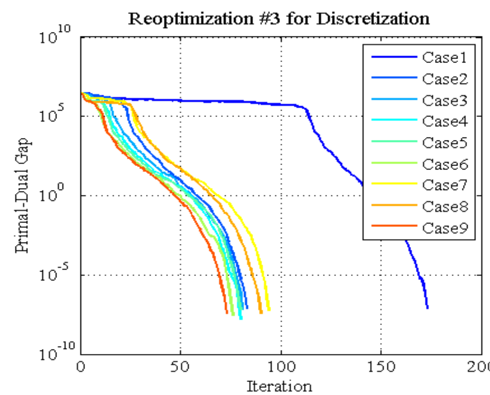


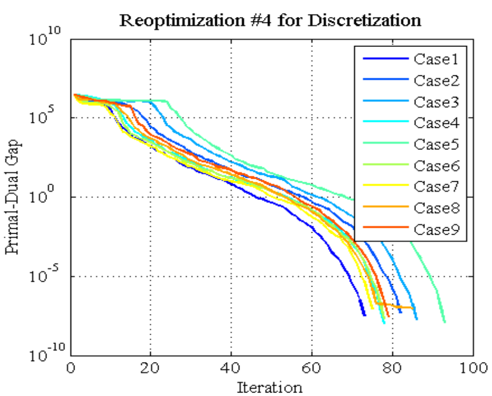
Convergence of Super-OPF on PJM System

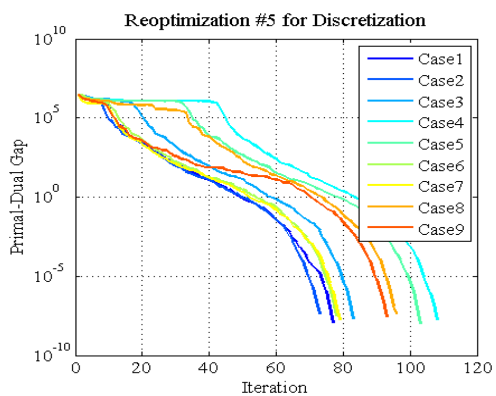
Generation cost minimization (reoptimization after discretization)











Issues with Current Generation of Optimal Power Flow

- Optimal power flow solution is NOT a global optimal solution
- Solvers only compute one (local) optimal solution while there are multiple local optimal solutions
- Each OPF solution corresponds to one location marginal pricing (which OPF solution is the right one ?)
- Current solvers are still not sufficiently robust

Challenges

$$\min C(x)$$

Subject to: $h(x)=0$

$g(x)\le 0$

However, security-constrained OPF can not be expressed as the above analytical form:

i. Power balance equations: $h(x)=0$

ii. Voltage limit constraints: $\underline{x}\le x\le \overline{x}$

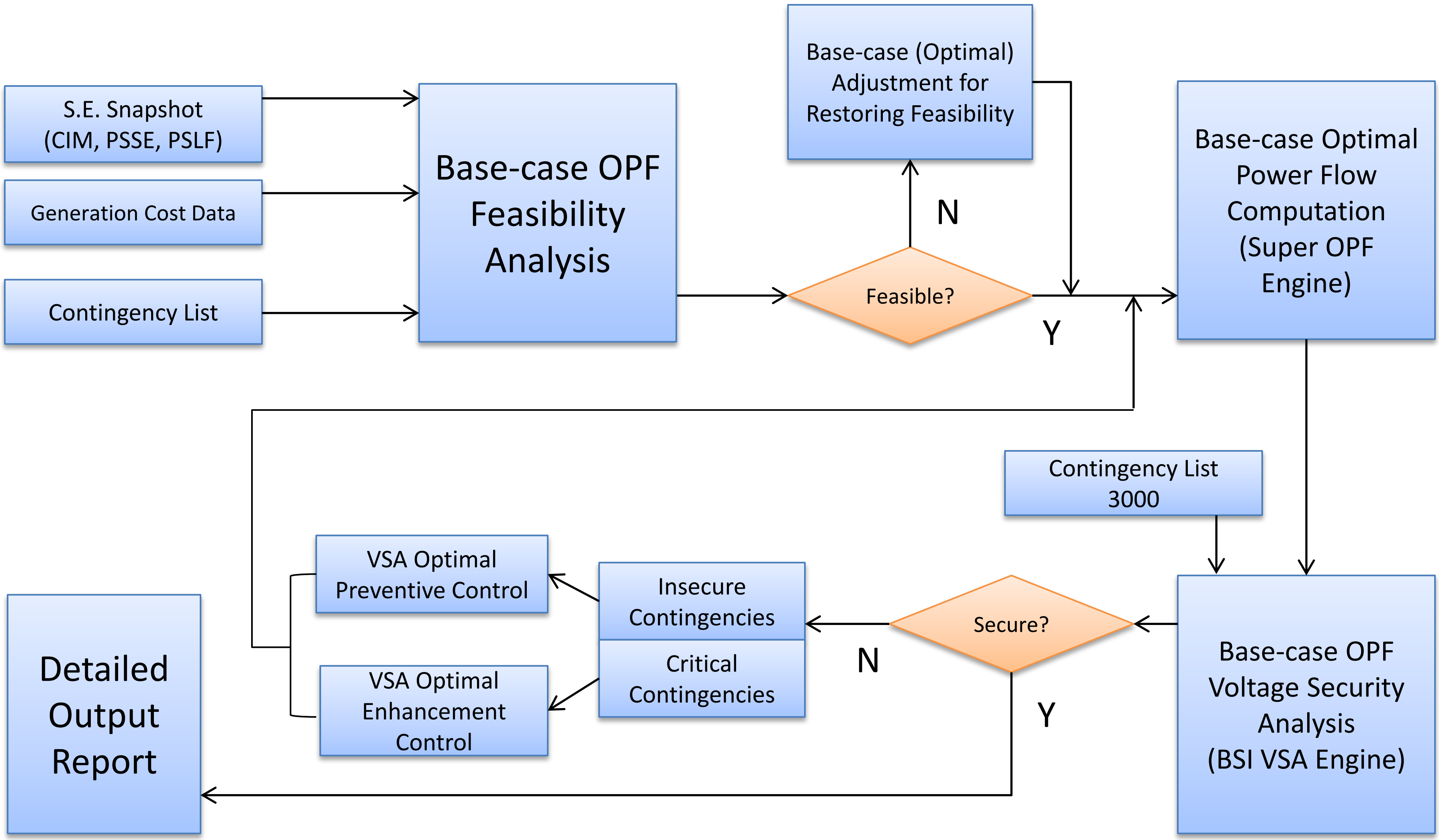
iii. Thermal limit constraints: $g(x)\le 0$

iv. Transient-stability constraints: ???

v. Voltage stability constraints: ???

Super-OPF-VS (Voltage Stability)

1. Input 2. Feasibility Check 3. Ensuring Feasibility 4. Computation Engine



7. Output Report 6. VSA Enhancement 5. VSA Check

Super-OPF Contingency Analysis

CAISO 6534-Bus System

Initial Power Flow → Super-OPF Solution → BSI VSA Preventive Control

CAISO System 1062 N-1 Contingencies

0	0	3010.028	-20.088	0	3010.028
1	21	-0.305	-0.305	3	0.000
2	34	-0.218	-0.218	6	0.000
3	691	-0.100	-0.100	17	0.000
4	8	-0.066	-0.066	1	0.000
5	35	-0.060	-0.060	5	0.000
6	24	-0.056	-0.056	2	0.000
7	36	-0.015	-0.015	4	0.000
8	639	1353.153	-9.031	12	2347.311
9	281	1884.894	-12.579	10	1397.252
10	282	1884.894	-12.579	11	1397.252
11	491	2715.639	-18.123	14	2481.276
12	492	2715.639	-18.123	13	2481.276
13	561	2716.463	-18.129	15	2485.825
14	521	2730.695	-18.224	16	2503.604
15	572	2789.834	-18.619	41	2789.834
16	864	2791.814	-18.632	42	2791.814
17	863	2791.814	-18.632	43	2791.814
18	628	2805.778	-18.725	44	2805.778
19	690	2807.721	-18.738	18	2619.026
20	684	2822.604	-18.837	47	2822.604

Load margin: 3010MW
Objective (loss): 2793.6MW
7 insecure contingencies

0	0	4840.000	-32.301	0	4840.000
1	21	-0.285	-0.285	3	0.000
2	34	-0.253	-0.253	5	0.000
3	35	-0.071	-0.071	4	0.000
4	8	-0.050	-0.050	1	0.000
5	24	-0.042	-0.042	2	0.000
6	561	4373.778	-29.189	16	4304.356
7	491	4386.839	-29.276	15	4284.000
8	492	4386.839	-29.276	14	4284.000
9	691	4404.315	-29.393	20	4404.315
10	521	4407.446	-29.414	17	4316.588
11	690	4437.187	-29.612	23	4437.187
12	281	4494.606	-29.996	18	4381.087
13	282	4494.606	-29.996	19	4381.087
14	639	4508.597	-30.089	21	4413.366
15	428	4544.092	-30.326	25	4483.436
16	432	4544.092	-30.326	26	4483.436
17	431	4544.092	-30.326	27	4483.436
18	430	4544.092	-30.326	28	4483.436
19	429	4544.092	-30.326	29	4483.436
20	501	4555.107	-30.399	30	4511.405

Load margin: 4840MW
Objective (loss): 1642.8MW
5 insecure contingencies

0	0	4840.161	-32.302	0	4840.161
1	561	4376.021	-29.204	23	4303.308
2	492	4389.094	-29.291	22	4282.792
3	491	4389.094	-29.291	21	4282.792
4	521	4409.681	-29.429	24	4315.462
5	690	4436.558	-29.608	25	4436.558
6	691	4509.283	-30.094	6	2244.418
7	431	4546.083	-30.339	26	4482.817
8	430	4546.083	-30.339	27	4482.817
9	429	4546.083	-30.339	28	4482.817
10	428	4546.083	-30.339	29	4482.817
11	432	4546.083	-30.339	30	4482.817
12	501	4556.994	-30.412	31	4510.822
13	509	4564.158	-30.460	32	4514.185
14	281	4571.971	-30.512	14	2416.908
15	282	4571.971	-30.512	15	2416.908
16	506	4577.873	-30.551	33	4534.745
17	662	4580.186	-30.567	38	4550.269
18	663	4580.186	-30.567	39	4550.269
19	456	4585.097	-30.599	34	4536.546
20	455	4585.097	-30.599	35	4536.546

Load margin: 4840MW
Objective (loss): 1674.6MW
No insecure contingency

Super-OPF Contingency Analysis

PJM 13183-Bus System

PJM System 6894 N-1 Contingencies

0	0	4900.904	1143.944	0	4900.904
1	2708	-0.200	-0.200	15	0.000
2	3628	-0.134	-0.134	4	0.000
3	1742	-0.126	-0.126	7	0.000
4	3528	-0.120	-0.120	14	0.000
5	1757	-0.119	-0.119	10	0.000
6	6096	-0.100	-0.100	8	0.000
7	5756	-0.100	-0.100	2	0.000
8	5162	-0.094	-0.094	13	0.000
9	2228	-0.064	-0.064	3	0.000
10	2025	-0.049	-0.049	9	0.000
11	2453	-0.038	-0.038	1	0.000
12	3619	-0.038	-0.038	5	0.000
13	4877	-0.034	-0.034	12	0.000
14	1999	-0.030	-0.030	6	0.000
15	3599	-0.021	-0.021	19	0.000
16	1543	-0.015	-0.015	11	0.000
17	3917	4385.884	1023.731	24	790.538
18	124	4870.774	1136.912	34	7034.131
19	2238	4895.449	1142.671	27	4047.664
20	2806	4895.868	1142.769	29	4657.256

Load margin: 4901MW
Objective (loss): 5589.3MW
16 insecure contingencies

0	0	4305.851	1005.050	0	4305.851
1	3628	-0.098	-0.098	3	0.000
2	1742	-0.097	-0.097	6	0.000
3	1757	-0.079	-0.079	8	0.000
4	3528	-0.078	-0.078	9	0.000
5	1999	-0.061	-0.061	5	0.000
6	3619	-0.030	-0.030	4	0.000
7	2025	-0.030	-0.030	7	0.000
8	2228	-0.025	-0.025	2	0.000
9	3599	-0.011	-0.011	1	0.000
10	238	4150.848	968.870	23	6278.033
11	380	4203.134	981.074	22	6117.726
12	527	4265.406	995.609	28	6835.490
13	718	4267.237	996.037	27	6796.001
14	124	4294.967	1002.509	25	6677.155
15	1630	4299.961	1003.675	13	498.673
16	5142	4301.633	1004.065	11	232.497
17	5143	4301.634	1004.065	10	232.497
18	648	4302.521	1004.272	29	6846.455
19	1214	4303.292	1004.453	16	2873.253
20	2015	4303.294	1004.453	14	1537.550

Load margin: 4306MW
Objective (loss): 3293.0MW
9 insecure contingencies

0	0	4298.985	1003.447	0	4298.985
1	1742	-0.049	-0.049	1	0.000
2	5952	4117.302	961.040	11	3280.010
3	238	4138.999	966.104	16	6337.589
4	380	4191.245	978.299	15	6162.814
5	6563	4201.527	980.699	25	6925.380
6	527	4255.147	993.215	21	6852.554
7	718	4256.856	993.614	19	6810.427
8	667	4274.275	997.680	24	6906.906
9	317	4274.946	997.836	26	6958.633
10	124	4287.687	1000.810	18	6719.880
11	5142	4293.659	1002.204	4	232.497
12	5143	4293.663	1002.205	3	232.497
13	2015	4293.894	1002.259	6	1535.457
14	3628	4295.075	1002.534	5	621.551
15	1214	4295.432	1002.618	8	2871.496
16	568	4299.906	1003.662	28	6970.977
17	647	4301.207	1003.966	17	6697.015
18	648	4304.292	1004.686	22	6893.834
19	639	4304.703	1004.782	30	6990.219
20	6468	4307.001	1005.318	2	232.497

Load margin: 4299MW
Objective (loss): 3293.8MW
1 insecure contingencies

Major technical Accomplishments

1. A four-stage, multi-level, adaptive homotopy-enhanced Interior Point OPF solver has been developed and it is composed of four stages for robustness and efficiency, This four-stage Solver has been evaluated on practical power systems.

2. A patent application of the four-stage, multi-level adaptive, homotopy-enhanced Interior Point will be filed.

3. Develop a novel solution method to handle the discrete control variables in SuperOPF.

4. Implement the novel solution method in the commercial-grade core SuperOPF-discrete .

Major technical Accomplishments

5. Evaluate the commercial-grade core SuperOPF-discrete on the PJM system model, a 13,000-bus EMS models in PSS/E data format. The simulation results are encouraging.

6. Develop a commercial-grade core SuperOPF-VS (voltage stability) software equipped with a commercial voltage stability solver capable of handling the voltage stability constraint of a large set of contingencies, such as 2500 contingencies.

7. Evaluate the commercial-grade core SuperOPF-VS on the PJM system model, a 13,000-bus EMS models in PSS/E data format. The simulation results are encouraging.

Major technical to be accomplished

8. Design the paths of developing a commercial-grade implementation of all the key functions of SuperOPF in the context of co-optimization framework that correctly accounts for contingencies, ancillary services, static and dynamic constraints in determining both dispatch and price.

9. Develop a commercial-grade core SuperOPF-Contingency software equipped with an optimal load shedding solver capable of handling up 10,000 loads.

10. Develop a comprehensive Automatic Voltage Control (AVC) formulation using the SuperOPF Framework.

11. Develop an effective solution methodology for solving the AVC formulation.